2024 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM)
July 15-19, 2024. Boston, MA, USA

Differentially Flat Reference Models for Bipedal Walking Robots

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***Abstract*— Bipedal Walking Robots have been controlled using simple reference models such as those based on inverted pendulums or spring-loaded inverted pendulums. These models are computationally tractable but significantly simplify the actual bipedal robot dynamics. The focus of this work is to develop a motion planning and control method based on intermediate complexity dynamic models that are closer to the full dynamic model in comparison with simpler inverted pendulum models and at the same time are computationally more tractable than full dynamic models. The proposed models exhibit the property of differential flatness that makes motion planning and control significantly tractable.**

I. INTRODUCTION

Motion planning and control of under-actuated bipedal walking robots involve solving nonlinear differential equations along with discrete events (impacts). The complexity further increases due to motion constraints such as positive ground normal reaction for supporting leg, ground clearance for swinging leg, foot placement constraints etc. One way this complexity has been handled in literature is by employing simple reference models such as those based on Inverted Pendulums or Spring-Loaded Inverted Pendulums (SLIP) [1] [2] [3]. These models are computationally tractable but significantly simplify the dynamics of the bipedal walking robots. Approaches based on full dynamic models, such those based on hybrid zero dynamics [4] work quite well, however, these methods rely on numerical computations to a greater degree. This work focuses on developing a reference model with intermediate complexity for motion planning and control of bipedal walking robots. The goal is for these intermediate complexity reference models to be closer to the actual dynamics of bipedal walking robots in comparison to SLIP or other inverted pendulum-based models while being computationally more tractable compared to full dynamic models. In the proposed model, robot links are assumed to have distributed mass and inertia with some special conditions on their center of mass similar to those in [5] for models without a torso. These models exhibit the property of Differential Flatness, which allows the analytical formulation of a parameterized, dynamically feasible family of trajectories in terms of outputs and their derivatives. Once a family of such trajectories are available, numerical optimization routines can be used to pick a trajectory from this family of trajectories that optimizes specific criteria such as energy consumption and maximum torque requirements while satisfying motion constraints such as positive ground reaction, minimum heel clearance, no-slip condition etc. The novelty of the present work is that differential flatness has been explored for a more complicated dynamics model with a torso, which was absent in [5].

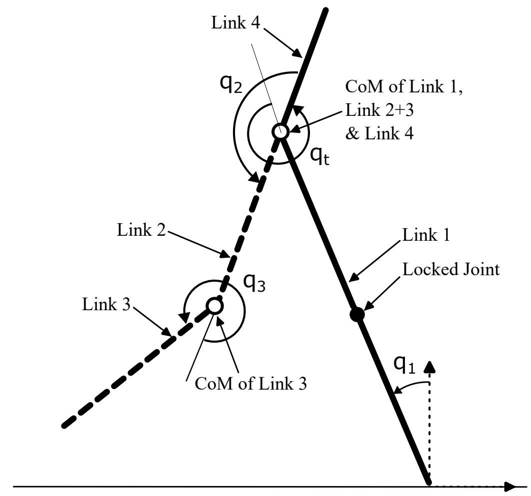

Fig. 1. Schematic of the five-link robot considered with conventions. The stance leg is shown as a dotted leg.

II. FIVE-LINK BIPED

The flatness-based design methodology is implemented on a five-link biped under-actuated planar biped 1. The bipedal walking robot consists of a torso with its center of mass at the hip joint. Two identical legs, namely, stance leg and swing leg, with their center of mass at the hip joint. Each leg has a knee joint that connects the two leg links. The lower link has its center of mass at the knee joint, as shown in Fig. 1. The knee and hip joints are actuated, whereas the ankle joints are unactuated. Both the knee joints have locking mechanisms that lock the joints. At any given instant, only one leg (stance leg) is in contact with the ground while the other leg (swing leg) swings in the air. The knee joint of the stance leg is locked during the single support phase. Legs interchange their roles instantaneously when the swing leg hits the ground. A complete periodic gait consists of two continuous swing phases (4-link phase and 3-link phase) separated by two discrete events (knee impact and heel impact), as shown in Fig. 2. The 4-link phase is where the knee joint of the swing leg is unlocked, and the knee joint of the stance leg is locked, whereas in a 3-link phase, both

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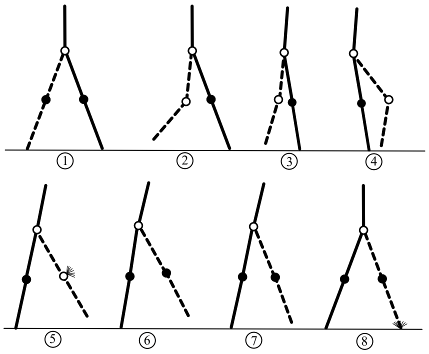

Fig. 2. Gait of five-link biped numbered 1-8 in order. Biped's gait starts with a double stance(1), followed by a 4-link swing phase (2-4), an instantaneous knee impact(5), a 4-link swing phase (6-7) and an instantaneous heel impact (8). The swing leg and support leg interchange their roles at the ground impact (8-1).

the knee joints are locked. The dynamics of these continuous swing phases and discrete impact events are derived using standard approaches (such as the energy method) available in the literature.

III. MOTION PLANNING BASED ON DIFFERENTIAL FLATNESS

Using the output variables presented in (1) and (2) it can be shown that the dynamics in swing phase is differentially flat in the four-link and three-link phases respectively. The configuration variables used here are depicted in Fig. 1 and ϵ_{ij} are the non-dimensional inertial parameters. Differential flatness is ensured by checking that these outputs are such that their relative degree equals the number of states in the four-link as well as three-link phases. These outputs are called flat outputs. A diffeomorphism between the original state space and flat output space can also be derived (due to the page limit for this extended abstract these equations/details could not be included here but will be communicated through a full publication).

$$\begin{aligned} y_{11} &= q_1 + \epsilon_{11}q_2 + \epsilon_{12}q_3 + \epsilon_{13}q_t \\ y_{12} &= \epsilon_{11}q_1 + \epsilon_{11}q_2 + \epsilon_{12}q_3 + \epsilon_{11}q_t \\ y_{13} &= \epsilon_{12}q_1 + \epsilon_{12}q_2 + \epsilon_{12}q_3 + \epsilon_{12}q_t \end{aligned} \tag{1}$$

y_{11} has a relative degree of four, y_{12} and y_{13} have a relative degree of two each, while the system has eight state variables in the 4-link phase. Similarly, y_{21} has a relative degree of four and y_{22} has a relative degree of two, while the system has six state variables in the 3-link phase.

$$\begin{aligned} y_{21} &= q_1 + \epsilon_{21}q_2 + \epsilon_{22}q_t \\ y_{22} &= \epsilon_{21}q_1 + \epsilon_{21}q_2 + \epsilon_{21}q_t \end{aligned} \tag{2}$$

The y_{ij} are chosen as polynomial functions of time and some coefficients of the polynomial are chosen a priori such that the trajectories are periodic over a gait cycle. The rest of the free coefficients are used as parameters of numerical optimization. An SQP-based optimization routine is used to modulate these parameters so that the motion constraints are satisfied.

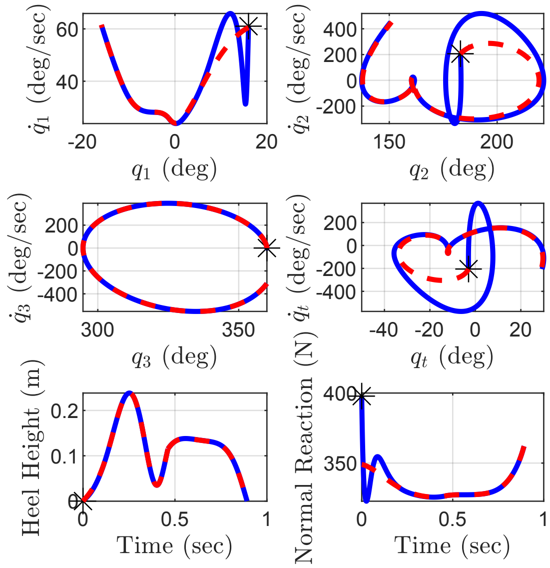

Fig. 3. Planned trajectories and tracking results for the five-link biped (dash line - planned trajectory, solid line - actual trajectory, * start of trajectories) along with motion constraints.

IV. RESULTS AND CONCLUSION

A family of dynamically feasible walking trajectories is obtained using the above mentioned parametrization of flat-output trajectories. A trajectory minimizing input norm is selected (Fig. 3) using SQP based numerical optimization method while satisfying motion constraints. Figure 3 shows tracking of this walking trajectory using a full-state feedback controller in flat-output space (solid lines are the planned trajectories and dashed lines are the actual trajectories). In this figure, the start point of the trajectories is at the heel-impact denoted by an $*$. In these tracking simulation results, initial angular velocity errors were imposed on the system which the controller was able to diminish within one gait cycle. We continue to work towards using this intermediate complexity reference model for motion planning and controller design with biped dynamics not exactly satisfying the center-of-mass constraints.

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