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Articulated Body Inertia (ABI) Forward Dynamics Computation Scheme for Floating Systems

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***Abstract*— This poster presents a forward dynamics computation method called ABI for the robotic system with a floating base, such as drone, underwater, and satellite manipulators. The proposed method differs from the conventional ABI [1], in that it can be applied to the robotic system with the floating base.**

Articulated Body Inertia (ABI) Method for Floating Systems

The articulated body inertia (ABI) algorithm was proposed in a very clever way in [1], which was based on the Gauss elimination method, however, the conventional ABI algorithm could compute only the fixed-grounded robotic systems. This poster shows that it can be extended to the floating robotic systems by computing the base body acceleration.

As a first step, the backward recursion begins with the articulated body inertia and the articulated body bias of the last body, i.e. $A_K^A := A_K$ and $b_K^A := B_K V_K$, as follows:

$$A_{i-1}^A := A_{i-1} + {}^{i-1}\mathrm{Ad}_i^{-T} A_i^a \, {}^{i-1}\mathrm{Ad}_i^{-1} \tag{1a}$$

$$b_{i-1}^A := b_{i-1} + {}^{i-1}\mathrm{Ad}_i^{-T} \, b_i^a, \tag{1b}$$

and

$$\widetilde{E}_i^{\#} = (\widetilde{E}_i^T A_i^A \widetilde{E}_i)^{-1} \widetilde{E}_i^T \tag{1c}$$

$$Y_i = I - A_i^A \widetilde{E}_i \widetilde{E}_i^{\#} \tag{1d}$$

$$h_i = -{}^{i-1}\mathrm{ad}_i \, {}^{i-1}\mathrm{Ad}_i^{-1} V_{i-1} + \dot{\widetilde{E}}_i \dot{\theta}_i \tag{1e}$$

$$A_i^a = Y_i A_i^A \tag{1f}$$

$$b_i^a = Y_i(b_i^A - F_i) + A_i^a h_i + A_i^A (\widetilde{E}_i^{\#})^T \tau_i \tag{1g}$$

for $i = K, K-1, \cdots, 1$. Through the above backward recursion, we have obtained a series of articulated body inertia matrices and bias vectors as follows:

$$(A_K^A, b_K^A) \quad \rightarrow \quad (A_{K-1}^A, b_K^A) \quad \rightarrow \quad \cdots \quad \rightarrow \quad (A_0^A, b_0^A)$$

As a second step, the base body acceleration is determined by using the articulated base body inertia and the articulated base body bias of the base body, i.e. A_0^A and b_0^A, as follows:

$$\dot{V}_0 = \left(A_0^A\right)^{-1} \left(F_0 - b_0^A\right), \tag{2}$$

where the computation of base body acceleration differs from the conventional ABI proposed in [1]. As the last step, the forward recursion begins with the base body acceleration $\dot{V}_0$ to obtain the joint acceleration $\ddot{\theta}_i$ as follows:

$$H_i = {}^{i-1}\mathrm{Ad}_i^{-1} \dot{V}_{i-1} + h_i \tag{3a}$$

$$\ddot{\theta}_i = (\widetilde{E}_i^T A_i^A \widetilde{E}_i)^{-1} (\tau_i - \widetilde{E}_i^T (b_i^A - F_i) - \widetilde{E}_i^T A_i^A H_i), \tag{3b}$$

$$\dot{V}_i = H_i + \widetilde{E}_i \ddot{\theta}_i, \tag{3c}$$

for $i = 1, 2, \cdots, K$. For the verification and comparison, let us assume the initial conditions that the base transformation $T_0 = I_4$, the base twist $V_0 = (1,1,1,0,0,0)$, the joint position $\theta = (0,0,0,\frac{\pi}{2},\frac{\pi}{2},\frac{\pi}{2},0)$, the joint velocity $\dot{\theta} = (\frac{\pi}{4},\frac{\pi}{4},\frac{\pi}{4},0,0,0,\frac{\pi}{4})$, the joint torque $\tau = (1,1,1,1,1,1,1)$ and the base control wrench $F_c = (0,0,0,1,1,1)$. The comparison results are suggested in Table, the proposed ABI shows the same results acquired from the conventional RNE and the Pinocchio [2] known as the state-of-the-art.

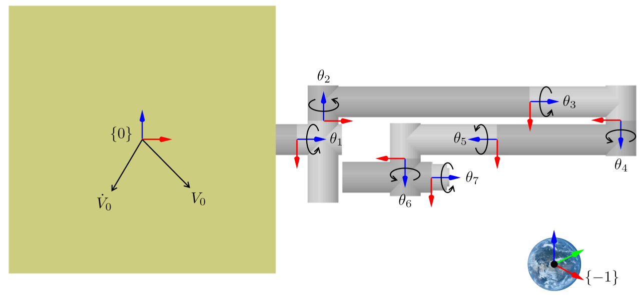

Fig. 1. Robotic system example with the floating base: satellite manipulator in space, where the global reference frame and the base body frame are represented by $\{-1\}$ and $\{0\}$, respectively.

Algorithm	Proposed ABI	RNE	Pinocchio
$\dot{V}_0 \in \mathbb{R}^6$	$\begin{bmatrix} 0.0211 \\ -0.0048 \\ -9.8320 \\ 0.0550 \\ 0.3696 \\ 0.0434 \end{bmatrix}$	$\begin{bmatrix} 0.0211 \\ -0.0048 \\ -9.8320 \\ 0.0550 \\ 0.3696 \\ 0.0434 \end{bmatrix}$	$\begin{bmatrix} 0.0211 \\ -0.0048 \\ -9.8320 \\ 0.0550 \\ 0.3696 \\ 0.0434 \end{bmatrix}$
$\ddot{\theta} \in \mathbb{R}^7$	$\begin{bmatrix} -1.1044 \\ 0.8633 \\ 0.3399 \\ 1.9729 \\ 0.0200 \\ 4.4880 \\ 112.8072 \end{bmatrix}$	$\begin{bmatrix} -1.1044 \\ 0.8633 \\ 0.3399 \\ 1.9729 \\ 0.0200 \\ 4.4880 \\ 112.8072 \end{bmatrix}$	$\begin{bmatrix} -1.1044 \\ 0.8633 \\ 0.3399 \\ 1.9729 \\ 0.0200 \\ 4.4880 \\ 112.8072 \end{bmatrix}$

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References

[1] R. Featherstone, *Rigid Body Dynamics Algorithms*, Springer, 2008.
[2] Pinocchio, https://github.com/stack-of-tasks/pinocchio, 2024