Robust Quadrupedal Locomotion through Asymptotic Stabilization of H-LIP on Dynamic Rigid Surfaces with General Vertical Motion

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***Abstract*—We present Late Breaking Results on achieving robust quadrupedal locomotion by stabilizing a hybrid-linear inverted pendulum (H-LIP) stepping on surfaces with vertical motion. Our framework analyzes H-LIP stability, derives feasible and stable footsteps using quadratic programming (QP) based methods, and generates real-time trajectories for locomotion on surfaces with uncertain vertical motion. We design an optimization-based torque control law and validate our framework through simulations and hardware experiments. The validation results on hardware demonstrate robust locomotion on surfaces with uncertain and unknown motion, including external disturbances and uneven terrain.**

I. Background

The quadrupedal robots show promise for real-world applications [1]. However, achieving provably stable and robust locomotion on surfaces with dynamic motions poses challenges due to hybrid and explicitly time-varying dynamics [2]. Our previous research has addressed legged locomotion on surfaces with periodic and accurately known motion [3]–[5]. The proposed work addresses locomotion on a rigid surface with general (periodic or aperiodic), vertical, uncertain motions.

II. Method

This section provides a brief overview of the proposed framework. The overall framework is illustrated in Fig. 1.

H-LIP Model and Stability Analysis. We introduce a time-varying H-LIP model to describe the essential robot dynamics associated with legged locomotion on a rigid surface with a vertical motion. We also derive the sufficient stability condition for the H-LIP model under a discrete-time foot-placement control law. Details of these analytical results are provided in [6].

QP-based Optimization of Foot-Stepping Control Gains. To enforce the proposed stability condition and necessary kinematic constraints (e.g., maximum step length), we formulate the following QP to optimize in real-time the gains of the foot-placement controller:

$$\min_{\mathbf{K}} \quad \frac{1}{2}\mathbf{KSK}^T + \mathbf{Kc} \qquad \text{s.t.} \qquad \mathbf{EK}^T < \mathbf{b}, \tag{1}$$

where $\mathbf{K} \in \mathbb{R}^{1\times 2}$ represents the control gain, $\mathbf{S} \in \mathbb{R}^{2\times 2}$ denotes the Hessian matrix of the cost function, $\mathbf{c} \in \mathbb{R}^{2\times 1}$ is the gradient vector of the cost function, $\mathbf{E} \in \mathbb{R}^{6\times 2}$ is a matrix used to form the linear constraints representing the kinematic feasibility and stability condition, and $\mathbf{b} \in \mathbb{R}^{6\times 1}$ is a vector of constraint bounds.

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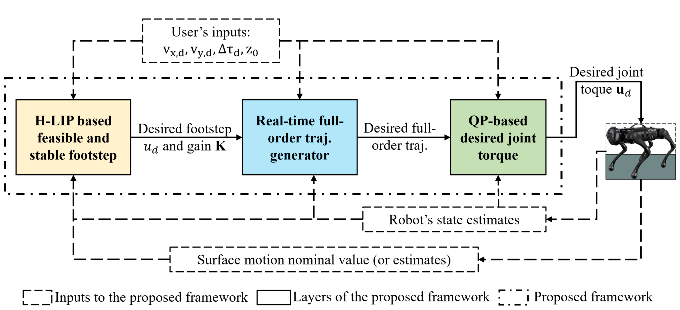

Fig. 1. Illustration of the overall proposed framework.

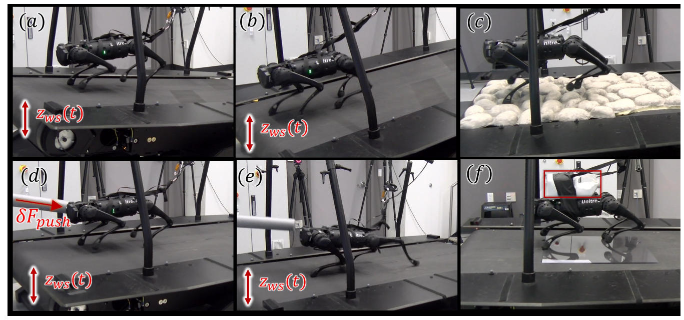

Fig. 2. Hardware experiments demonstrating robust locomotion: (a) and (b) uncertain surface motion, (c) walking on rocky terrain, (d) and (e) recovery from an external push, and (f) loaded walking (fluid weighing 30% of the robot's mass) on a glass plate. The experiment video is available at https://youtu.be/S7ysMQp0Vyo.

III. Results

Hardware experiments on a Unitree Go1 robot trotting on a rocking treadmill, which possesses a general, vertical, unknown motion, confirm the locomotion robustness under the proposed control approach, as demonstrated through the snapshots of the experiment videos in Fig. 2. The experiment video is available at https://youtu.be/S7ysMQp0Vyo.

IV. Conclusion

This abstract summarizes a reduced-order model based quadruped control method that exploits the provable stabilization of a time-varying H-LIP model to achieve robust underactuated trotting on a vertically oscillating surface. Various hardware experiments confirm the robustness of the proposed method under different types of uncertainties, including uncertain surface motion, surface unevenness, intermittent pushes, and external loads. Our future work will focus on integrating the proposed model-based approach and reinforcement learning to enable rapid and provable adaptation to general classes of real-world uncertainties beyond those considered in this study.

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