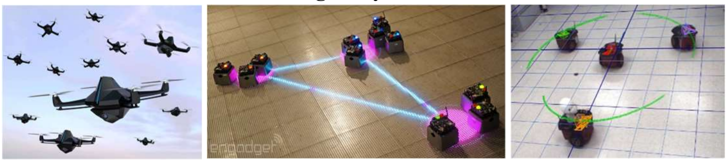
Formation Analysis of Dynamic Multi-agent Systems Controlled by a Generalized Cyclic Pursuit Mechanism

Taeheon Kwak, Yeongjae Kim and Tae-Hyoung Kim
Department of Mechanical Engineering, Chung-Ang University

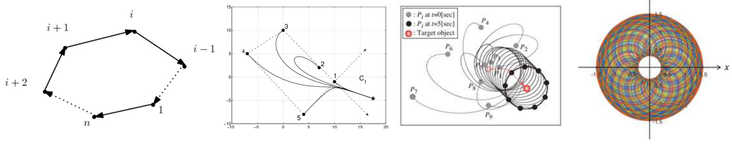
1. Introduction

▪ **Distributed control of multi-agent system for formation control**



- Since distributed control of multi-agent systems can be used in many different areas, numerous studies have been conducted.
- Particularly, various tasks can be carried out through formation control.

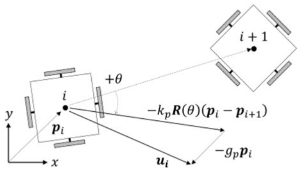
▪ **Cyclic pursuit: Formation control scheme**



- Cyclic pursuit, in which agent i pursues indexed $i+1$ (modulo n), has the benefit of minimizing communication.
- The generalized cyclic pursuit(GCP) can be used to create different formation patterns.

2. System description

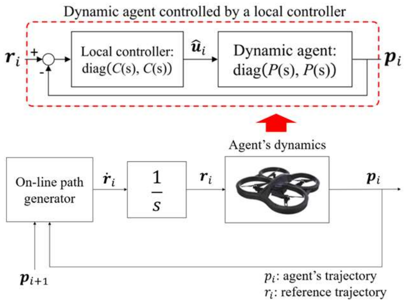
▪ **Graphical representation and mechanism of GCP**



$$\dot{\boldsymbol{r}}_i(t) = u_i(t)$$
$$u_i(t) = -g_p \boldsymbol{p}_i(t) - k_p \boldsymbol{R}(\theta)(\boldsymbol{p}_i(t) - \boldsymbol{p}_{i+1}(t))$$
$$\boldsymbol{R}(\theta) = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

g_p, k_p : constant control gains
θ: constant offset angle

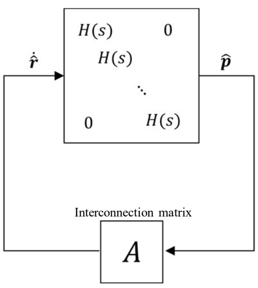
▪ **Block diagram of dynamic agent**



- The figure on the left shows the block diagram of the i-th agent.
- We consider the dynamics of agents not considered in previous studies.
- Set each agent as a feedback system with dynamics $P(s)$ and local controller $C(s)$.
- Then, each agent has a structure that follows $\boldsymbol{r}_i$, which is a reference position input.

3. Methodology: Generalized frequency variable

▪ **Block diagram of overall system**



$$H(s) = \frac{1}{s} \cdot \frac{P(s)C(s)}{1 + P(s)C(s)}$$

MAS dynamics

$$\boldsymbol{A} = -g_p \boldsymbol{I} - k_p \boldsymbol{L} \otimes \boldsymbol{R}(\theta)$$

$\boldsymbol{I}$: Identity matrix
$\boldsymbol{L}$: Circulant matrix
$\boldsymbol{R}(\theta)$: Rotation matrix

- The integrator and the dynamics of each agent are represented by the transfer function $H(s)$.
- $\boldsymbol{A}$ represents the interconnection matrix, indicating that each agent communicates through GCP.

▪ **Generalized frequency variable and domains**

$T(s) = \left(\frac{1}{H(s)}\boldsymbol{I} - \boldsymbol{A}\right)^{-1} = (\phi(s)\boldsymbol{I} - \boldsymbol{A})^{-1}$: Overall transfer function

$\phi(s)$: Generalized frequency variable

Hara, S., Tanaka, H. & Iwasaki, T. (2013). Stability analysis of systems with generalized frequency variables. IEEE Transactions on Automatic Control, 59(2), 313-326.

$\Omega := \phi(\mathbb{C}_+) = \{\zeta \in \mathbb{C} \mid \exists s \in \mathbb{C}_+ \ s.t. \ \phi(s) = \zeta\}$,
$\hat{\Omega} := \phi(\hat{\mathbb{C}}_+) = \{\xi \in \mathbb{C} \mid \exists s \in \hat{\mathbb{C}}_+ \ s.t. \ \phi(s) = \xi\}$,
$\Omega^c := \mathbb{C} \setminus \hat{\Omega}$,

These domains are used to perform the multi-agent system stability analysis.

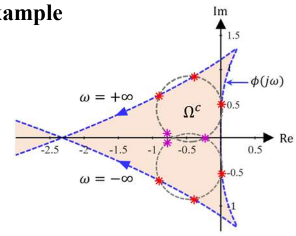
where $\mathbb{C}_+ = \{s \in \mathbb{C} \mid Re(s) > 0\}, \hat{\mathbb{C}}_+ = \{s \in \mathbb{C} \mid Re(s) \geq 0\}$

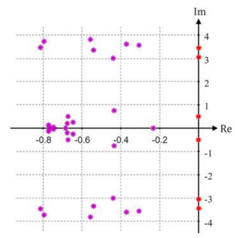
4. Stability analysis with graphical approach

▪ **Relationships between eigenvalue distribution of A based on domains and closed-loop(CL) pole of $T(s)$**

- If all the eigenvalues of $\boldsymbol{A}$ lie inside the domain Ω^c, all the closed-loop poles of $T(s)$ lie in the left-half plane(LHP) of the complex plane.
- If an eigenvalue of $\boldsymbol{A}$ lies inside Ω, the corresponding closed-loop pole of $T(s)$ exists in the right-halp plane(RHP) of the complex plane.
- If an eigenvalue of $\boldsymbol{A}$ lies on $\phi(j\omega)$, the corresponding closed-loop pole of $T(s)$ exists on the $j\omega$-axis of the complex plane.

▪ **Example**





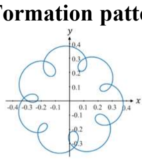
- Three pairs of eigenvalues exist on $\phi(j\omega)$ and the remaining eigenvalues exist inside Ω^c(left figure), so three pairs of CL poles exist on $j\omega$-axis, and the rest exist in LHP(right figure).

5. Analytical approach to formation analysis

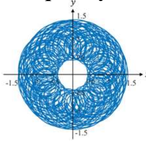
▪ **Steady-state response of i-th agent with multiple $j\omega$-axis poles**

$$\boldsymbol{p}_{\infty,i}(t) = \sum_{k=1}^{m} a_k \begin{bmatrix} \cos(\omega_k t + \psi_{k,i}) \\ \sin(\omega_k t + \psi_{k,i}) \end{bmatrix}$$

▪ **Formation pattern according to the frequency ratio**

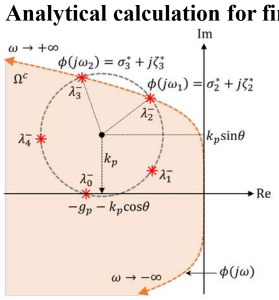


- closed curve when system has rational frequency ratio



- dense curve when system has irrational frequency ratio

▪ **Analytical calculation for finding frequency component**

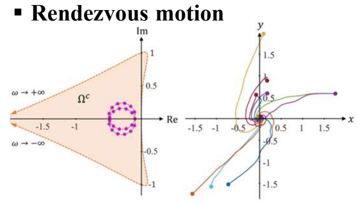


1. Find σ_ℓ^* and ζ_ℓ^* that satisfy following equation
$$(\sigma_\ell^* + g_p + k_p cos\theta)^2 + (\zeta_\ell^* - k_p sin\theta)^2 - k_p^2 = 0$$
2. Find ω_k that satisfy following equations simultaneously
$$Re[\phi(j\omega_k)] = \sigma_\ell^*, Im[\phi(j\omega_k)] = \zeta_\ell^*$$

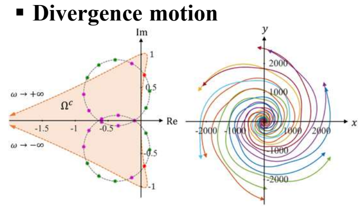
Rational function

6. Simulation results

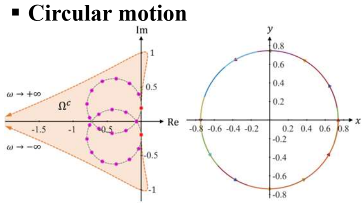
▪ **Rendezvous motion**



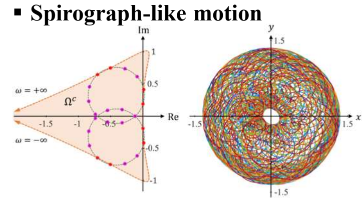
▪ **Divergence motion**



▪ **Circular motion**



▪ **Spirograph-like motion**



- The eigenvalue of $\boldsymbol{A}$ and the domain Ω^c are shown in the left figure for each simulation, while the response of the multi-agent system is displayed in the corresponding right figure.
- The effectiveness of the suggested analysis methodology is demonstrated by the simulation results.