

Formation Analysis of Dynamic Multi-Agent Systems Controlled by a Generalized Cyclic Pursuit Mechanism*

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Abstract—Circular formation control is a strategy for movement coordination in multi-agent systems that includes having a set of agents move in a circle while keeping a specific distance between one another. This strategy is often used when multiple agents need to move in a coordinated manner, for example, to entrap a target, perform patrol and surveillance operations, and search for sources in a sensed environment.

Meanwhile, the cyclic pursuit mechanism has garnered attention from numerous researchers due to its inherent advantages, including minimal communication requirements for connectivity. A cyclic pursuit mechanism is a representation of a collection of n leaderless agents, each labeled with a number, where agent i chases its specific neighbor, indexed $i + 1$ (modulo n). Cyclic pursuit may be described in graph theory as a directed cycle graph, where the nodes stand in for agents and the directed edges for information flow. One area of study in cyclic pursuit is formations that achieve rendezvous or maintain a fixed or variable inter-agent distance while rotating a stationary or moving target.

The generalized cyclic pursuit (GCP) mechanism, a modified cyclic pursuit mechanism, is used in another section to create non-circular formation patterns. It was shown that such a method might lead to movements in a collection of agents that are epicyclic or spirograph-like. Each agent in the GCP approach follows its leading neighbor along a line of sight that has been offset by a common angle. In addition to the relative position information between agents, the pursuing mechanism that results uses an agent's absolute position as a stabilizing term. The eigenvalue distribution of an interconnection matrix resulting from the GCP structure, which determines how a neighboring agent $i + 1$ influences the trajectory of agent i , determines the motions of agents involving rendezvous, diverging spirals, circular patterns, and epicyclic patterns. Additionally, the analytical investigation of the non-circular epicyclic or spirograph-like movements displayed by a collection of agents under the multiple imaginary-axis eigenvalue distribution of an interconnection matrix.

Nonetheless, it can only be applied to a particular category of multi-agent systems where each agent is modeled as a single integrator. The reason for this is that the closed-loop poles of GCP-controlled multi-agent systems are equivalent to the

distribution of eigenvalues in the interconnection matrix of the system only when each agent is modeled as a single integrator. As a result of this fact, a new analysis of the formation patterns attained by GCP-controlled dynamic multi-agent systems is required, which is the focus of interest in this study.

The control-theoretic analysis of the collective behavioral patterns of multi-agent systems is presented here using graphical and analytical approaches. Each agent is supposed to have a general continuous-time linear time-invariant (LTI) model for its dynamics, and the GCP mechanism controls how each agent moves. In terms of the agent's dynamics and the tuning parameters of the GCP scheme in use, the proposed techniques clearly demonstrate the process underlying the occurrence of various formation patterns, such as rendezvous, diverging spirals, circular, and spirograph-like patterns.

The detailed features of this study are as follows. First, the generalized frequency variable (GFV) is used to show how a dynamic multi-agent system, whose collective behavior is guided by the GCP scheme, may be formulated into the LTI system framework. Then, we examine the stability, instability, and marginal stability of the dynamic multi-agent systems under discussion using a graphical framework based on the stability analysis of a general LTI system with GFV. Second, although the suggested graphical analysis framework is highly proficient in anticipating rendezvous or diverging spiral motions of GCP-controlled dynamic multi-agent systems, it may prove challenging to precisely predict whether a group of dynamic agents will move in a circular or spirograph-like fashion. Therefore, an analytical tool is developed to overcome the limitations of the graphical technique. Third, we describe the steady-state formation trajectories of a swarm of GCP-controlled dynamic agents by explicitly taking into account their general continuous-time LTI model and the case where there are more than two pairs of closed-loop poles on the imaginary axis of the complex plane. Additionally, we discuss the analytical standards for evaluating the spirograph-like closed and open orbits that correspond with the dynamic agents' steady-state trajectories. Lastly, the study presents comprehensive simulation results that demonstrate the validity of the developed analytical methodologies.

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