Robust Optimal H_∞ Control for Active Suspension System Using Input Saturation Function

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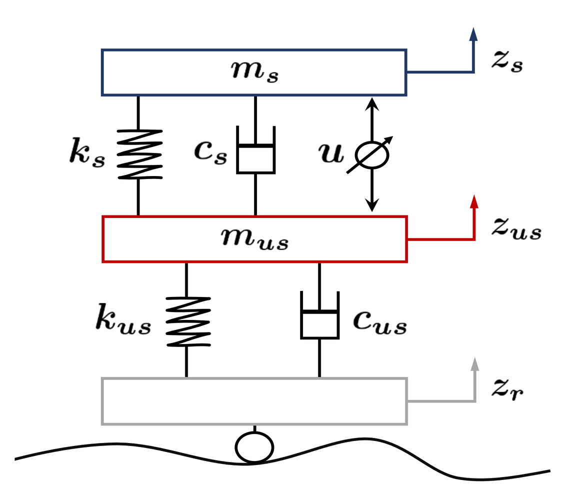
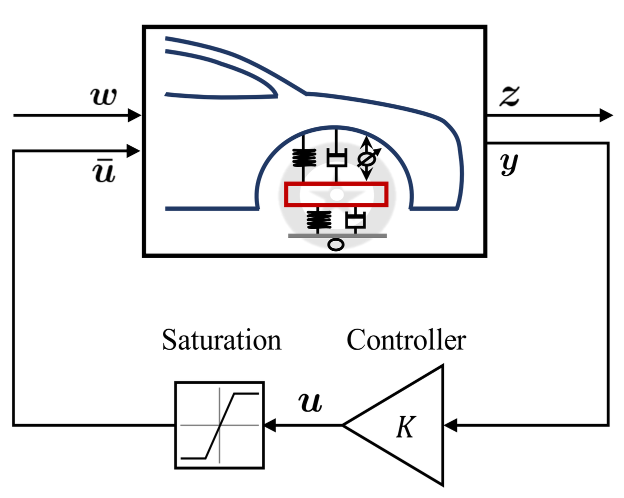
INTRODUCTION

- This study proposes **the optimal H_∞ control methodology for systems constructed with an input saturation function**.
- The vehicle is a system that is influenced by random road disturbances, and ride comfort is related to the vibration of the human body.
  → **Optimal H_∞ control** is selected as the control method to improve ride comfort via active suspension system.
- Input saturation is an important design specification in practical control systems, and it may cause performance degradation such as time delay, increased overshoot, vibration deterioration, and even affect the stability of the systems.
  → The controller is designed **based on systems with input saturation functions** to address the actuator saturation problem.
- The optimization problem for H_∞ controller design is formulated by bilinear matrix inequalities, making it an NP-hard problem.
  → In this study, a **metaheuristic optimization algorithm** is developed as an alternative to solve the optimization problem with bilinear matrix inequality.

METHODOLOGY

❖ Active Suspension System Using Saturation Function





✓ *State Space Representation*

$$x(t) = [z_s - z_{us},\ \dot z_s,\ z_{us} - z_r,\ \dot z_{us}]^T$$

$$w(t) = [\dot z_r]$$

$$\bar{u}(t) = \begin{cases} -u_{lim}, & \text{if } u < -u_{lim} \\ u(t), & \text{if } -u_{lim} \le u(t) \le u_{lim} \\ u_{lim}, & \text{if } u > u_{lim} \end{cases}$$

$$\dot x(t) = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_{us}} & \frac{c_s}{m_{us}} & -\frac{k_{us}}{m_{us}} & -\frac{c_s + c_{us}}{m_{us}} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ -1 \\ \frac{c_{us}}{m_{us}} \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_{us}} \end{bmatrix} \bar{u}(t)$$

✓ *Controlled Output Vector for Improvement of Ride Comfort*

$$z(t) = [\ddot z_s] = \begin{bmatrix} -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{m_s} \end{bmatrix} \bar{u}(t)$$

❖ Optimization Problem Formulation for Controller Design

$V = x^T P x$ (Lyapunov candidate function)

$z^T z - \gamma^2 w^T w + \dot V < 0$ (Constraint condition related to H_∞ performance)

▪ CONDITION 1 $(C1)$

$$\begin{bmatrix} \Xi & PB_w & PB_u & C_1^T D_u & \frac{1+\delta}{2} C^T K^T D_u^T D_u \\ B_w^T P & -\gamma^2 I & 0 & 0 & 0 \\ B_u^T P & 0 & -\varepsilon_1 I & 0 & 0 \\ D_u^T C_1 & 0 & 0 & -\varepsilon_2 I & 0 \\ \frac{1+\delta}{2} D_u^T D_u K C & 0 & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0$$

where,

$$\Xi = C_1^T C_1 + \frac{1+\delta}{2} C^T K^T D_u^T C_1 + \frac{1+\delta}{2} C_1^T D_u K C + \varepsilon_2 \left(\frac{1-\delta}{2}\right)^2 C^T K^T K C$$
$$+ \left(\frac{1+\delta}{2}\right)^2 C^T K^T D_u^T D_u K C + \varepsilon_3 \left(\frac{1-\delta}{2}\right)^2 C^T K^T K C + \left(\frac{1-\delta}{2}\right)^2 C^T K^T D_u^T D_u K C$$
$$+ A^T P + PA + \frac{1+\delta}{2} C^T K^T B_u^T P + \frac{1+\delta}{2} P B_u K C + \varepsilon_1 \left(\frac{1-\delta}{2}\right)^2 C^T K^T K C$$

▪ CONDITION 2 $(C2)$

$$\begin{bmatrix} \left(\frac{u_{lim}}{\delta}\right)^2 & KC \\ C^T K^T & \frac{P}{\rho} \end{bmatrix} \ge 0$$

✓ *Design Problem of H_∞ Controller Based on System Using Saturation Function*

minimize γ,

subject to $(C1)$ and $(C2)$ with $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, $\varepsilon_3 > 0$, and $P > 0$.

❖ Procedure for Solving Bilinear Matrix Inequality Problems

(STEP 1) : Determine the unknown controller matrices using the design variables of optimization algorithm

$\vec{x}_i^k = [x_{i,1}^k,\ x_{i,2}^k,\ x_{i,3}^k,\ x_{i,4}^k]^T$

→ $K := \begin{bmatrix} x_{i,1}^k & x_{i,2}^k & x_{i,3}^k & x_{i,4}^k \end{bmatrix}$ → Unknown variables are only ε_1, ε_2, ε_3, and P. → $(C1)$ becomes a linear matrix inequality and **can be readily solved by LMI solvers**.

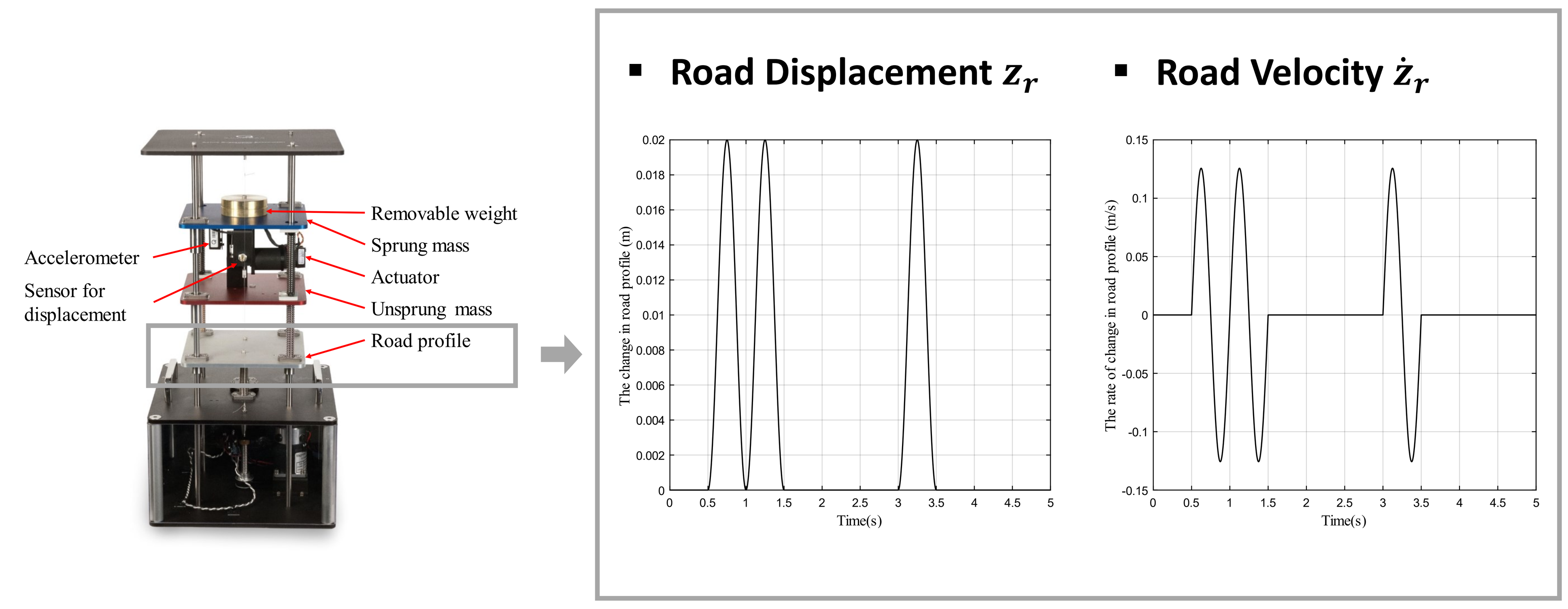
(STEP 2) : Calculate the fitness value γ

Solve matrix inequality conditions by using LMI solvers → $\mathcal{L}(\vec{x}_i^k) := \gamma$

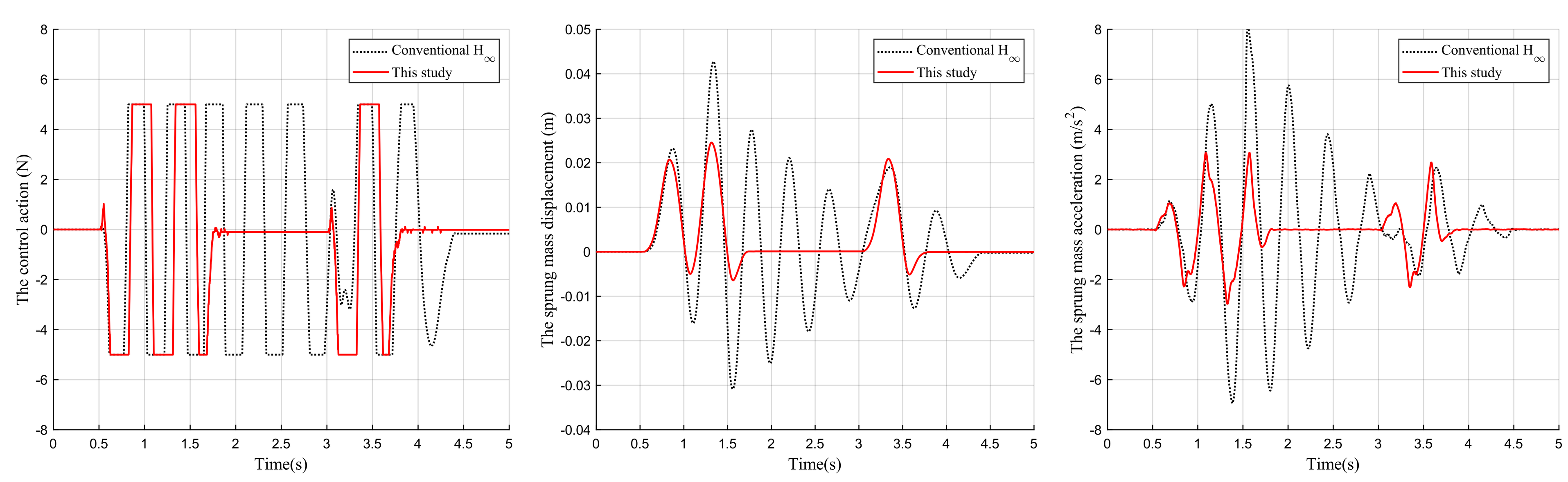
(STEP 3) : Update design variables through metaheuristic optimization algorithm

EXPERIMENT RESULTS

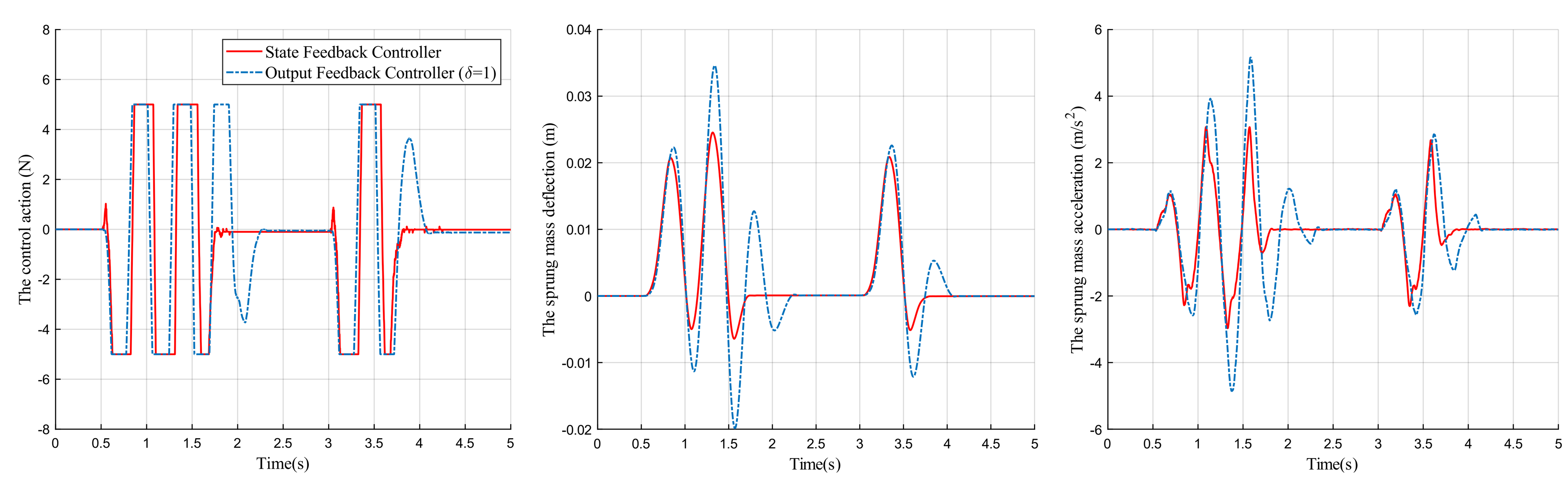
❖ Experimental Validation Using Quanser's Active Suspension Platform



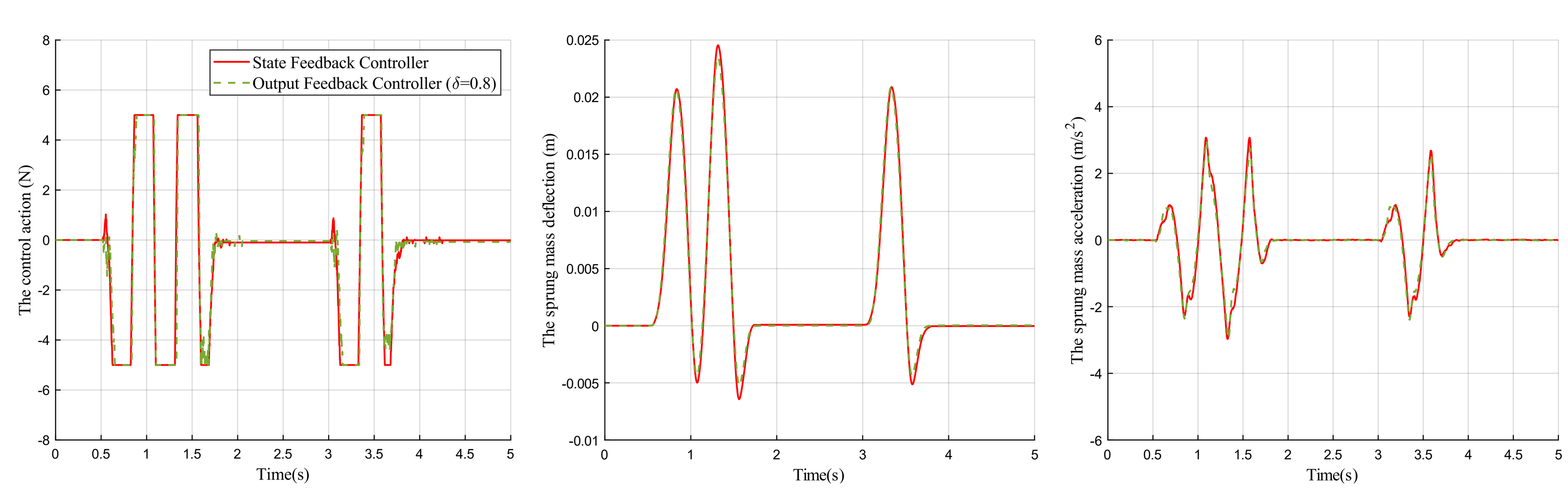
▪ **RESULT 1 : Comparison with Conventional H_∞ Controller**



▪ **RESULT 2 : H_∞ Controller for Output Feedback Control System**



▪ **RESULT 3 : Robustness of the Proposed H_∞ Controller against Input Saturation**



CONCLUSION

- In this paper, an optimal H_∞ controller design problem was formulated for systems constructed with input saturation functions in order to directly address the actuator saturation issue.
- Through the employment of a metaheuristic algorithm, the difficulty of the optimization problem with bilinear matrix inequalities was surmounted.
- Full-state feedback controllers as well as output feedback controllers could be designed using the proposed algorithm.
- Experimental results conducted under actuator saturation using Quanser's active suspension platform have validated the performance of the controllers designed using the proposed methodology.
- It is also validated that modification of the δ value affects the robustness of the proposed controller against input saturation.