

Robust Optimal H_∞ Control for Active Suspension System Using Input Saturation Function*

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Abstract—The suspension system of a vehicle is one of the essential components that keeps the vehicle and the people inside it safe. The suspension systems are typically divided into passive, semi-active, and active categories. Compared to the passive suspension system, the active suspension system allows for better ride comfort and handling stability by independently controlling each suspension. Therefore, the active suspension system is constructed to improve ride comfort by absorbing vibrations and shocks transmitted from the road surface to the vehicle body, maintain tire-to-road traction associated with driving maneuverability, or steerability, and ensure the stability of the vehicle body under a variety of driving conditions.

Among the various controller design requirements of an active suspension system, ride comfort is considered the main point of view. To achieve the design purpose of enhancing ride comfort, the comfort-oriented H_∞ control for the active suspension system is adopted. In the H_∞ control design procedure, the H_∞ norm of the transfer function from uneven road disturbance to vehicle body acceleration is usually used to specify the ride comfort performance. The H_∞ norm of the transfer function and its upper limit can be expressed as linear matrix inequality (LMI) conditions that can be formulated as a straightforward optimization problem while handling Lyapunov stability analysis simultaneously. Due to this simplicity of design, the optimal H_∞ controller design problem for enhancing ride comfort is developed into the LMI optimization problem that minimizes the upper limit of the H_∞ norm. On the other hand, actuator input saturation is also an essential consideration in the practical design of control systems because it is a common nonlinear problem resulting from the limited power of the actuator. In closed-loop systems designed without consideration for actuator saturation, input saturation can cause performance degradation such as time delay, increasing overshoot, lengthening settling time, increasing steady-state error, aggravating oscillation, and even affect the stability of the system. Nonetheless, most existing research exploits linear mathematical models to characterize the behavior of systems with nonlinear saturation functions.

In recent decades, numerous efforts have been made to handle the input saturation problem in the control design process. Consequently, a strategy is proposed to design the controller by

solving the LMI optimization problem formulated based on the system described by the nonlinear input saturation function. However, in this control strategy, the differential equations of state variables are described by the input saturation function, but it is only applicable to systems in which the controlled output variables are defined solely by the state variable. Unfortunately, the vehicle body acceleration is selected as the controlled output for improving ride comfort in the active suspension system, and its equation is formulated by an input variable. Therefore, the H_∞ controller design problems have to be formulated on the basis of the system in which both state variables and input variables characterize the controlled output. However, this controller design problem appears as a bilinear matrix inequality (BMI) optimization problem, which is an NP-hard problem. In contrast to LMI optimization problems, which can be solved numerically using methods such as efficient convex optimization algorithms, it is highly challenging to find a feasible solution set for BMI optimization problems. Thus, comfort-oriented H_∞ control for active suspension systems with input saturation functions appears to be a challenging field of research; however, few attempts have been made in this direction, which has motivated our present research.

In conclusion, this study proposes an H_∞ control synthesis for solving the actuator saturation problem and conducts an experimental study of H_∞ control for the active suspension system of the quarter car. In the H_∞ control design procedure for the active suspension system, actuator saturation is directly handled by introducing a dynamic model with a saturation function. To improve ride comfort for passengers, the acceleration of the vehicle body is selected as a controlled output, and the H_∞ norm of the transfer function from disturbance to controlled output is optimized. Based on Lyapunov stability theory, the control synthesis problem is formulated as a non-convex bilinear matrix inequality. This design difficulty is overcome by the proposed single-objective distributed quantum-behaved particle swarm optimizer, which efficiently explores the optimal controller that provides the minimum upper limit of the H_∞ norm. The simulation and experimental tests are performed using the Quanser's active suspension system platform and the road profile generated by trigonometric functions. The results demonstrate the effectiveness of the proposed H_∞ controller.

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