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Dynamic Inversion for Wheeled Inverted Pendulum with Extra Joint using Singular Perturbation Technique

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***Abstract*—This study introduces a dynamic inversion based trajectory planning algorithm for wheeled inverted pendulum (WIP) robots that incorporates a linear joint atop the WIP body. We utilize the singular perturbation technique to obtain an asymptotic series solution for the inverse joint trajectory, while considering kinematic and dynamic constraints.**

***Index Terms*—Wheeled Inverted Pendulum, dynamic inversion, trajectory planning, redundancy**

I. Inversion Algorithm and Simulation Result

Consider the longitudinal motion of a WIP with an extra linear joint shown in Fig.1 where the output variable $\psi(t)$, with its desired value $\psi_d(t)$, is the center of mass of the WIP, the travel distance is denoted by $x(t)$, and $\theta(t)$ and $l(t)$, respectively, denote the tilt angle and the linear joint movement. The output kinematic constraint and internal dynamic constraint form a set of two differential equations with three joint variables, which is, by nature, destined to be unbounded by the forward integration.

$$\begin{bmatrix} 1 & r_1-\gamma\theta l & \gamma \\ D_1 & D_2 & D_3 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \\ \ddot{l} \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{1}{\mu r_c}(r_2\theta+\gamma l) \end{bmatrix} = \begin{bmatrix} \ddot{\psi}_d(t) \\ 0 \end{bmatrix},$$

where $\mu = \frac{1}{r_c(m_1+m_2)g}$ may be regarded as a singular parameter (being relatively big compared to other parameters in the system dynamics), $r_c = \sqrt{r_2^2+l^2}$ and $\gamma = m_2/(m_1+m_2)$ are some derived parameters, and D_i (i = 1, 2, 3) denotes the coefficient of acceleration variables in the internal dynamics. To obtain a stable inversion solution, a series form of solution is assumed such that $x(t) = \sum_{i=0}^{\infty}\mu^i x_i^i(t)$, $\theta(t) = \sum_{i=1}^{\infty}\mu^i\theta_i^i(t)$, and $l(t) = \sum_{i=1}^{\infty}\mu^i l_i^i(t)$. Substituting the assumed series forms into the above constraint equations and then collecting the zero-th order terms of μ's power yield the general zero-th order inverse solution:

$$\ddot{x}_0 = \ddot{\psi}_d \quad \& \quad [\theta_0 \; l_0]^T = \mathbf{J}^\dagger\ddot{\psi}_d + \lambda N(\mathbf{J}),$$

where λ is an arbitrary constant and $\mathcal{N}(\mathbf{J})$ is the null space of $\mathbf{J}$ defined as $\mathbf{J} := \frac{1}{D_1 r_c}[r_2 \quad \gamma]$. The non-uniqueness of the inversion solution is due to having a redundant degree of freedom in the system. We may impose an additional functional or safety-related requirement to specify the undetermined constant λ. For example, a solution can be found by zeroing the tilting angle (θ_0) or by giving condition $l_0 = g_d(t)$ to create useful safe posture during the travel. If the zero-th order solution is not sufficiently precise, continue to obtain higher order solutions by following the similar procedure.

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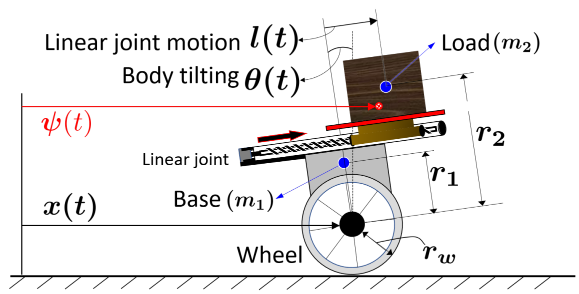

Fig. 1: Schematic of a WIP system with an extra linear joint

For validation, the dynamic inversion solution is obtained with $\ddot{\psi}_d(t)$ defined by a 7-th order polynomial function to travel 6[m] for 6[s] while linear joint to produce $l(t) = 0.1\sin(2\pi t)$. As shown in Fig. 2 the zero-th order solution satisfies the kinematic and dynamic constraints very much.

As a result, this study validates that the proposed trajectory planning algorithm effectively addresses the dynamic inversion problem and delivers stable solutions.

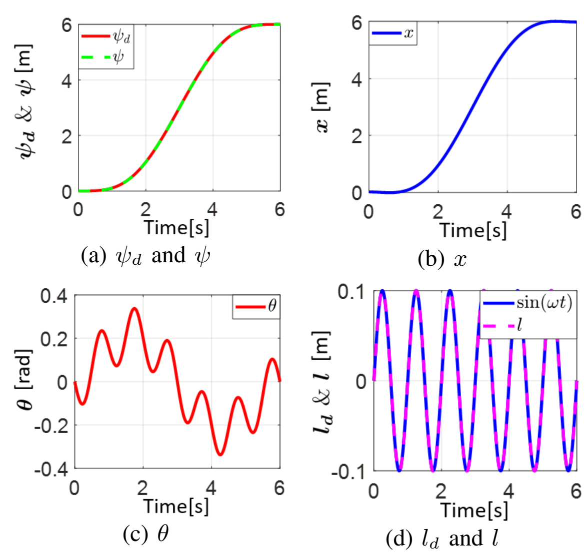

Fig. 2: Zeroth-order inversion solution for a forward motion