# Effect of Coil Size on Efficiency of Wireless Power Transfer with Magnetic Field Resonant by Halbach Array

1<sup>st</sup> Takayuki Oba Graduate school of Intergrated design Engineering Keio University Yokohama, Japan oba@sum.sd.keio.ac.jp 2<sup>nd</sup> Yuhei Tomioka Graduate school of Intergrated design Engineering Keio University Yokohama, Japan tomioka@sum.sd.keio.ac.jp 3<sup>rd</sup> Takahiro Nozaki Department of System Design Engineering Keio University Yokohama, Japan nozaki@sd.keio.ac.jp

Abstract—In recent years, wireless power transfer (WPT) has become popular from the viewpoint of easy charging. Among them, magnetic field resonance WPT is being researched and developed due to its high efficiency. However, there are magnetic fluxes that cannot be used for power supply and are leaking out in magnetic field resonance WPT. Therefore, this paper applies a special magnet arrangement, a Halbach array, for transmission coils. The Halbach array transmission coils can concentrate the magnetic fluxes on one side. This feature allows the magnetic fluxes to be concentrated to increase the amount of magnetic fluxes chained to the receiving coil. It has a significant effect on efficiency. Since the size of the transmission and receiving coils also affects efficiency, this paper changes the size of the coils and analyzes its effect on efficiency. The efficiency of the Halbach array transmission coils will be confirmed by comparison with conventional transmission coils.

*Index Terms*—Near-field wireless power transfer, Halbach array, coil size, transfer efficiency.

# I. INTRODUCTION

WPT is a technology that supplies power without a wired connection and has been put to practical use in industrial, medical, and other fields [1] [2]. There are several types of WPT and the magnetic field coupling type is used to charge electric vehicles (EV) and smartphones [3] [4]. Among the magnetic field coupling type, the magnetic field resonance type has attracted much attention because of its high efficiency and is mainly used in EV charging.

Reducing power losses and improving efficiency are important in WPT. Especially, changes in mutual inductance have a significant impact on the transfer efficiency. Therefore, various studies have been conducted to improve the transfer efficiency. One method to solve this problem is impedance matching [5]. T. C. Beh et al. proposed an automated impedance matching system. The automated impedance matching system uses variable inductors and capacitors to adjust the impedance according to changes in mutual inductance. However, this method has the disadvantage of increasing the size of the circuit, because it requires switches, drive circuits, and controls to change the parameters. Frequency tuning is another solution and is varying the operating frequency of the power supply on the primary side [6]. However, in order to prevent interference between devices that use high frequencies, WPT is limited to a certain frequency band and the frequency can only be adjusted within a specified range. Other solution is to achieve zero voltage switching in the inverter MOSFETs using a switched control capacitor [7]. W. Li et al. proposed a double-side self tuning *LCC/S* system. Switching control capacitors are applied to the primary and secondary sides to accommodate changes in mutual inductance. However, it is necessary to identify sequential coil parameters to control the switch. These three methods adjust circuit parameters and cannot aggregate magnetic flux.

There are also studies that improve the transfer efficiency by devising the structure of coils. There is a method in which a relay coil is inserted between the transmitter and receiver coils [8]. Although this approach improves the transfer efficiency at long distances, the maximum efficiency at short distances is reduced due to losses in the relay coil. As an alternative, transmission coils using a Halbach array has been proposed [9] [10]. This method uses four side coils around the transmission coil to aggregate the magnetic flux. The Halbach array is considered to be an effective method for WPT to EV, which are expected to become widely used. However, the box shape of the previous study makes it unsuitable for supplying power to EV where the transmission coils are embedded in the ground.

This paper compares the efficiency of the new Halbach array of transmission coils and conventional transmission coils. It is designed to be embedded in the ground and has a flat transmission surface. Three simulations are performed with different sizes of the transmitting coils and the receiving coil. Coil size considerations are given for the coil size required for EV based on simulation results. This paper also consider the size of the side coils based on the theoretical equation.

This paper is organized as follows. Section II explains the Halbach array. In section III, the modeling of the circuit of the the magnetic field resonance WPT. Section IV presents the modering of the Halbach array transmission coils. Section



Fig. 1. Magnetic flux density of Halbach array.



Fig. 2. Transmitter.



Fig. 3. Circuit of S-S type WPT.

TABLE I: Parameters of circuit.

V [V]	Voltage				
I [A]	Current				
L [H]	Inductance				
C[F]	Capacitance				
$R [\Omega]$	Resistance				
$R_L \ [\Omega]$	Load resistance				
M [H]	Mutual inductance				
$\omega  [rad/s]$	Angular frequency				
$\eta$ [%]	Efficiency				
$\bigcirc_1$	Primary side				
$\bigcirc_2$	Secondary side				

V compares the efficiency of Halbach array transmission coils with conventional transmission coils through simulation. Section VI discusses coil size and side coil length. In section VII, this paper is summarized.

## II. HALBACH ARRAY

A Halbach array is an arrangement of permanent magnets first discovered by Mallinson as a one-side flux in 1973 [11] and proposed by Halbach in 1980 [12]. Fig. 1 shows the magnetic flux density of the Halbach array. This arrangement allows the magnetic flux on the lower side to be concentrated on the upper side, thus producing a strong magnetic field on the upper side and a weak magnetic field on the lower side. It is used in applications such as magnetic levitation systems [13] and nuclear magnetic resonance systems [14] because of its ability to aggregate the surrounding magnetic field. This array of permanent magnets is applied to the WPT transmission coils.

Fig. 2(a) shows the outline of the Halbach array transmission coils and Fig. 2(b) shows the outline of the conventional transmission coils. The Halbach array transmission coils consist of five square coils, which are connected in series. Conventional transmission coils in Fig. 2(b) generate equal magnetic fields at the upper and lower sides. Therefore, the lower magnetic flux is not used for charging. On the other hand, by passing a current through the coil in the direction of the arrow in Fig. 2(a), a magnetic flux can collect in the center of the transmission coils. Hence, the lower magnetic flux not used for charging can collect the magnetic flux in the center of the transmission coils. This increases the amount of magnetic fluxes chained to the receiving coil, which can be expected to improve efficiency. The structure of Fig. 2(a) is also useful for applications where the distance from the transmitter coil to the receiver coil is short, such as electric vehicles, because the surface of the transmitter coils is flat.

## III. CIRCUIT MODELING

Fig. 3 shows a circuit of series-series (S-S) type WPT. S-S type is a method of connecting an inductor and a capacitor in series on both the primary and secondary sides. It can supply power with high efficiency among magnetic field resonance types. In this paper, the five transmission coils of the Halbach array are considered as one coil, and the mutual inductance of these five is not considered.

Table I shows the parameters of the circuit. From Kirchhoff's law, the circuit equations of Fig. 3 can be expressed as

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$$V_1 = (jS_1 + R_1)I_1 + j\omega M I_2 \tag{1}$$

$$0 = (jS_2 + R_2 + R_L)I_2 - j\omega MI_1$$
(2)

$$S_1 = \omega L_1 - \frac{1}{\omega C_1} \tag{3}$$

$$S_2 = \omega L_2 - \frac{1}{\omega C_2}.$$
(4)

By solving the circuit equations (1) and (2), the currents in the circuit are expressed as

$$I_{1} = \frac{R_{2} + R_{L} + jS_{2}}{(R_{1} + jS_{1})(R_{2} + R_{L} + jS_{2}) + (\omega M)^{2}} V_{1}$$
(5)

$$I_2 = -\frac{j\omega M}{(R_1 + jS_1)(R_2 + R_L + jS_2) + (\omega M)^2} V_1.$$
 (6)



Fig. 4. Two parallel copper wires.

Efficiency is the ratio of the power consumed by the load resistance to the power consumed by the primary and secondary sides. Therefore, the efficiency can be calculated as

$$\eta = \frac{(\omega M)^2 R_L}{\left\{ (R_2 + R_L)^2 + {S_2}^2 \right\} R_1 + (\omega M)^2 (R_2 + R_L)}.$$
 (7)

Under conditions of resonance, the reactances of the coil and capacitor cancel each other out. when the angular frequency is the resonant frequency, the efficiency can be expressed as

$$\eta_0 = \frac{(\omega_0 M)^2 R_L}{(R_2 + R_L) \{R_1 R_2 + R_1 R_L + (\omega_0 M)^2\}},$$
 (8)

where  $\omega_0$  is

$$\omega_0 = \frac{1}{\sqrt{L_1 C_1}} = \frac{1}{\sqrt{L_2 C_2}}.$$
(9)

Since the resistance R is a measurable parameter and the angular frequency  $\omega_0$  is a configurable parameter, the only unknown parameter is mutual inductance M. Efficiency can be improved by increasing the mutual inductance from (8).

#### **IV. MUTUAL INDUCTANCE MODELING**

This section derives the mutual inductance of the transmission coils in the Halbach array shown in Fig. 2(a). The coils in Fig. 2(a) are divided into the center coil, the coils at both ends, and the side coils.

#### A. Two parallel copper wire

Two parallel copper wires are shown in Fig. 4. From Neumann's formula, the inductance of the two parallel copper wires in Fig. 4 can be expressed as

$$M_{i} = \int_{a_{2i}}^{b_{2i}} \int_{a_{1i}}^{b_{1i}} \frac{\mu_{0}}{4\pi\sqrt{(x_{1} - x_{2})^{2} + d^{2}}} \mathrm{d}x_{1} \mathrm{d}x_{2}, \qquad (10)$$





Fig. 6. Case of coils at both ends.

where d represents the distance between two copper wires and i represents the index. By solving the integral, the inductance of the two parallel copper wires in Fig. 4 can be calculated as

$$M_{i} = \frac{\mu_{0}}{4\pi} \bigg( \sqrt{(a_{2i} - a_{1i})^{2} + d^{2}} - \sqrt{(a_{2i} - b_{1i})^{2} + d^{2}} + \sqrt{(b_{2i} - b_{1i})^{2} + d^{2}} + \sqrt{(b_{2i} - b_{1i})^{2} + d^{2}} + (a_{1i} - a_{2i}) \log \left| a_{2i} - a_{1i} + \sqrt{(a_{2i} - a_{1i})^{2} + d^{2}} \right| + (a_{2i} - b_{1i}) \log \left| a_{2i} - b_{1i} + \sqrt{(a_{2i} - b_{1i})^{2} + d^{2}} \right| + (b_{1i} - b_{2i}) \log \left| b_{2i} - b_{1i} + \sqrt{(b_{2i} - b_{1i})^{2} + d^{2}} \right| + (b_{2i} - a_{1i}) \log \left| b_{2i} - a_{1i} + \sqrt{(b_{2i} - a_{1i})^{2} + d^{2}} \right| \bigg|$$

$$(11)$$

### B. Case of center coil

Fig. 5 shows the case of center coil. The mutual inductance between square coils in the case of Fig. 5 can be expressed as

$$M = 4 \left( M_1 - M_2 \right). \tag{12}$$

Of the two parallel copper wires in the receiving coil, the mutual inductance of the near copper wire is  $M_1$  and the mutual inductance of the far copper wire is  $M_2$ . The sign is positive when the currents are in the same direction and negative when they are in opposite directions.

# C. Case of coils at both ends

The case of coils at both ends is shown in Fig. 6. The mutual inductance between square coils in the case of Fig. 6 can be calculated as

$$M = (M_1 - M_2) - 2(M_3 - M_4) - (M_5 - M_6).$$
(13)



Fig. 7. Case of side coils.

 $M_3$  and  $M_4$  represent the mutual inductance between the lateral copper wires of the transmission coil and the parallel copper wires of the receiving coil.  $M_5$  and  $M_6$  represent the mutual inductance between the longitudinal copper wires on the left side of the transmission coil and the parallel copper wires of the receiving coil.

## D. Case of side coils

Fig. 7 shows the case of side coils. The mutual inductance between square coils in the state of Fig. 7 can be expressed as

$$M = (M_1 - M_2) - (M_7 - M_8).$$
(14)

 $M_7$  and  $M_8$  are the mutual inductance between the copper wire on the lower side of the transmission coil and the copper wire of the parallel receiving coil.

#### E. Halbach array transmitter

The  $t_1$  in Fig. 5, Fig. 6, and Fig. 7 represent the size of the transmission coils and  $t_2$  represents the size of the receiving coil.  $t_3$  in Fig. 7 represents the height of the side. In this paper, the lengths of  $t_1$  and  $t_3$  are the same because they are square coils. From (12), (13), (14), the mutual inductance of Fig. 2(a) can be calculated as

$$M = 4mn (M_1 - M_2) + 4ml (M_1 - M_2) - 4ml (M_3 - M_4) - 2ml (M_5 - M_6) - 2ml (M_7 - M_8),$$
(15)

where m represents the number of turns of the receiving coil and n represents the number of turns in the center of the transmission coil, l represents the number of turns at both ends of the transmission coils and the side coils.

# V. SIMULATION

This section compares the efficiency of the Halbach array transmission coils and conventional transmission coils. Two coil sizes, 100 mm and 150 mm are prepared and three simulations are performed for cases with 100mm transmission coils and 100 mm receiving coil, 150 mm transmission coils and 150 mm receiving coil, 150 mm transmission coils and 150 mm receiving coil. The air gap is run from 10 mm to 150 mm,

TABLE II: Parameters of simulation.

	Fig. 9		Fig. 10		Fig. 11	
	Halbach.	Conv.	Halbach.	Conv.	Halbach.	Conv.
$t_1 \; [mm]$	100	100	150	150	150	150
$t_2 \text{ [mm]}$	100	100	100	100	150	150
$L_1 \ [\mu H]$	76.37	73.56	122.54	119.61	122.54	119.61
$L_2 \ [\mu H]$	28.95	28.95	28.95	28.95	49.22	49.22
$C_1$ [nF]	45.91	47.66	28.61	29.31	28.61	29.31
$C_2$ [nF]	121.09	121.09	121.09	121.09	71.233	71.233
$R_1 \ [\Omega]$	1.668	1.624	2.411	2.412	2.411	2.412
$R_2 \ [m\Omega]$	570.07	570.07	570.07	570.07	915.80	915.80
$R_L [\Omega]$	10	10	10	10	10	10
$V_1$ [V]	10	10	10	10	10	10
m	10	10	10	10	10	10
n	5	10	5	10	5	10
l	5	0	5	0	5	0



Fig. 8. Models created in JMAG-Designer.

varying the spacing by 10 mm. The operating frequency of the simulation is set to 85 kHz. Table I shows the parameters used in this study. The simulations were performed using JMAG-Designer. The model created in JMAG-Designer is shown in Fig. 8.

Simulation results for 100mm transmission coils and 100mm receiving coil are shown in Fig. 9. Fig. 9 shows that the Halbach array transmission coils are more efficient when the air gap is less than 40 mm. Fig. 10 shows a comparison of efficiency at 150 mm transmission coils and 100 mm receiving coil. In Fig. 10, the Halbach array transmission coils outperform the conventional method when the airgap is 90 mm or less. A comparison of the Halbach array transmission coils for 150 mm transmission coils for 150 mm transmission coils and 160 mm receiving coil and 160 mm receiving coil are shown in Fig. 10. Fig. 11 shows that the Halbach array transmission coils are more efficient than the conventional method when the air spaper shown in Fig. 10. Fig. 11 shows that the Halbach array transmission coils are more efficient than the conventional method when the air spaper shown in Fig. 10. Fig. 11 shows that the Halbach array transmission coils are more efficient than the conventional method when the air spaper shown in Fig. 10. Fig. 11 shows that the Halbach array transmission coils are more efficient than the conventional method when the air spaper shown in Fig. 10. Fig. 11 shows that the Halbach array transmission coils are more efficient than the conventional method when the air spaper shown the air spaper shown in Fig. 10 mm transmission coils are more efficient than the conventional method when the air spaper shown the air spaper shown in Fig. 10 mm transmission coils are more efficient than the conventional method when the air spaper shown in Fig. 10 mm transmission coils are more efficient than the conventional method when the air spaper shown in Fig. 10 mm transmission coils are more efficient than the conventional method when the air spaper shown in Fig. 10 mm transmission coils are more efficient than the conventional method when the air spaper shown in Fig. 10 mm transmission coils are more shown in Fig. 10 mm transmission coils are more efficient than the conventional method when the air spaper shown the air s

## VI. DISCUSSION

## A. Coil size

From the comparison between Fig. 9 and Fig. 10, increasing the size of the transmission coils increases the distance at which it is more efficient than the conventional method. In addition, the decay of efficiency with distance becomes slower. However, the maximum efficiency is reduced. Comparison of Fig. 10 and Fig. 11 show that increasing the size of the receiving coil decreases the distance at which the efficiency is higher than that of the conventional method. In this study, the Halbach array transmission coils outperform conventional



Fig. 9. Comparison of Halbach array transmission coils and conventional transmission coils  $(t_1=t_2=100 \text{ mm}).$ 



Fig. 10. Comparison of Halbach array transmission coils and conventional transmission  $coils(t_1=150 \text{ mm}, t_2=100 \text{ mm}).$ 

transmission coils by up to 90 mm. However, since the air gap of electric vehicles is a few hundred millimeters [15], the coil size needs to be considered. Based on the above considerations, it is necessary to increase the transmission coils size. Simulation is performed using the derived theoretical equation (15).

Fig. 12 shows the simulation results when the size of the transmission coils is increased to 400 mm from the conditions shown in Fig. 11. The Halbach array of transmission coils outperforms the conventional method for air gaps below 290 mm. It was confirmed that the bus can be extended from 90 mm in Fig. 10 to 290 mm in Fig. 12. However, electric vehicles require a more efficient power supply. Fig. 13 shows the simulation results from Fig. 12 with a power receiving coil size of 300 mm. The airgap, which is exceeded by the efficiency of the Halbach array transmission coils, was reduced to 256 mm. On the other hand, the maximum efficiency increased to 77 %. The size of the receiving coil to be mounted on the vehicle and the size of the transmission coil to be embedded in the road is an issue for future study. Since the further improvement of efficiency is necessary, the size of the receiving coil when mounted on a car and the size of the transmission coil to



Fig. 11. Comparison of Halbach array transmission coils and conventional transmission  $coils(t_1=t_2=150 \text{ mm}).$ 



Fig. 12. Comparison of Halbach array transmission coils and conventional transmission  $coils(t_1=400 \text{ mm}, t_2=150 \text{ mm}).$ 

be embedded in the road should be considered. Optimization of efficiency is necessary because of constraints such as the size of the vehicle and the cost of embedding the coils in the ground. There is also room for future research on improving efficiency through the circuit parameters such as impedance matching and frequency tuning.

# B. Side coil

Five square coils are used in this study. However, the size of the side coils is not studied. Therefore, the side coils are changed from square to rectangular to compare the efficiency. The size of the side coils is varied to 200mm and 600m for the value of  $t_3$  in Fig. 7. Comparison of efficiency by side coil size is shown in Fig. 14. Fig. 14 shows that the efficiency is higher when the side coil height is small. The decrease in the resistance of  $R_1$  due to the reduced amount of copper wire used is thought to be the reason for the improved efficiency. The decrease in resistance is greater than the increase in mutual inductance due to the decrease in the fifth term on the right side of (15). The height of the side coils needs to be considered in the future, as it is a combination of the cost of embedding the coils in the ground.



Fig. 13. Comparison of Halbach array transmission coils and conventional transmission  $coils(t_1=400 \text{ mm}, t_2=300 \text{ mm}).$ 



Fig. 14. Comparison of efficiency by side coil size.

#### VII. CONCLUSION

This paper proposes a transmission coil using a Halbach array to improve the transfer efficiency in WPT with magnetic field resonance. Simulations of the proposed method and the conventional method were performed to compare the efficiency. It was confirmed that the proposed method achieves higher efficiency than the conventional method at close distance. Increasing the size of the transmitter coil improved the transmission distance and increasing the size of the receiver coil improved the efficiency. Efficiency improved when side coil height was reduced. Experiments will confirm the effectiveness of the Halbach array of power transmission coils in the future.

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