# Contextual Anomaly Detection in Hot Forming Production Line using PINN Architecture

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Abstract— This paper presents a physics-informed neural network (PINN) architecture for contextual anomaly detection in a hot forming production line. It enhances widely used proximity- or distribution-based anomaly detection approaches for industrial processes through the consideration of contextual process data. The physical model is built using a priori process knowledge and thermodynamic equations. This model is then injected into the loss function of a neural network. The network is trained on data from the production line and constantly regularized by the physical loss term. Within inference, the PINN predicts the resulting temperature of the produced blank given the contextual process data. The anomaly detection is performed using the unsupervised local outlier factor algorithm on the error between actual and predicted blank temperature. This makes it possible to assess whether the achieved product temperature appears normal or abnormal based on the database. The main advantage of this novel approach is that it can detect contextual anomalies that remain otherwise undiscovered.

Keywords—contextual anomaly detection, hybrid modeling, physics informed neural network, process control, hot forming, informed modeling

# I. INTRODUCTION

In times of narrow production windows, high quality requirements and growing emphasis on energy efficiency, the demands on production monitoring systems are increasing. Usually, series production systems are monitored using static value thresholds. In case the value exceeds the specified threshold, an alarm is triggered, prompting either an operator inspection or a production interruption. Measures are then taken to eliminate the defect. In addition to static production monitoring, statistical quality control is often carried out, in

which randomly selected products are subjected to a detailed and often destructive quality inspection. While the specific causes of production errors can vary, they can usually be attributed to several key factors. Discovering the characteristic relational dynamics of these key factors is the challenge of contextual anomaly detection. Contextual anomaly detection is a powerful data analysis technique used to identify anomalies or outliers in data based on their contextual information. Unlike traditional anomaly detection methods that solely rely on statistical analysis, contextual anomaly detection takes into account the relationships and dependencies between given data points within a given context [1]. By considering the context in which an observation occurs, this approach can effectively distinguish between normal variations and abnormal behaviors or events. This makes it particularly useful in detecting anomalies in complex systems such as production environments, where understanding the context is crucial for accurate anomaly identification.

Within the scope of this paper a contextual anomaly detection technique is developed on a given use-case of a press hardening production line. Press hardening, also known as hot stamping or hot forming, is an advanced manufacturing process used in the automotive industry to produce highcomponents with exceptional strength. lightweight mechanical properties. This innovative technique involves heating a blank of sheet metal to a high temperature and then rapidly stamping it into a die using a hydraulic press. The intense heat and pressure enable the material to undergo phase transformation, resulting in a fully hardened part with improved strength, durability, and crashworthiness. Press hardening offers several advantages over traditional forming and heat treatment methods. The process allows for precise shaping of intricate and complex geometries, ensuring tight tolerances and dimensional accuracy. Moreover, the rapid cooling during the stamping process helps achieve high levels of strength and hardness, making the parts lightweight vet

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incredibly strong. This not only enhances the overall performance of the component but also contributes to fuel efficiency and reduced emissions of the produced vehicles.

The press hardening process requires meeting several key requirements to achieve successful martensitic phase transformation. First, the initial state of the blank needs to be fully austenitized typically at a temperature of approximately 950°C. Next, during the pressing cycle, the blank should be rapidly cooled down at a rate of at least 60K/s. This rate is essential for the martensitic phase transformation [2]. The phase transformation stops at a final temperature of around 280°C. The cooling is stopped at a final product temperature of around 150°C -250°C ensuring full transformation. The entire process should take less than 20 seconds. The cooling mechanism is typically realized using water-cooled dies or dies, the heat is transferred from the blank to the cooling water via the dies, rapidly reducing its temperature.

As of [2–4] the main factors that can affect the quality of the final part hardening are:

- Material properties of the blank and its coating
- Ambient influences such as heat and dust
- Degradation of the production machine
- Human error

Since these factors are difficult to monitor directly, process supervision needs to analyze available data sources for any signs of anomalies or artifacts that may be caused by one or more of these disturbance factors.

## II. RELATED WORKS

As of [5, 6] an anomaly or outlier is defined as datapoints that do not conform with the rest of the datapoints, so that it might be generated by a different mechanism. On the other hand, a contextual anomaly considers only the surrounding datapoints to determine if it deviates from the expected pattern [6, 7]. The context defining surrounding might be given by a timely or situational relation. In Fig. 1 it is shown that the difference between a global and a contextual anomaly detection lies within the preprocessing steps, the final detection methods can be the same. Thus, the modeling of the contextual reference point is a crucial step for contextual anomaly detection.

Database



Fig. 1. Difference between global and contextual anomaly detection

## A. Context modeling approaches

Within the literature there are several approaches on the contextual modeling, of which many rely on various types of neural network models. In [1] a LSTM network is used to build an inverse process model giving the contextual scope of the tested datapoint. CARMONA uses a window based approach to dynamically set the given context of time-series datapoints [8]. There Temporal Convolutional Networks embed the suspect and context window. SCHLEGL uses a generative adversarial network to generate suitable reference images for anomaly detection [9]. For univariate time-series neural basis expansion analysis called N-BEATS show promising results in modeling process behavior [10]. There several fully connected deep learning blocks are used to split the given time-series is its governing frequencies. The approach has shown great results on long horizon forecasting challenges. OLIVARES have enhanced this approach to also regard exogenous variables resulting in the N-BEATSx method [11]. ZHOU uses a deep neural network autoencoder in combination with a principal component analysis to perform robust reconstruction of images [12]. It is tested on a noisy MNIST dataset and challenged against an Isolation Forest algorithm.

Another increasingly popular approach is the integration of physical models into neural networks leading to so-called Physics Informed Neural Networks (PINN) or Physics Guided Neural Networks (PGNN) [13-15]. The goal is to penalize network solutions that are physically inconsistent, bridging the gap between theory-based models and pure data-based models. Whereas most examples refer to physical processes these approaches can include any formal expressed process relation. KARPATNE ET AL. describe different architectures theory-guided initialization and theory-guided e.g. regularization [16]. GÖTTE AND TIMMERMANN uses the PINN approach for system identification for the purpose of control layout [17]. SCHÖN enhances the approach to a Multi-Objective Physics-Guided Recurrent Neural Network for identification of non-autonomous systems [18].

# B. Anomaly detection approaches

Despite the advances in the field of neural networks, within the anomaly detection method palette, pure statistical and traditional unsupervised approaches remain powerful in this sector. Within the scope of this work, only anomaly detection algorithms leading to a specific classification of inlier/outlier or normal/abnormal are regarded. Commonly they are classified as unsupervised and supervised methods.

# 1) Supervised methods

Supervised methods require a fully labeled training dataset. The difference in comparison to a conventional classification task is that the class of abnormal datapoints is generally sparsely populated leading to an extremely imbalanced dataset [7]. Sometimes anomalies are deliberately injected into a given set of normal datapoints in order to obtain a labeled data set. It is questionable whether the injected anomalies represent possible anomalies realistically. Among the supervised methods popular approaches are support vector machines with its variations [19], k-Nearest-Neighbor [20], Hidden Markov Models [21] and decision trees [7].

#### 2) Unsupervised methods

As anomalies are expected to occur rarely and are defined as not conforming with the norm, labels are usually difficult to obtain. This factor emphasizes the need of unsupervised methods for anomaly detection of real-world data. Among the most popular methods there are Isolation Forest [22], Local Outlier Factor (LOF) [23], K-means clustering [24] and Gaussian Mixture Models [25]. All these methods have in common that they cluster the dataset and generate a feature based on the distance to either a cluster centroid or a mean density within the cluster. The final classification of a datapoint as normal or abnormal is done through a threshold value. As a first approach this threshold value is set during the training process to meet a certain given contamination ratio. As there is no label within the dataset the contamination needs to be guessed leading to uncertain evaluation results. Counter measures can be sparsely obtaining labels from given data to boost the certainty about the threshold value.

#### **III. MODELING THE HOT FORMING PROCESS**

As a first step within the contextual anomaly detection a contextual model that represents the process behavior needs to be built. Within the scope of this paper the promising PINN approach is applied. The architecture consists of a neural network where a first principle physical model is integrated as an additional loss term within the training process. At first the physical model is described.

#### A. Building the physical model

The physical model is based on [26] with some adjustments. The general setup of the phases is shown in Fig. 2. During the press phase the heat energy is transferred from the blank to the cooling water via the press die. During the waiting phase the blank is taken out leaving the die and cooling pipes to exchange their heat energy. The available sensor data of the whole process are the following:

- Temperature of cooling water  $T_{CW}$  at intake and outflow positions
- Blank temperature  $T_B$  at start and end of pressing phase (through thermografic image)

The missing or unknown physical values are:

- Temperature of both die parts  $T_D$
- Temperature of the blank  $T_B$  during the pressing phase
- Temperature of cooling water  $T_{CW}$  within the die parts



Fig. 2. Heat-Exchange models. Pressing phase (left), waiting phase (right) as of [26]

For the press phase, the simplified governing ordinary differential equations (ODE) are (1) and (2). Parameters are *m*: mass of part,  $c_p$ : material specific heat capacity,  $\alpha$ : heat transfer coefficient and A: contact surface of both materials. Indices are B: Blank, D: Die, CW: Cooling Water.

$$\dot{T}_B = \frac{\alpha_1 A_1}{m_B * c_{p,B}} \cdot (T_D - T_B) \tag{1}$$

$$\dot{T}_{D} = \frac{\alpha_{1}A_{1}}{m_{D}*c_{p,D}} \cdot (T_{D} - T_{B}) - \frac{\alpha_{2}A_{2}}{m_{D}*c_{p,D}} \cdot (T_{D} - T_{CW})$$
(2)

During the waiting phase the blank is removed, leaving the die to be cooled down by the cooling water. The differential equation of the die temperature reduces to (3).

$$\dot{T}_D = -\frac{\alpha_2 A_2}{m_D * c_{p,D}} \cdot (T_D - T_{CW}) \tag{3}$$

The main simplifications of these modeling equations are that the heat exchange with the ambient air is neglected, and the temperature of the cooling water is assumed to be constant whereas in real world the cooling water temperature rises along the contact surface. Similarly, all temperature gradients within the die and the blank material are neglected.

For simulation the ODE-system can be solved as an initial value problem. The difficulties with the application of the given formula for simulating the press temperature lie within the identification of the parameter and the initial values for both the representative blank and die temperature. As in [26] one solution can be to identify the values through optimization. Therefor the press is taken in a situation where it has experienced a long waiting phase. It is assumed that the die temperature is equal to the cooling water temperature. The optimization identifies the parameter in four blocks of the products  $\alpha_1 A_1, \alpha_2 A_2, m_T c_{p,T}, m_B c_{p,B}$ . Practically these values need to be obtained through experiments as in [2]. The problem with the optimization approach is that the simulation error adds up during runtime and there is no comparison with the temperature states of the production plant. The main reason for this behavior is that it is not possible to detect the core temperature of the die at runtime. The result of the last simulation step of the previous phase is simply assumed to be the initial value of the following phase. Within the scope of this paper another approach is introduced.

This approach is using the overall heat energy balance for the pressing phase (5) and for the waiting phase (6) where E is the heat energy of the corresponding part based on (4).

$$E = m * c_p * T \tag{4}$$

$$E_{B,Start} + E_{D,Start} = E_{B,End} + E_{D,End} + E_{CW}$$
(5)

$$E_{D,Start} = E_{D,End} + E_{CW} \tag{6}$$

With the use of the assumed start temperature of the blank and the thermographic image from the blank taken at the end of the pressing phase both the start and end energy can easily be calculated. The total energy consumed by the cooling water energy can be calculated using (7). There the temperature difference of the incoming and outflowing water is multiplied by the specific heat capacity and the mass flow rate. This energy flow rate is then integrated over the corresponding phase duration.

$$E_{CW} = \int \dot{m} * c_p * \Delta T_{CW,in,out} \tag{7}$$

With the help of the energy balance the residual energy of the die at the end of the pressing phase can be obtained. Furthermore, it is possible to calculate the die energy during the waiting phase using (3) and (6), as this is the only unknown component in the equation (8-9). For simplification the effective temperature of the cooling water is assumed to be the mean between intake and outflow temperature.

$$\dot{E}_{CW} = -\dot{E}_D \tag{8}$$

$$\dot{m}_{CW} * c_p * \Delta T_{CW,in,out} = -\alpha_2 A_2 (T_D - T_{CW})$$
(9)

#### B. Physics Informed Neural Network architecture

The vanilla PINN architecture uses a simple fully connected neural network consisting of a few rather small hidden layers of around 32-128 nodes each. The physical model is integrated into the loss term through evaluation on some training points sampled over the investigated input spectrum. These injected loss components are usually called physics loss and boundary loss [27]. This way the model can be fitted not only on the given training datapoints but also on simulation-based training points and even the expected gradient at these points. The result is a more mature network solution compared to classical neural networks that are only trained on given datapoints. In recent time several different PINN architectures have been proposed in the literature. BRUDER AND MIKELSONS describe a more general approach combining physical models and neural networks in order to construct a grey box vehicle model [28]. As mentioned, GÖTTE use a more serialized approach whilst SCHÖN use a recurrent network architecture. MOSELEY use a more complex structure using several neural networks that focus on different subdomains each. The overall solution is obtained by chaining the single network solutions piece by piece [27]. MENG also splits the initial problem into smaller parts using a number of small PINN models in a multi-staged manner for long-time integration [29]. BAJAJ introduces a gaussian process smoothing to overcome robustness issues with noisy or corrupted data [30]. The press hardening process consists of a series of pressing and waiting phases so the time horizon for the simulation of a single phase is bounded. As discussed in section A the available measurement data during each phase is limited to only the cooling water temperatures. The data loss can therefore only address this information. The blank temperature is just available at the start and end of the pressing and is therefore regarded within the boundary loss (BC Loss) component. The die temperature is not available as sensor data and remains to be estimated through the physical model. The ODE-Loss tries to balance the heat exchange between the cooling water, blank and die components. Thus, the PINN model needs to heavily rely on the physical information. To experiment with generalization capabilities of the neural network, different scales of detailed physical models are implemented within the ODE-Loss component.



Fig. 3. Base PINN architecture for press hardening modeling

The base PINN architecture used within this paper is shown in Fig. 3. For the final anomaly detection, the estimation error is transformed into an outlier score using the unsupervised LOF algorithm as discussed in section II.B.2). The parameter network performs parameter identification of the heat capacity of the blank and die components as well as the heat transfer coefficient between blank and die. The identification is integrated within the error backpropagation during the training process of the main network.

## IV. EVALUATION OF PINN MODEL BASED ANOMALY DETECTION FOR PRESS HARDENING PROCESSES

## A. Proof of concept on simulation database

First, the basic functionality of the model architecture is verified using a simulation example. For this purpose, a series of cooling processes are simulated using the previously described physical model. The resulting temperature profiles are recorded as data for training. Then, a PINN model is constructed and trained using this data set. The example demonstrates the extent to which the model can learn the behavior of the physical model under optimal conditions. The result of such a test run is shown in Fig. 4. There, the true temperature curve is shown as a solid line. From this true curve some training data points are collected and marked with star symbol. The PINN model is then trained using this data under the evaluation of the physical model on triangle marked locations. The final PINN response is shown as a dashed line. Upon analysis of Fig. 4, it is evident that the model accurately predicts the characteristic behavior of the temperature curves within the time range of the data set. However, when extrapolating to a further time range (t>10s), the results, especially the curve of the blank temperature, deviate significantly from the physically expected behavior. Consequently, the model is not capable of achieving generalization beyond the training range. This discrepancy suggests a limitation in the model's ability to capture the process dynamics of the physical model. There both temperatures would asymptotically meet at a point of complete temperature alignment. Since the duration of the pressing phase is fixed, the lack of extrapolation capability is not a major drawback of the approach.



Fig. 4. Result of PINN model on simulation data

#### B. PINN construction on given press hardening database

Transitioning to training with real data puts the dependency of the PINN model on data support points to the test. This is due to the lack of measurements regarding the curve of the blank and die temperatures during the pressing phase. The available measurements are limited to a complete coverage of the cooling water temperatures, as well as the start and end temperatures of the blank. Furthermore, estimates for the start and end temperature of the press are made using Equation (9). The transition from start to end temperature of blank and die need to be learned by the PINN model only based on the given physical relations.



Fig. 5. Result of PINN model on exemplary real process data

The dataset contains 67,000 press cycles in total. Within this dataset, the average final temperature of the blank is 113°C with a standard deviation of 9.9°C. The calculated starting temperature for the top die is 19.1°C, and 18.5°C for the bottom die, with a standard deviation of 2.3°C each. An 80% train-test split is applied. When examining the PINN model's performance on real data, it becomes apparent that the missing data points within the pressing phase significantly expand the freedoms of the main network in finding solutions. The network now only considers the energy balance introduced by the physical model. As a result, as shown in the example in Fig. 5, the curve exhibits higher variations compared to the simulation example in Fig. 4.

The evaluation of model performance is conducted using the Mean Absolute Error (MAE). It should be noted that the test dataset is supposed to contain anomalies, which distort the MAE value. The background is that the PINN model is intended to predict the normal behavior of the blank temperature. In the case of an anomaly, it is desired behavior that the model error is large to identify the anomaly. For this reason, in addition to the MAE metric, a robust version of the error metric called Robust MAE is also utilized. Like Robust Scaling, this metric reduces the underlying error vector by the lower and upper 10% of the values.

TABLE I. PERFORMANCE METRICS OF PINN MODEL

Error Std.	MAE	<b>Robust MAE</b>
9.88°C	7.51°C	7.32°C

#### C. Anomaly Detection on PINN results

For the subsequent contextual anomaly detection, the LOF algorithm is used. This method enhances the well-known k-Nearest Neighbor algorithm by considering the local density of the data point distribution. The anomaly classification threshold varies dependent of the local data density. In areas of low density, the threshold value is greater compared to areas with high local data density. The results of the LOF analysis can be seen in Fig. 6. In addition to some global outliers, a few local anomalies can also be identified. As with most unsupervised anomaly detection methods, the overall threshold is adaptable to the expected anomaly contamination of the dataset.



Ground Truth Value (scaled)

Fig. 6. Result of LOF Anomaly Analysis

#### V. CONCLUSION AND OUTLOOK

This paper describes the development of a Physics-Informed Neural Network model for contextual anomaly detection in hot forming processes. The model utilizes fundamental thermodynamic equations for heat transfer and storage in solid materials to learn the process of cooling and hardening during hot forming based on available start and end temperature measurement data. The resulting model can estimate the final temperature of the blank based on given contextual process data, which serve as the basis for subsequent contextual anomaly detection. One notable drawback of this approach is the sensitivity of the model to the distribution of weights and biases at the initialization stage. Since no measurement data is available during the pressing phase, the model is guided solely by the underlying physics equations. One possible approach to mitigate this behavior is to initially train the model on simulation data and then transfer it to real data. This approach will be further explored in future investigations. Another potential approach is the use of a Neural Ordinary Differential Equation. In a Neural ODE, the physical model not only influences the training process but also becomes an independent component of the model that is used during inference. Another possibility for expansion is the use of a Physics Enhanced Latent Space Variational Autoencoder, where the thermal image data can be represented by a variable-resolution latent space enabling to incorporate more details of the blank than just die mean temperature. This could potentially further improve the level of detail within the anomaly detection analysis.

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