Longitudinal Motion Modeling and Experimental Verification of a Microrobot Subject to Liquid Laminar Flow

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Abstract—This study presents an untethered magnetic manipulation technique for controlling a microrobot position under high rate laminar flows up to 4.5 mL/min. An increase in flow rate exponentially increases the drag force on the microrobot and negatively impacts its positioning accuracy. Increasing the longitudinal force generated by the microrobot’s driving apparatus helps in overcoming the disruptive effects of the fluid flow and increases longitudinal motion stability. To this end, we propose a magnetic configuration with two ring-shaped magnets, one above and the other below the microfluidic channel. This configuration causes the magnetic field lines emanating from the ring-shaped magnets to converge on both sides of the microrobot. Thus, the magnetic trapping forces that hold the microrobot in position are increased. To the best of our knowledge, no prior study exists on investigating the longitudinal motion for high flow velocities (> 5 mm/s). Investigating the longitudinal forces (along the x-axis) that affect a magnetically-driven microrobot is a novel research topic that has many potential application areas such as cell research, micromanipulation, and lab-on-a-chip systems. The microrobot’s dynamical motion is modeled as a second-order system, and using this model as a guideline, we demonstrate the ability of a microrobot in a square-shaped microfluidic channel (900 µm × 900 µm) to follow a linear trajectory with a relative velocity up to 132.6 mm/s. A straight and longitudinal trajectory of 4000 µm has been successfully followed in the same and opposite directions to the flow for different flow rates (1-4.5 mL/min) and different robot speeds (10-50 mm/s).

I. INTRODUCTION

Recently, microrobots have been used widely for a variety of applications such as cell-driven drug transportation [1], artificial insemination [2], measuring systems [3], microsurgery [4] and cellular treatment [5]. In particular, microrobots have been used in targeted drug transportation applications within a microfluidic channel and noted for its many advantages in this domain [6]. Among these advantages are: ease of use for biomedical applications, energy efficiency, and minimal collateral damage at the operated region [7]. Smaller length scales of microfluidic environments render the inertial terms of viscous force and surface tension significant. Thus, these terms should be taken into account in microfluidic applications [8]. However, at small length scales, precise position control of the microrobot becomes more important [9]. A comparison of different propulsion methods for precise positioning of microrobots can be found in a study conducted by Kosa et al. [10].

Different microrobot levitation techniques have been employed to achieve precise control ability in challenging conditions. Examples of such techniques are: electromagnetic [11], [12], [13], [14], [15], permanent magnet-based [16], ultrasonic [17], air cushion [18], thermally actuated [19], MRI driven [20] and optically driven [21] levitation. These techniques have been used under a variety of working conditions such as in air [22], in different liquid environments [23], and in channels with different flow characteristics [11]. In these studies, the effects of drag force and Re (Reynolds number) on the microrobots during their locomotion and motion profiles were examined. On the other hand, force measurement on a microswimmer was also studied for the different Reynolds numbers [24].

Many microfluidic applications, such as cell separation and sorting, cell immobilization, and CTC (circulating tumor cell) detection require a continuous flow to be present. For a microrobotic platform to be functional in the context of such applications, it needs to be able to withstand the disturbance due to flow inside the channel. Such a platform holds the potential to be used as an analysis platform upon which many techniques can be developed. A microrobot, which is capable of precise positioning in a time-varying flow regime, can investigate large amounts of sample in a very short time and enable cell reuse for downstream applications [25], [26], [27], [28]. Levitation and orientation of a microrobot under higher flow rates in lab-on-a-chip platforms have the potential to be used in cell-culture and cell-harvesting applications. In previous studies on the use of microrobots inside microchannels, the fluid flow was either stationary or had a very low velocity (~ 557.4 mm/s at up to 35 mL/h) [29], [30]. Since the levitation techniques used in these studies can’t generate sufficient longitudinal forces on the microrobot, they can’t resist the exponentially increasing drag force of the flow in higher velocities. Therefore, the development of a levitation technique that can generate higher longitudinal forces is needed to increase the precision of locomotion and stabilize the robot against higher flow rates.

Khalil et al. studies the locomotion of an object in a microfluidic channel in the creeping flow regime (Re < 1) [29]. Their electromagnetic configuration for levitation was used to examine the effects of different controller techniques on the robot’s speed, rise time, and motion sensitivity. Although rate of the applied flow is low (< 0.6 mL/min), the positioning error is reported as ≈16 µm. Belharet and his team manipulated a spherical microrobot (φ 500 µm) in water and glycerin with different viscosity values under laminar flow regime [23]. Applied flow rates varied in the range of 2.8 mL/min and 10 mL/min and the reported...
positioning error was approximately 100\µm. This margin of error renders the accurate positioning of the microrobot impossible. Sanchez and his team used permanent magnets to control a catalytic microrobot movement in a 150\µm wide microfluidic channel [31]. They were able to achieve a longitudinal velocity of 78\µm/s when the velocity of the applied flow was 73\µm/s. However, this technique requires the use of a catalyst for self-propulsion, which compromises the viability of the environment for cell studies and reduces the application’s repeatability. Sitti et al. showed that a robot with dimensions of 250\µm × 130\µm × 100\µm can produce a longitudinal force of 52 nN when it is moved in contact with the surface [32]. When driven in non-contact mode, the robot could only produce a longitudinal force of 1.7 nN.

**Significance:** As a contribution to the literature, we propose a novel permanent magnet (NdFeB) based micromanipulation method which produces higher longitudinal forces compared to previously reported techniques and enables the positioning and locomotion of a microrobot in laminar flow settings ($Re < 2000$). A microrobot driving configuration with pyrolytic graphite and two ring-type permanent magnets increased longitudinal forces acting on an untethered microrobot. The basis of the proposed configuration was the concentration of the magnetic field force lines on the microrobot and formation a force vector on both sides of the robot’s vertical axis. We report our microrobot’s motion characteristics at different flow rates with simulation results and confirm them with experimental results. The proposed methodology enables microrobots to carry out manipulation tasks that require continuous-flow microfluidic applications. The ability to withstand longitudinal disturbance forces will allow more robust and effective cell manipulation methods to be developed.

**II. MATERIALS AND METHODS**

In this section, the proposed micromanipulation technique is described in detail. This technique is based on the interaction of a ferromagnetic microrobot, two ferromagnetic outer magnets and a diamagnetic pyrolytic graphite layer. It enables the positioning of a microrobot inside a microfluidic channel in which a continuous laminar flow is present. In Figure 1-A shows the experimental setup in an isometric view. Initially, the microrobot is levitated at the designated starting position, and then a steady-state flow is applied to the channel using a syringe pump. Displacement of the microrobot is then measured by a laser displacement sensor. In Figure 1-B, close-up view of the levitation mechanism is
A sketch that shows the primary forces acting on the microrobot for the proposed dynamic model. Here the fluidic forces are modeled as a damping element, and the magnetic forces are modeled as spring elements due to the stabilizing nature of the pyrolytic graphite.

A model of the proposed levitation technique is given in Figure 2. All calculation steps and assumptions to obtain the corresponding mathematical model are given in detail in this section. Also, free-body diagram of the system, calculation of the forces acting on the microrobot (Figure 3) and longitudinal motion model (along the x-axis) are described. Throughout the manuscript, longitudinal motion is the motion of the microrobot along the x-axis, which corresponds to the length of the robot and vertical motion is motion along the z-axis, which acts in the direction of gravitational forces.

### A. CAD Design

In the proposed microrobot manipulation technique, the flow inside the channel is generated by a syringe pump. The position of the microrobot is measured using the laser displacement sensor. The CCD camera, coupled with the optical microscope, is used to obtain the channel’s side-view. The motorized linear stage controls the vertical position of the upper and lower magnets. Controlling the vertical position allows us to adjust the microrobot’s levitation height inside the channel. A detailed view is shown in Figure 1-B. The pyrolytic graphite layer generates a repulsive force on the microrobot due to its diamagnetic properties ($\mu_r = 0.999991$). The lower magnet is used to increase the longitudinal forces on the robot. With the lower magnet present, the vertical force that the upper magnet needs to generate is higher. The servomotors are used to control the orientation of the upper and lower magnets in 2-DOF, which enables us to control the orientation of the microrobot.

### B. Free Body Diagram

A free body diagram of the proposed technique during longitudinal motion (along the x-axis) is presented in Figure 3. In this figure, the two lifter magnets were positioned above and under the channel and aligned along the z-axis. They are N52 grade (Remanence $B_r = 1.43T$, no coating, axially magnetized, Hangzhou YangYi Magnetics Co., Ltd) NdFeB and have dimensions of $20\mu m \times 40\mu m \times 8mm$. N52 grade was used due to its higher concentration of magnetic flux compared to other grades. The N52 grade robot which is called “carrier magnet” has dimensions of $250\mu m \times 250\mu m$. The microfluidic channel has a square-shaped cross-section with the dimensions of $900\mu m \times 900\mu m$. The pyrolytic graphite on the surface is $7mm \times 30mm \times 0.5mm$.

It was expected that the levitated microrobot would not align itself in its initial position due to the drag force caused by the flow applied along the longitudinal axis (-x axis). Thus, microrobot displacement is measured along with the upper lifter and lower lifers magnets center, which is shown as “centerline” in Figure 3. The microrobot would have a certain displacement during flow, $\tau$, relative to the magnets’ centers. The angles between the upper lifter magnet and lower lifter magnet with the “centerline,” which is due to this displacement, were denoted as $\alpha$ and $\beta$, respectively.

In Figure 3, $F_p$ represents the diamagnetic repulsive force generated by pyrolytic graphite, $F_m$ the magnetic force, $F_b$ the buoyant force, $F_g$ the gravitational force. The drag force, $F_d$, is shown at a distance, $\tau$, from the microrobot center. This offset is a result of the drag force profile acting on the microrobot due to the flow in the channel. This profile, which has a parabolic characteristic, produces different drag forces at different points on the robot surface (Supplementary materials Figure 1 and Figure 2). By averaging these velocities the microrobot surface; a single resultant force can be obtained which is is formed a few microns (denoted as $\tau$).
TABLE I: Parameters of the microrobotic setup

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_r )</td>
<td>Robot density</td>
<td>7400kg/m³</td>
</tr>
<tr>
<td>( m_r )</td>
<td>Robot mass</td>
<td>9.081 \times 10^{-8} kg</td>
</tr>
<tr>
<td>( V_r )</td>
<td>Robot volume</td>
<td>1.2272 \times 10^{-11} m³</td>
</tr>
<tr>
<td>( A_c )</td>
<td>Projected surface area</td>
<td>4.9087 \times 10^{-8} m²</td>
</tr>
<tr>
<td>( F_b )</td>
<td>Buoyant force</td>
<td>0.7703\mu N</td>
</tr>
<tr>
<td>( F_g )</td>
<td>Gravitational force</td>
<td>0.8904\mu N</td>
</tr>
<tr>
<td>( \mu_r )</td>
<td>Magnetic permeability</td>
<td>0.999991</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>Vacuum permeability</td>
<td>1</td>
</tr>
<tr>
<td>( B_r )</td>
<td>Remanence field</td>
<td>1.43T</td>
</tr>
<tr>
<td>( \chi )</td>
<td>Magnetic susceptibility (⊥)</td>
<td>450 × 10^{-6}</td>
</tr>
</tbody>
</table>

...down from the center of gravity of the microrobot. Details of the calculation of this force is given in Section II-C. According to free body diagram buoyant and gravitational forces are, \( F_b = V_r(\rho_r - \rho_f)g \) and \( F_g = m_r g \), and the moment force is:

\[
M_r = F_d r = I_r \ddot{\theta}
\]

where \( I_r = m_r r^2 \) and \( \ddot{\theta} \) represents orientation angle of microrobot to the pyrolytic graphite surface and \( \theta \) represents angular velocity. The orientation angle of the microrobot can be calculated as

\[
\dot{\theta}(t) = \int \ddot{\theta}(t) = \int \frac{F_d}{m_r r^2}(t)^2
\]

To calculate \( \ddot{\theta} \), \( F_d \) and \( r \) must be known. Detailed calculations are shown in Section II-C. Also, for the calculation of \( F_m \) and \( F_p \), no explicit analytic formulas were reported [33]. However, detailed calculation steps and the analysis of magnetic forces exerted on microrobots in a fluid medium, was previously reported [8], [34], [35]. The robot was located above pyrolytic graphite and balanced with two lifter magnets above and below the channel as described in Section II-A. In this model, the effects of flow and additional lower lifter-magnet were also added to the mathematical model in accordance with previous works. Brownian and atomic forces are neglected in our model since their effects on the microscale are insignificant (< 1nN) [36].

C. Drag Force Calculation

For calculating the drag force, the cross-sectional area of the microrobot during longitudinal movements should be used. Reynolds number was determined based on (3) by considering flow rate and microrobot speed.

\[
Re = \rho_f D v / \mu
\]

Here, density of the fluid is denoted by \( \rho_f = 998.29 \text{ (kg/m}^3 \text{ at 25°C)} \), hydraulic diameter by \( D = 0.9 \text{ mm} \) for square-shaped channel, fluid velocity by \( v \), and dynamic viscosity by \( \mu = 0.001003 \text{ Pas} \). Laminar flow conditions begin to deteriorate when \( Re > 2000 \), for which the fluid velocity is calculated as \( v = 2232.7 \text{ mm/s} \). For fluid velocities below this value, obtained Reynolds number guarantees that the flow regime stays in the laminar region. Drag force coefficient \( c_d \) was calculated in (4).

\[
c_d = 24 \frac{d_A}{Re} 
\begin{array}{c}
1 + \frac{0.15}{Re} \left( \frac{d_A}{d_n} \right)^{0.687} \\
0.42 \left( \frac{d_A}{d_n} \right)^2 \\
1 + 4.25 \times 10^{-4} \left( \frac{d_A}{d_n} \right)^{-1.16} \\
\end{array}
\]

(4)

where \( d_A = \sqrt{4A_c / \pi} \) denotes spherical diameter, \( d_n = \sqrt{6V_r / \pi} \) denotes nominal diameter, and \( c \) denotes the surface sphericity (shape factor) [33]. Calculating these values accordingly, we obtain \( d_A / d_n = 0.8736 \) and \( c = 0.5531 \). Using these values and substituting (3) in (4) for different velocities depending on the respective \( Re \) value gives the graph shown in Figure 4-A. Since we are working in a laminar flow regime, variations in the \( c_d \) value are large at lower flow rates (\( c_d \approx 0.49, Re > 1000 \)). Therefore, assuming that the working speed of the robot is in the range of 0–100 mm/s, the relevant region of the \( c_d \) graph is shown in more detail at Figure 4-B. Drag force can be calculated by using the \( c_d \), which is calculated in this figure and for different microrobot speeds, \( v_r \), according to (5).

\[
F_d = 0.5c_d \rho_f A_c v_r |v_r|
\]

(5)

To determine the robot orientation described in Section II-B, the distance between the microrobot center and the...
center of drag force, \( r \), needs to be calculated (as shown in Figure 3). This calculation is done for 100 \( \mu m \) levitation height, and 1-5 mL/min flow velocity where no-slip boundary conditions were applied (Supplementary material Figure 1). Additionally, mean velocity values are calculated for each flow rate. The \( r \)-value is assumed to be unaffected by changes in flow rate because the standard deviation of this parameter over each flow rate value was determined as \( \pm 0.089 \) mm/s. The maximum error value was thus calculated as \( \% 0.3 \), which is a negligible value (Supplementary material Figure 2).

D. Magnetic Force Calculation

Various formulas in the literature can be used for determining the interactions between identical magnets; however, they can’t be utilized for non-identical magnets, as in the case of our configuration [37]. The following procedure was used for the calculation of the magnetic forces between the carrier magnet and the lifter magnets. Firstly, the ring-shaped lifter magnets (Figure 5-A) were both assumed as two disc-shaped magnets. The outer diameter of one of these magnets was equal to the outer diameter of the original magnet. The outer diameter of the other magnet was equal to the inner diameter of the original magnet. The magnetic field around the microrobot was calculated for both disc-magnets. The geometric mean of the magnets’ resulting magnetic fields was then taken to determine the particular magnetic field of a ring-magnet. As such, the magnetic force acting on the microrobot (Figure 5-B) was determined by taking the mathematical integration of the magnetic field magnitude over the microrobot body (9).

The lifter and carrier magnets were illustrated in Figure 5-A and 5-B respectively. The remanence magnetic field changes with the distance from magnets’ center and can be formulated as in (6) and (7). The magnetic field on the symmetry axis of an axially magnetized ring magnet, \( B_h \) [38] is,

\[
B_h = \frac{B_r}{2} \left( \frac{D_r + z}{\sqrt{R^2 + (D_r + z)^2}} - \frac{z}{\sqrt{R^2 + z^2}} \right)
\]

where \( z \) is the distance from the center of the magnet. The magnetic field on the symmetry axis of an axially magnetized cylinder magnet, \( B_h \) [38] is,

\[
B_h = \frac{B_r}{2} \left( \frac{D + z}{\sqrt{R^2 + (D + z)^2}} - \frac{z}{\sqrt{R^2 + z^2}} \right)
\]

To ensure the stability of the microrobot at the levitation point, following criteria (8) must be met,

\[
\nabla^2 U = -\frac{\chi V}{2\mu_0} \nabla^2 B^2 > 0
\]

where potential energy is represented by \( U \), vacuum permeability by \( \mu_0 \), magnetic susceptibility by \( \chi \), and volume by \( V \). Only a diamagnetic material can satisfy this stability condition [39]. The attractive magnetic force, which was applied by a lifter magnet, acting on the microrobot, was expressed in (9) in a volumetric form [8], [34], [40],

\[
F_m = \frac{\nu_r - 1}{2\mu_0 \mu_r} \iiint \nabla B^2 dv
\]

By using parameters in Table I, the net magnetic force required for the levitation of the microrobot on \( z \)-axis can be calculated by (10),

\[
F_t = F_p + F_{m,u,z} \cos \alpha - F_{m,l,z} \cos \beta
\]

where \( F_t = 0.120 \) \( \mu N \). To obtain \( F_t \), it was necessary to move the upper and lower lifter magnets vertically to the pyrolytic graphite surface in the configuration shown in Figure 1-B. However, if these two magnets were equidistant from the microrobot, the total magnetic force would be 0 N. The net magnetic force acting on microrobot can be calculated by using (6), (7), (8), (9), and it is shown as a surface graph in Figure 6. Here, the \( x \)-axis represents the lower magnet’s displacement from its initial position. The \( y \)-axis represents the initial distance between upper and lower magnets and pyrolytic graphite’s upper surface at the beginning of the parameter sweep with a constant offset value. In the simulations, the upper and lower magnets initially had the same distance to the microrobot such that the net magnetic force on it was zero. Then, the lower magnet was displaced downward with \( 1 \) \( \mu m \) steps in the range of 0-10 mm. This displacement causes the net force on the microrobot to increase exponentially at each step. The \( F_t \) value, calculation of which is given in (10), corresponds to the net magnetic force required to achieve levitation. The microrobot can be levitated at every point on the surface graph shown in Figure 6 corresponding to this value (0.120 \( \mu N \)) on the \( z \)-axis. Thus, it can be observed that levitation can be achieved for different initial distance and offset values.
Fig. 6: The surface graph obtained as a result of the net magnetic force calculation is shown. The x-axis is the distance between the magnets and the graphite; the y-axis shows the offset applied to the lower positioned lifter magnet; the z-axis shows the total magnetic force values. The bar above the figure shows the color map of the force change between 0 – 0.8 μN.

E. Mathematical Model

In this section, a dynamic motion model of the longitudinal motion of the microrobot is given. The conceptual diagram of the microrobot manipulation technique is illustrated in Figure 1-B. The model of the proposed technique is illustrated in Figure 2. This system yields a second-order mathematical model for which the free-body diagram is given in Figure 3. In this model, the microrobot is modeled as a mass. The microrobot and fluid interactions are modeled as a damping element. The interactions between magnets, pyrolytic graphite, and microrobot are modeled as a spring element. Damping and spring elements represent the liquid media’s hydrodynamic structure and the interactions of permanent magnets with diamagnetic material, respectively.

The spring nature of the magnetic forces can be understood by observing the vertical levitation distance between graphite and microrobot. When microrobot moves downwards (z-direction in Figure 2), pyrolytic graphite generates a higher lifting force on it. Thus, the microrobot can move to its stable position. Similarly, if the microrobot moves upwards, a decrease in the lifting force can compensate for this disturbance and pull the robot downwards.

When the flow is applied through the microrobot, it causes a fluidic pressure on its surface. So the microrobot moves longitudinally (through -x-axis) to the centerline. As the microrobot translates to this pressure, its movement is damped by the water due to its viscosity. The effect of fluidic medium on magnetic interactions can be neglected since the DI-water’s magnetic permeability is almost identical to that of air ($\mu_r = 0.999991$). Consequently, drag force exerted on microrobot’s surface due to applied flow can be modeled using a damping element. Since the system’s dynamic model is constructed using mass-damping-spring elements, the transfer function of a general second-degree system can be used to model the system’s dynamic response, as shown in (11), where $\tau$ is the displacement on the x-axis.

$$\ddot{\tau}(t) + \frac{c_s}{m_r} \dot{\tau}(t) + \frac{2k_s}{m_r} \tau(t) = F_0(t) \quad (11)$$

Here, $m_r$, $c_s$, $k_s$ and $F_0$ represents mass, damping, spring coefficients, and disturbance respectively. For this dynamic system; damping ratio, $\zeta$ (12) and natural resonance frequency, $\omega_n (rad/s)$ (13), can be calculated as,

$$\zeta = \frac{2k_s}{2\sqrt{k_sm_r}} \quad (12)$$

$$\omega_n = \sqrt{\frac{k_s}{m_r}} \quad (13)$$

where microrobot mass is $m_r = 9.081 \times 10^{-8}$ kg. Viscous damping coefficient acting on a particle within the channel which has a height of $h = 900 \mu m$, can be determined using (14) [41].

$$c_s = \mu A_c/h = 5.4705 \times 10^{-8}(Ns/m) \quad (14)$$

where the projected surface area of the microrobot $A_c = 4.9087 \times 10^{-8} m^2$. According to (11), $k_s$ is required to determine the dynamic response of the system. In this case, the relationship between the longitudinal force acting on the robot and the robot distance to upper and lower magnets center, $\tau$, should be calculated. In this way, microrobot moves away from the center of the lifter magnets up to a certain distance, and it gives the longitudinal force as shown for the fixed levitation height $h = 100 \mu m$ in Figure 7. In this figure it can be seen that the longitudinal magnetic force has a linear characteristic for small displacements (up to 7.5 mm). This plot has an exponential characteristic when the microrobot moves away from the center of the lifter magnet greater than 7.5 mm. So, the second spring coefficient should be used by curve fitting in the first order. That is why we use two spring coefficients for displacements lower than 7.5 mm and between 7.5 mm and 15 mm by using (15).

$$k_s = F_l/\tau \quad (15)$$

where $F_l$ is the longitudinal force which can be seen in Figure 7. In our case, $k_s = 21.675 \text{ nN/mm}$ and $k_s = 17 \text{ nN/mm}$ is calculated, respectively. After determinations of $c_s$ (14) and $k_s$ (15) are completed, we can assume that a single external force is applied and equations of the microrobot motion for both values of $k_s$ are obtained from (11) as,

$$\ddot{\tau}(t) + 0.602\dot{\tau}(t) + 238.69\tau(t) = F_0(t) \quad (16)$$

$$\ddot{\tau}(t) + 0.602\dot{\tau}(t) + 187.13\tau(t) = F_0(t) \quad (17)$$

where the maximum disturbance amplitude is $F_0$ and robot mass, $m_r = 9.081 \times 10^{-8}$ kg. To obtain the transfer function of the robot motion, time domain equations in (16) and (17) should be converted to the frequency domain. When initial
The longitudinal force on the microrobot increases linearly as it moves away from the center of the lifter magnets to a certain distance. The longitudinal magnetic force can be modelled as a spring element due to its linear relationship with displacement. In light of these results, we see that for displacements lower than 7.5 mm, the spring coefficient can be taken as $k = 21.675$ N/mm. If microrobot displacement is measured greater than 7.5 mm, the spring coefficient is switched to $k = 17$ N/mm. If microrobot displacement is measured greater than 7.5 mm, the spring coefficient is switched to $k = 17$ N/mm.

The responses of these transfer functions to different disturbance amplitudes, $F_0$, due to fluid flow are given in Figure 8. The natural resonance frequency of (16) and (17) and the settling time of the system were calculated as $\omega_n = 2.458$ Hz, $\omega_n = 2.177$ Hz and $t_s = 3.145$ sec respectively.

The drag force values for different robot velocities are calculated using (5). Subsequently, the $r$ value is substituted in (1) and $\theta$ values are calculated using (2). The orientation angles for flow velocities between 1-4.5 mL/min with 0.5 mL/min steps are calculated as $0.643^\circ$, $0.598^\circ$, $0.579^\circ$, $0.578^\circ$, $0.571^\circ$, $0.489^\circ$, $0.494^\circ$ and $0.436^\circ$ respectively. These angles were calculated for the robot’s steady-state dynamic solution. As such, the theoretical longitudinal positioning error was determined as $11.015 \pm 3.231$ nm. Additionally, a positioning error of $1.161 \pm 0.178 \mu m$ was calculated in the z-axis due to the deterioration in the parallelity of the microrobot to the surface. However, since the position of the mass center of the robot and the height of it’s geometrical center doesn’t change the effect of this orientation during the robot’s movement can be ignored.

III. SIMULATION RESULTS

In this section, drag force coefficient, $c_d$, was obtained by using a Finite Element Method (FEM) based program, COMSOL® in order to confirm the analytical results (Figure 4). Overlapping of analytical and simulation results show the accuracy of the proposed model. For this reason, it is important to determine the relationship between Re and $c_d$ for calculating the drag force more accurately since the rest of the parameters are known in (20). A parametric and time-dependent analysis was performed for different flow rates from 0 mL/min to 10 mL/min. In this analysis, fluidic flow around the microrobot was simulated in a square-shape channel (900 $\mu m \times 900$ $\mu m$). No-slip boundary condition is considered in the channel walls (Hydrodynamic entrance length is also given in supplementary material Figure 3). All simulations conducted with a workstation with an AMD Threadripper 1950x (16 real cores) processor with 96 GB RAM and a Windows-10 64-bit operating system using the software COMSOL® Multiphysics Version 5.3 (CPU License No: 17076072).

In this analysis, to compare $c_d$ with the analytical results, the microrobot position was kept constant in the channel. The microrobot is assumed to be stationary in the channel. It is a valid simplification since $c_d$ is only affected by the relative velocity between the fluid and microrobot when the cross-sectional area is fixed. Therefore, analyzing a range of flow rates also accounts for when the robot moves inside the channel with the same relative velocity. Assuming the robot is stationary helps reduce the computational time and decrease the complexity of analysis. Drag force coefficient can be calculated by integrating surface tension, $F_d$, on the robot cross-sectional area, and rest in (20).

$$c_d = 2F_d/\left(\rho_f A_c v_r |v_r|\right)$$  \hspace{1cm} (20)

In Figure 9, the variation of the flow rate in the channel (longitudinal direction) was isometrically shown when the flow rate was selected as 2.5 mL/min. The right and left ends of the channel parallel to the y-z axes refer to the
The analysis result for a flow rate of 41.6 mm/s is shown. The velocity field reaches its maximum at the center of the channel due to the no-slip boundary condition (B) and (C).

inlet and outlet, respectively. No-slip boundary conditions and gravity are also considered. The black arrows on the figure showed the flow direction, and the characteristic of flow and the velocity vectors at the level of the microrobot was shown in the top view (B) and front view (C) in detail. In (C), it can be observed that the velocity profile acting on the microrobot has exponential characteristics, and it generates a non-symmetrical drag force on the cross-sectional body of the robot. In this way, a different magnitude of the drag force is produced on each point on the robot surface. That is why drag force is shown acting on the robot a few microns down from its center of gravity in Figure 3. The change of drag force coefficient with Re is shown in Figure 10, which gives the results of the parametric analysis shown in Figure 9. The parametric analysis was performed for every 0.5 mL/min step in the range of 0–10 mL/min. At each step, \( F_d \) and \( v_r \) were substituted in (15) and \( c_d \) was calculated. The results obtained as such are shown in Figure 10. According to the theoretically calculated \( c_d \) values shown in Figure 4-B, it can be observed that there was an absolute average error value of ±0.0388 (2.26%) in simulation. On the other hand, both theoretical and simulation results were converged to the same \( c_d = 0.49 \) at higher Re (Re>2000).

The microrobot’s dynamic behavior inside the fluidic channel is simulated using COMSOL® to determine its motion pattern and compare it with the analytical results (Supplementary material video-1). In addition to Figure 9, the moving mesh, deformable-body, laminar flow, and fluid-structure interaction modules of the COMSOL® are utilized in longitudinal motion simulation (Supplementary material Figure 4 and detailed Figure 5.)

**IV. EXPERIMENTAL RESULTS**

After analytical and simulation results were obtained, experiments were conducted to analyze microrobot longitudinal motion in the laminar flow regime with the proposed setup. The microrobot’s longitudinal motion characteristics and immobilization response were studied when the lifter magnets moved towards or against the flow direction. In section II-E, the dynamic behavior of the microrobot was studied based on the \( \tau \) and longitudinal force calculations. Here, the microrobot motion and its accuracy at different speeds were investigated under different flow rates. In the first experiment, the microrobot longitudinal motion characteristics were evaluated at different flow rates ranging from 0.5 mL/min to 4.5 mL/min with 0.5 mL/min intervals when microrobot velocity is zero (Figure 11). An optical microscope system records the longitudinal position of the microrobot, and its displacement is measured using the laser displacement sensor given in Figure 12. Due to the nature of streamflow and magnetic field, the displacement increased exponentially for higher flow rates. The experiment was repeated 10 times to demonstrate the repeatability of the proposed methodology. Although the microrobot was resistant to flow at 4 mL/min, levitation stability began to deteriorate at this flow rate. For this reason, the detachment point of the microrobot was also determined. It was calculated that the distance from the detachment point to the starting position was 15152.2 \( \mu \) m. At this point, upper lifter magnet is 19.7 \( \mu \) m and lower carrier magnet is 22.8 \( \mu \) m away from the graphite surface. All experiments were performed with the robot at a constant levitation height of 100.0 \( \mu \) m (Supplementary material Figure 6).

Here, the oscillation response of the microrobot depending on varying flow rates was reported. At flow rates higher than 2.5 mL/min, an oscillation response with increasing amplitude and a less stable motion was observed as shown in Figure 11-E, F, G, H. Because of non-contact motion and low environment stiffness, the vibrational response of the microrobot approaches the resonance frequency of the system [42]. From the robotic perspective, another reason
Fig. 11: The oscillation amplitude of the microrobot subject to laminar flow are presented. When the microrobot reaches its final position, it exhibits an oscillatory motion characteristic due to the interaction of drag and magnetic forces. The oscillation amplitude varies depending on the applied flow rate. The amplitude of oscillation relative to the resting position is shown in (A) 1mL/min, (B) 1.5mL/min, (C) 2mL/min, (D) 2.5mL/min, (E) 3mL/min, (F) 3.5mL/min, (G) 4mL/min, (H) 4.5mL/min. For lower flow rates the oscillation amplitudes were insignificant (Supplementary material video-2).

for not holding microrobot at a single location under a constant stream flow was that our system did not have any active or passive controller. The continuous oscillation hindered our ability to position the microrobot precisely. The microrobot had less stable characteristics due to higher oscillation amplitudes at higher flow rates.

The second experiment was conducted where the microrobot was moved 4000 µm longitudinally towards and against the flow. It is shown from the side-view in Figure 13. Initially, the microrobot was in a levitation state at the starting position. The first oscillation was observed with the flow, and it was successfully moved to 4000 µm away from its initial position. After 4000 µm displacement, the microrobot demonstrated the same oscillation interval. By the same scenario, Table II was obtained by measuring the longitudinal displacement of the microrobot subject to different flow rates when it was intended to move 4000 µm. Each value at the intersection of each row & column was averaged over 10 experiments. Here, there is no data available for when the flow speed reaches the detachment point (4.5 mL/min) and microrobot speed reaches 50 mm/s. In Table II, this point was represented as "x".

For different microrobot speeds (10 – 50mm/s), each distance data was grouped under the respective flow rates. These rates were then shown in Figure 14 with means ± SD (Standard Deviation) values. Data were normalized by flow rate, expressed as means ± SD, and compared to the microrobot speed by 1-way ANOVA and unpaired T-test (Statview v.5.0, SAS, Cary NC). A P value of < 0.05 was considered significant. The results show no major difference between the displacements obtained from different flow rates since P was found as 0.46. The minimum error measured for the longitudinal displacement of 4000 µm for flow rates higher than 2.5mL/min was 0.951% at 2.5mL/min and the maximum was 2.106 % for 4mL/min. Comparison of the displacement values determined from the analytical calculations, simulation results, and experimental measurements and error values in comparison with the experimental results are given in Table III.

V. CONCLUSION

This study investigates the motion profiles of an untethered microrobot subject to laminar flow. For a microrobot that is intended to be moved towards or against the flow direction, the total net magnetic force’s longitudinal component plays a critical role. According to the analytical, simulation, and experimental results, the magnetic field was intensified by our new micromanipulation method. Consequently, the microrobot successfully followed a linear trajectory of 4mm length back and forth at a flow rate of 4.5mL/min (92.6mm/s) and with a velocity of 40 mm/s. At a velocity of 50mm/s, the levitation of the microrobot was distorted. The distance of this point, at which detachment occurred, to the starting point, was calculated as 15152.2µm. The results

<table>
<thead>
<tr>
<th>Flow Rate (mL/min)</th>
<th>Microrobot Speed (mm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4015.4 4024.9 4004.1</td>
</tr>
<tr>
<td>1.5</td>
<td>3996.6 3988.8</td>
</tr>
<tr>
<td>2</td>
<td>4017.3 4010.2</td>
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<tr>
<td>2.5</td>
<td>4084.9 4020.7</td>
</tr>
<tr>
<td>3</td>
<td>4054.1 4042.5</td>
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<td>4032.5 4026.1</td>
</tr>
<tr>
<td>4</td>
<td>4081.8 4069.1</td>
</tr>
<tr>
<td>4.5</td>
<td>4071.9 4054.1</td>
</tr>
</tbody>
</table>

Table II: Microrobot 4000µm longitudinal motion experimental results

Fig. 12: Displacement of the microrobot for different flow rates as measured by the laser displacement sensor.
TABLE III: Comparison of the displacement values determined from the analytical calculations, simulation results and experimental measurements are given at the upper part of the table. Error values in comparison with the experimental results are given at the bottom part of the table.

<table>
<thead>
<tr>
<th>Flow rate (mL/min)</th>
<th>Analytical</th>
<th>FEM Simulation</th>
<th>Experimental</th>
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<tbody>
<tr>
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</tr>
<tr>
<td>1</td>
<td>1119.7</td>
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<table>
<thead>
<tr>
<th>Error (%)</th>
<th>Analytical</th>
<th>FEM Simulation</th>
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</tr>
<tr>
<td>4</td>
<td>6.98</td>
<td>6.92</td>
</tr>
</tbody>
</table>

ACKNOWLEDGMENT

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REFERENCES


Fig. 13: Presents a side view of this experiment at a flow rate of 2.8mL/min and a microrobot speed of 50mm/s. From the initial position, the microrobot traveled 4mm in the longitudinal axis. From its initial position to the final position, the microrobot performed oscillatory characteristics.

Fig. 14: Standard deviation values of all measured longitudinal motion results with means for microrobot speeds from 10mm/s to 50mm/s are expressed. Although different flow rates and microrobot velocities are applied, displacement and error amounts are similar according to $P < 0.05$ by 1-way ANOVA (p=0.46).

