A Novel Rotational Actuator With Variable Stiffness Using S-shaped Springs

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Abstract—In this paper, we propose a novel rotational variable stiffness actuator that uses S-shaped springs as elastic elements. The S-shaped springs are designed based on the principle of variable beam length, which allows for an easily customizable and wide-ranging stiffness regulation from a minimum value to near-infinite. The stiffness can be smoothly regulated online by rotating the S-shaped springs, resulting in a compact stiffness regulation mechanism. Moreover, a planetary gear differential is used to drive the stiffness regulation mechanism such that both the stiffness control motor and the main drive motor can be placed at the proximal part. This configuration effectively reduces the reflected inertia of the moving parts, which helps to achieve a compact design and improve the control performance. The proposed actuator ranks high among designs of the similar output power in terms of the power-to-weight ratio and the power-to-volume ratio; therefore, the compactness of the proposed design is verified. Besides, the stiffness regulation speed of the proposed design also ranks high among the designs. Consequently, the actuator can be used for fast stiffness regulation. For robust position tracking under different operating conditions, a disturbance observer is used to estimate the mismatched lumped disturbance caused by the load torque and model uncertainties. To further improve the control performance, a sliding mode control term is introduced to compensate for the disturbance estimation error. The stability of the closed-loop system is proved using the Lyapunov method. Experimental results under different operating conditions have verified the validity of the novel actuator and the proposed controller.

Index Terms—Stiffness regulation mechanism, S-shaped spring, trajectory tracking, variable stiffness actuator.

I. Introduction

COMPLIANT actuators have been widely used in applications where safe human-robot interaction or shock absorption is critical. The serial elastic actuator is a common approach to achieve actuator compliance due to such merits as easy implementation and large energy storage capacity [1]. However, its performance (e.g., in terms of dynamic behaviour and energy storage capability) heavily relies on the stiffness of the fixed spring, which severely limits its adaptability to different applications. As an emerging way to overcome this limitation, variable stiffness actuators (VSAs) can achieve optimal behaviors for a wide range of applications by adjusting the spring stiffness [2], [3]. They are at least valuable, if not irreplaceable, for shock absorption, cyclic movements, and explosive movements [2], [4], [5].

Various working principles, such as equilibrium controlled stiffness, bio-mimetic antagonistic controlled stiffness, structure-controlled stiffness, and mechanically-controlled stiffness, can be used to regulate the stiffness [6]. Antagonistic controlled stiffness method is inspired by the human muscle system [7]. In this method, position control and stiffness regulation actuators are placed in an opposed manner at the joint, and each actuator can pull the joint and exert torques in one direction. When both motors pull the joint, stiffness increases due to the increasing actuator stiffness. Representative designs of this kind include the BAVS [8] and FAS [9]. Although superior performance can be achieved with ingenious designs using this method [10]–[12], energy is needed to regulate the stiffness even when the joint is not loaded [13], and it is generally challenging to achieve near-infinite stiffness.

In contrast, the mechanically-controlled stiffness method allows stiffness regulation from a minimum value up to infinity, thereby ensuring wide-ranging adaptability for a diversity of applications [14]. Two well-known designs, the MACCEPA [15] and VS-Joint [16], are prominent representatives of this principle. In both designs, the compliance and equilibrium position can be independently controlled. However, compliance is regulated by adjusting the preload of the elastic element, which leads
to considerable energy consumption because the spring deflection is an additional load on the motor [6]. An effective way to solve this problem is to use the concept of the lever arm, in which the pivot point, the location of the spring, or the force point is moved to regulate the actuator stiffness. Therefore, stiffness can be adjusted without injecting and extracting energy into and from the elastic elements [17]. Designs based on this concept include the AwAS [18], AwAS-II [14], vsaUT [17], vsaUT-II [19], and CompAct-VSA [20]. In these designs, ball screw mechanism, planet gear mechanism or rack pinion mechanism is used to achieve the translational motion of the moving point, thereby making the stiffness regulation mechanism complex [21]. This limitation is alleviated by the Archimedean spiral relocation approach [22]. However, the stiffness motor is placed at the distal part, which increases the reflected inertia of the main drive system.

The principle of structure-controlled stiffness is another effective way to achieve wide-ranging stiffness regulation [6]. As a simple realization, a rotated leaf spring is used as an elastic element, the rotation of which helps to regulate the actuator stiffness by changing its effective moment of inertia [23], [24]. However, for continuous stiffness regulation, the mechanism should be designed to overcome the problem of lateral buckling [25]. An alternative implementation without considering the intricate buckling effect is to adjust the effective length of the beam [6]. This implementation allows for the independent control of the stiffness and the equilibrium position [26]. In [27], an energy efficient variant of this design is proposed. Two identical actuators are used in parallel to overcome the load torque, and the stiffness is determined by the angle differential of the two actuators. Therefore, both actuators do not need to apply control torques to maintain a constant stiffness. However, an appropriate load-sharing control strategy should be designed to evenly distribute the load torque between the two actuators, which makes the controller implementation an intricate work. A novel VSA using this principle with only one motor is proposed in [28], [29], which ensures low-energy-cost and large-range stiffness regulation. However, the equilibrium position of the actuator cannot be independently controlled, and therefore such designs are suitable for some specific applications, such as human augmentation and prosthetics.

The motivation of this paper is to design a compact and energy-efficient VSA based on S-shaped spring (S-VSA) with a capacity of wide-ranging stiffness regulation. The distinctive features of the proposed design are four-fold:

1) A novel S-shaped spring with two centro symmetric circular beams is proposed, and the stiffness is continuously regulated by adjusting the effective beam length. Compared with designs which regulate stiffness by converting rotation to translational motion [14], [15], [18], [21], [26], [29], [32], the effective beam length adjustment can be easily achieved by rotating the S-shaped spring in the proposed design, which leads to a compact stiffness regulation mechanism.

2) The joint can be completely stiff, which implies a capacity of wide-ranging stiffness regulation. Besides, the stiffness is customizable by reasonably modifying the design variables of the S-shaped spring.

3) When the load or the deflection is zero, the stiffness control motor does not need to apply an additional torque to counteract the spring preload. Therefore, the energy efficiency of the proposed design is higher than designs that regulate stiffness by counteracting the spring preload [8], [15], [32].

4) A planetary gear differential is used to drive the stiffness regulation mechanism such that both two motors can be placed at the proximal part. This configuration effectively reduces the reflected inertia of the moving parts than designs that make the stiffness regulation mechanism as a part of the main drive chain [14], [18], [20], [22], [26], [32], which helps to achieve a compact design.

PID controllers have been widely used for the control of VSAs due to its simplicity [14], [15], [18], [20], [22], [30]. However, these controllers cannot guarantee satisfactory performance in all operating conditions, especially when plant parameters change drastically [33]. In the proposed S-VSA, the reflected inertia of the moving parts are significantly reduced, and therefore better control performance can be expected. To fully exploit the potential of the proposed design and achieve robust control performance under different operating conditions, a nonlinear control strategy is proposed, in which a disturbance observer is used to estimate the mismatched lumped disturbance caused by the load torque and model uncertainties. The disturbance estimation error converges to a ball centered at the origin, the radius of which is determined by the observer gain. In addition, a sliding mode control term is introduced to compensate for the disturbance estimation error. The stability of the closed-loop system is proved using the Lyapunov method. Experimental results under different operating conditions have verified the validity of the novel actuator and the superiority of the proposed controller.

The proposed design has potential usages in numerous applications where adaptable compliance is critical, e.g., legged robots [34], [35], robotic prosthesis [36], [37], and human–robot interaction [31], [32]. The remainder of this paper is organized as follows. Section II presents the working principle and the mechanical design. Mathematical modeling and performance analysis are given in Section III. The control strategy is introduced in Section IV. Experimental studies are given in Section V and comparisons with previous designs are given in Section VI. Finally, conclusions are drawn in Section VII.
II. Design of the VSA

A. Working Principle

Fig. 1 shows the stiffness regulation principle of the proposed design. A novel S-shaped spring with two centrosymmetric circular beams is used as the elastic element. Without loss of generality, a typical actuator configuration with four equispaced S-shaped springs is considered in this paper as shown in Fig. 1(a). Each S-shaped spring is connected with the input frame by a revolute joint which rotates around the cylinder axis of the S-shaped spring. Besides, beams of each spring are in contact with the thrust plates which are fixed on the output frame. The output torque from the input frame passes through the S-shaped spring and becomes the driving torque for the output frame. According to the beam theory, the actuator stiffness can be continuously regulated by adjusting the effective length of the circular beam, which can be easily achieved just by rotating the proposed S-shaped spring around its own axis because of its circular outline. In addition, centrosymmetric beam arrangement ensures identical stiffness settings in both directions of rotation.

Fig. 1(b) shows the case when all the S-shaped springs rotate synchronously in the anticlockwise direction around their own axes. Compared with Fig. 1(a), the contact...
points between the beams and the thrust plates gradually move away from the free end of the beams, and therefore the effective beam length is decreased, implying increased stiffness. Conversely, the effective beam length is increased, thereby leading to decreased stiffness. As shown in Fig. 1(c), when S-shaped springs rotate to a position where the intermediate plates are in contact with the thrust plates, the actuator stiffness becomes near-infinite because the the intermediate plates and the thrust plates can be regarded to be rigid compared with the S-shaped spring. The actuator stiffness in this case depends on the structural flexibility of these plates. Fig. 1(d) shows some variants of the S-shaped spring. By modifying design variables of the spring (e.g., material, geometric shape of the beam, and spring number), the actuator stiffness can be easily customized. Therefore, the proposed stiffness regulation principle can be tailored for various applications.

B. Mechanical Design

Fig. 2(a) shows the drivetrain schematic of the proposed design, and Figs. 2(b)-(e) depict the internal structure of the drivetrain. The actuator consists of the following three parts.

1) Main drive mechanism. The main drive mechanism is shown in Fig. 2(c). The main drive motor is mounted on a base plate, and the motor torque is transmitted to the input frame through the worm drive (Parts: 1-2) and the succeeding gear train (Parts: 3-4). Then, the output torque of the input frame passes through the S-shaped springs and becomes the driving torque for the output frame.

2) Stiffness regulation mechanism. The stiffness regulation mechanism is shown in Figs. 2(d) and (e). The stiffness control motor is also mounted on the base plate, and it drives the central shaft (Part: 8) through the worm drive (Parts: 6-7). After the succeeding planetary gear train (Parts: 9-11) and the timing belt drive (Parts: 12, 14), the stiffness regulation torque is transmitted to the stiffness regulation shafts (Part: 13) that rotate together with the S-shaped springs relative to the input frame, and therefore the stiffness can be regulated.

3) Sensing system. As shown in Fig. 2(b), a contactless rotary encoder is mounted on the input frame to measure the angular displacement of the S-shaped spring $\theta_2$, which can be used to calculate the actuator stiffness. In addition, two magnetic ring encoders are used to measure the displacement of the input frame $\theta_1$ and that of the output frame $q$, respectively. The difference between $\theta_1$ and $q$ is the deflection of the actuator $\gamma$, i.e., $\gamma = \theta_1 - q$.

A prototype of the proposed S$^3$VSA is shown in Fig. 3 and its main specifications are listed in Table I.

Remark 1: The torque flow schematic of the proposed VSA is shown in Fig. 2(f). Both the main drive motor and the stiffness control motor are placed on the base. The stiffness control motor does not need to counteract the main drive motor because spring elastic force is approximately perpendicular to the direction along which the springs are rotated. More specifically, the load torque acting on the main drive motor will cause a resistant torque on the stiffness control motor, which is generally much smaller than the load torque.

Remark 2: When the deflection or the load is zero, to regulate the actuator stiffness, the stiffness control motor only needs to overcome the friction in the drivetrain and accelerate the reflected inertia of the stiffness regulation mechanism because the spring elastic force is approximately perpendicular to the direction along which the springs are rotated, i.e., the spring preload is not an additional load for the stiffness control motor. Therefore, the energy efficiency of the proposed S$^3$VSA is higher than designs that regulate the actuator stiffness by counteracting the spring preload [8], [15], [32].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight (with motors)</td>
<td>1.54</td>
<td>kg</td>
</tr>
<tr>
<td>Weight (without motors)</td>
<td>0.87</td>
<td>kg</td>
</tr>
<tr>
<td>Diameter</td>
<td>125</td>
<td>mm</td>
</tr>
<tr>
<td>Length</td>
<td>118</td>
<td>mm</td>
</tr>
<tr>
<td>Range of rotation</td>
<td>0-360</td>
<td>degree</td>
</tr>
<tr>
<td>Range of stiffness</td>
<td>12.12-infinity</td>
<td>Nm/rad</td>
</tr>
<tr>
<td>Max. continues torque</td>
<td>7.56</td>
<td>Nm</td>
</tr>
<tr>
<td>Peak output torque</td>
<td>22.7</td>
<td>Nm</td>
</tr>
<tr>
<td>Resolution of stiffness regulation</td>
<td>12.64</td>
<td>Nm/rad$^2$</td>
</tr>
<tr>
<td>Full range stiffness regulation time</td>
<td>0.1</td>
<td>s</td>
</tr>
<tr>
<td>Spring rotation range</td>
<td>0-120</td>
<td>degree</td>
</tr>
<tr>
<td>Max. deflection</td>
<td>$\pm12$</td>
<td>degree</td>
</tr>
<tr>
<td>Number of the springs</td>
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<td></td>
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<tr>
<td>Beam thickness</td>
<td>0.4</td>
<td>mm</td>
</tr>
<tr>
<td>Beam width</td>
<td>10</td>
<td>mm</td>
</tr>
</tbody>
</table>

1 The VSA datasheet describing the proposed design can be downloaded from: https://github.com/CX805819/Datasheets/blob/master/S3VSA.pdf.
III. Mathematical modeling of the VSA

A. Dynamic Modeling

The actuator dynamics can be modeled by [8], [38], [39]

\[
M \ddot{q} + C \dot{q} + G(q) + \tau_d(t) = \tau(\gamma, \theta_2)
\]
\[
J_1 \ddot{\theta}_1 + D_1 \dot{\theta}_1 + \tau(\gamma, \theta_2) = u_1
\]
\[
J_2 \ddot{\theta}_2 + D_2 \dot{\theta}_2 + \tau_r(\gamma, \theta_2) = u_2
\]

where \(q, \theta_1, \) and \(\theta_2\) are the angular displacements of the output frame, the input frame, and the S-shaped spring, respectively. \(M, J_1,\) and \(J_2\) are the reflected inertias of the output frame, the input frame, and the stiffness regulation mechanism, respectively, \(C, D_1,\) and \(D_2\) are the equivalent damping coefficients for the output frame, the input frame, and the S-shaped spring, respectively. \(G(q)\) is the gravitational torque acting on the output frame, \(\tau_d(t)\) is a lumped disturbance torque that includes the load torque and model uncertainties such as hysteresis, friction, and internal damping. \(\tau_r(\gamma, \theta_2)\) is the resistant torque on the stiffness control motor which represents the influence of the deflection \(\gamma, u_1\) and \(u_2\) are the equivalent driving torque acting on each motor, and \(\tau(\gamma, \theta_2)\) is the spring elastic torque given by

\[
\tau = K(\theta_2) \gamma
\]

where \(K(\theta_2)\) is the actuator stiffness.

B. Stiffness and Resistant Torque

Fig. 4 shows the schematic diagram of the ith spring \((N\) springs are used in total, \(i = 1\) to \(N\)\). As shown in Fig. 4(a), the driving torque for the output frame is clockwise, and the S-shaped spring is compressed with a deflection angle \(\gamma\). Using the Castigliano’s theorem [40], the deflection of a thin circular curved beam (see Fig. 4(b)) in the \(y\) direction is given by

\[
\Delta y_i = \frac{3F_i (2r - h)^3}{4Eb^3} \Omega(\theta_2)
\]

with

\[
\Omega(\theta_2) = \pi - (\theta_{20} + \theta_2) + \frac{1}{2} \sin [2(\theta_{20} + \theta_2)]
\]

where \(r\) is the radius of the spring outline, \(E\) is Young’s modulus of the beam, \(b\) and \(h\) are the width and thickness of the beam section, respectively, and \(\theta_{20}\) is the initial angular displacement of the spring.

The deflection of the beam in the angular direction at the contact point \(A\) can be given by [27]

\[
\Delta y_i = R \sin \gamma
\]

where \(R\) is the distance between the axis of the input frame and that of the S-shaped spring. Combining (3) and (4), the spring elastic torque \(\tau\) is given as

\[
\tau = \sum_{i=1}^{N} F_i R = \frac{4NER^2bh^3 \sin \gamma}{3(2r - h)^3 \Omega(\theta_2)}
\]

Then, the actuator stiffness \(K\) is obtained by

\[
K = \frac{\partial \tau}{\partial \gamma} = \frac{4NER^2bh^3 \cos \gamma}{3(2r - h)^3 \Omega(\theta_2)}
\]

It is clear from (6) that the actuator stiffness \(K\) can be easily customized by modifying the design variables, e.g., material \((E)\), geometric dimensions \((b, h, r, R)\), and number of springs \((N)\). Fig. 5 shows the influence of different design variables on the spring stiffness \(K\). It can
be seen that \( h \) and \( r \) have the most influence on \( K \), while \( b \) is correlated linearly with \( K \).

Remark 3: In the spring elastic torque model (5)-(6), many uncertainties such as hysteresis, friction, backlash, and internal damping are not considered. These terms would inevitably introduce modeling errors which are lumped to \( \tau_d(t) \) in (1). This is a reasonable simplification because the contribution of modeling errors is small as shown in the experimental studies. If very precise control is required when applying large output torque, these nonlinearities should be modeled explicitly.

Fig. 6(a) shows that the influence of spring angle \( \theta_2 \) and deflection \( \gamma \) on the actuator stiffness \( K \). It is shown that the stiffness can be regulated from a minimum value to infinity by adjusting \( \theta_2 \). Besides, the coupling between the stiffness \( K \) and the deflection \( \gamma \) is almost negligible, and \( K \) is mainly dependent on \( \theta_2 \). Fig. 6(b) shows the influence of spring angle \( \theta_2 \) and deflection \( \gamma \) on the spring elastic torque \( \tau \).

The elastic energy \( E_s \) in S-shaped springs is given by

\[
E_s(\gamma, \theta_2) = \int_0^\gamma \tau d\gamma = \frac{4N Er^2 bh^3}{3(2r-h)^3} \Omega(\theta_2) (1-\cos \gamma). \tag{7}
\]

When the deflection \( \gamma \) is not zero, the stiffness control motor needs to overcome a resistant torque \( \tau_r \) to regulate the stiffness. It is given by

\[
\tau_r(\gamma, \theta_2) = \frac{\partial E_s}{\partial \theta_2} = \frac{4N Er^2 bh^3 (1-\cos \gamma) 1-\cos[2(\theta_2 + \theta_2)]}{3(2r-h)^3} \Omega^2(\theta_2)
\]

Figs. 6(c) and (d) show the required energy \( E_s \) and the resistant torque \( \tau_r \) which needs to be provided by the stiffness control motor to regulate the stiffness \( K \) as a function of \( \gamma \) and \( \theta_2 \). It is shown that the resistant torque \( \tau_r \) is zero when the load is zero or the stiffness is infinite (i.e., \( \gamma = 0 \)). Besides, when the deflection is small, little energy and torque is needed to regulate the stiffness. Therefore, the proposed S\(^V\)SASA design consumes significantly less energy to regulate stiffness than designs that need to overcome the spring pretension.

IV. Controller Design

A. Disturbance Observer

The control task can now be summarized as follows: Given the desired trajectory of the output frame \( q_d \), the objective is to synthesize a bounded control input \( u \) such that the output \( q \) tracks \( q_d \) as closely as possible. To facilitate controller design, the following assumption is made:

Assumption 1: The lumped disturbance \( \tau_d(t) \) is continuous and satisfies

\[
\tau_{\text{min}} \leq \tau_d \leq \tau_{\text{max}}
\]

\[
|\dot{\tau}_d| \leq \mu
\]

where \( \tau_{\text{min}} \) and \( \tau_{\text{max}} \) are the lower and the upper bounds of \( \tau_d \), and \( \mu \) is the bound of \( \dot{\tau}_d \).

Remark 4: It is generally very difficult, if not impossible, to rigorously verify this assumption for the proposed VSA. This is mainly due to the fact that the actual load torque is usually unpredictable and it even exhibits nonlinear dynamics such as friction and hysteresis. However, the experimental results show that the assumption is a reasonable simplification and can be used to facilitate controller design.

As shown in (1), the unknown disturbance \( \tau_d \) significantly degrades the system control performance. To improve performance, a disturbance observer is designed as follows

\[
\begin{align*}
\dot{\xi} &= -k_o \xi + k_o(\tau - G \dot{q} + k_o M \dot{q}) \\
\dot{\tau}_d &= -k_o M \ddot{q} + \xi
\end{align*}
\]  

where \( \dot{\tau}_d \) is the estimated disturbance, \( \xi \) is an internal state variable of the disturbance observer, and \( k_o \) is the observer gain. The disturbance estimation error is defined as \( \hat{\tau}_d = \tau_d - \dot{\tau}_d \). Differentiating \( \hat{\tau}_d \) with respect to time, one gets

\[
\dot{\hat{\tau}_d} = \ddot{\tau}_d - k_o \ddot{\tau}_d.
\]

Define a Lyapunov-like function \( V_d \) as

\[
V_d = \frac{1}{2} \tau_d^2. \tag{8}
\]

Differentiating \( V_d \) with respect to time yields

\[
\dot{V}_d \leq -k_o \tau_d^2 + \dot{\tau}_d \dot{\tau}_d - k_o \tau_d^2 \leq -k_o V_d + \frac{\mu^2}{2k_o}.
\]

Therefore, the disturbance estimation error \( \hat{\tau}_d \) is bounded, and converges exponentially to a ball centered at the origin with radius \( \mu/\sqrt{2k_o} \). Therefore, the disturbance estimation error can be arbitrarily diminished by the observer gain \( k_o \).

B. Position Tracking Control

Define a position tracking error \( e_1(t) = q_d(t) - q(t) \) and a filtered tracking error \( e_2(t) = \dot{e}_1(t) + k q_1 e_1(t) \), where \( k q_1 \) is a control gain. Differentiating \( e_2 \) with respect to time and multiplying both sides of the resulting equality by \( M \) yields

\[
M \dot{e}_2 = M (\dot{q}_d + k q_1 \dot{e}_1) + G(q) + C \dot{q} + \tau_d - \tau. \tag{9}
\]

The virtual control torque \( \tau_u \) for \( \tau \) is then designed as

\[
\tau_u = M (\dot{q}_d + k q_1 \dot{e}_1) + G(q) + C \dot{q} + \dot{\tau}_d + k q_2 e_2 + \tau_{us}
\]

where \( k q_2 \) is a control gain. The sliding control term \( \tau_{us} \) is used to reject the disturbance estimation error and will be introduced later. Substituting the above equation into (9), one gets the tracking error dynamics of the output frame

\[
M \dot{e}_2 = -k q_2 e_2 - \tau_{us} + \dot{\tau}_d + \tau_u
\]

where \( \tau_u = \tau_u - \tau \) is the tracking error of \( \tau \).

The virtual control torque \( \theta_u \) for \( \theta \) is designed as

\[
\theta_u = K^{-1} \tau_u + q.
\]
Similarly, define a position tracking error \( e_3 = \theta_u - \theta \) and a filtered tracking error \( e_4(t) = e_3(t) + k_{q1} e_3(t) \), where \( k_{q1} \) is a gain. Differentiating \( e_4 \) with respect to time yields

\[
J_1 \dot{e}_4 = J_1 (\dot{\theta}_u + k_{q1} \dot{e}_3) + \tau + D_1 \dot{\theta}_1 - u. \tag{10}
\]

The actual control input \( u_1 \) is designed as

\[
u_1 = J_1 (\dot{\theta}_u + k_{q1} \dot{e}_3) + \tau + D_1 \dot{\theta}_1 + k_{q2} e_4 \tag{11}
\]

where \( k_{q2} \) is a control gain. Combining the above result with (10) yields \( J_1 \dot{e}_4 = -k_{q2} e_4 \).

Define the state variables as \( \mathbf{e} = [e_1, e_2, e_3, e_4]^T \), the system dynamics is given by

\[
\dot{\mathbf{e}} = \mathbf{A} \mathbf{e} + \mathbf{B} \tau_d - \mathbf{B} \tau_{us}
\]

\[
\mathbf{A} = \begin{bmatrix}
-k_{q1} & 1 & 0 & 0 \\
0 & -\frac{k_{q2}}{J_1} & \frac{K}{J_1} & 0 \\
0 & 0 & -k_{q1} & 1 \\
0 & 0 & 0 & -\frac{k_{q3}}{J_1}
\end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix}
0 \\
1 \\
0 \\
0
\end{bmatrix}.
\]

From the above equation, the eigenvalues of \( \mathbf{A} \) are \(-k_{q1}\), \(-k_{q2}/J_1\), \(-k_{q1}\), and \(-k_{q2}/J_1\), respectively. If the gains \( k_{q1}, k_{q2}, k_{q1}, \) and \( k_{q2}/J_1 \) are chosen to be positive, then the system \( \dot{\mathbf{e}} = \mathbf{A} \mathbf{e} \) is asymptotically stable. Therefore, for any symmetric and positive definite matrix \( \mathbf{Q} \in \mathbb{R}^4 \), the corresponding matrix \( \mathbf{P} \in \mathbb{R}^4 \) that solves the Lyapunov equation \( \mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} = -\mathbf{Q} \) is symmetric and positive definite [41]. The robust control term \( \tau_{us} \) is then designed as

\[
\tau_{us} = k_{q3} \text{sgn}(\mathbf{e}^T \mathbf{P} \mathbf{B}) \tag{12}
\]

where \( k_{q3} > \tau_{\text{max}} \) is a positive control gain.

Now, choose a Lyapunov function candidate as \( V = \mathbf{e}^T \mathbf{P} \mathbf{e} \). The main result of this study is summarized as follows.

Theorem 1: Consider the system (1) under Assumption 1 driven by the control law comprised of (8), (11) and (12). If the condition \( k_{q3} > \tau_{\text{max}} \) is satisfied, then the closed loop system achieves practical partial asymptotic stability in the sense that all signals involved are of \( L_\infty \) and the tracking error \( \mathbf{e}(t) \) exponentially converge to 0.

Proof. The time derivative of \( V \) is

\[
\dot{V} = \mathbf{e}^T (\mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A}) \mathbf{e} - 2 \mathbf{e}^T \mathbf{P} \tau_d - 2 \mathbf{e}^T \mathbf{P} \tau_{us} + 2 \mathbf{e}^T \mathbf{P} \mathbf{B} \tau_{us}.
\]

Applying (12) to the above result, one gets

\[
\dot{V} \leq -\rho \mathbf{e}^T \mathbf{Q} \mathbf{e} - 2 \mathbf{e}^T \mathbf{P} \mathbf{B} (k_{q3} + |2 \mathbf{e}^T \mathbf{P} | \tau_{\text{max}})
\]

\[
\leq -\rho \mathbf{e}^T \mathbf{Q} \mathbf{e} - 2 \mathbf{e}^T \mathbf{P} \mathbf{B} (k_{q3} - \tau_{\text{max}}).
\]

It follows from the above result that \( \dot{V} \leq -\rho \mathbf{e}^T \mathbf{Q} \mathbf{e} \). Thus, one gets \( \dot{V} \leq -\rho \mathbf{V} \) with \( \rho = \lambda_{\text{min}}(Q)/\lambda_{\text{max}}(P) \), where \( \lambda_{\text{min}}(Q) \) and \( \lambda_{\text{max}}(P) \) are the minimal eigenvalue and the maximum eigenvalue of \( P \), respectively. It can be seen from the above result that all closed-loop signals are of \( L_\infty \) and the tracking error \( \mathbf{e}(t) \) exponentially converge to 0.

\[\square\]

V. Experimental studies

In this section, experiments are carried out on an experimental platform (see Fig. 7) to verify the validity of the proposed design. Two identical BLDC motors (EC-4 pole 30, 200W, Maxon) with corresponding drivers (Gold Twitter G-TW15, Elmo) are used to drive the prototype. The motor drivers are which can provide a maximum power output 1200W. The driver for the main drive motor operates in the cyclic synchronous torque mode, while that for the stiffness control motor operates in the cyclic synchronous position mode. The gear ratios for the main drive mechanism and the stiffness regulation mechanism are 80 and 15, respectively. Moreover, two 19-bit absolute encoders (AKSIM, Renishaw) are used to measure the angular displacements of the input frame \( \theta_1 \).
and the output frame $q$, respectively. An encoder (RM08, Renishaw) with a resolution of 4096 cpr is used to measure the S-shaped spring displacement $\theta_2$. The proposed control strategy is implemented in the MATLAB/xPC target environment, and the control loop rate was set at 1 kHz. In general, two cases are considered in the experimental studies:

- Load-free case. There is no external load torque acting on the output frame.
- Load case. The load mass is 0.35 kg, and the length of the arm is 0.25 m.

A. Actuator Stiffness Performance

The relationship between the actuator torque and the deflection $\gamma$ is measured for different sets of the spring angular displacement $\theta_2$. The actuator torque is measured by a force sensor and the deflection $\gamma$ is obtained from encoder measurements. The dashed lines in Fig. 8 show the theoretical stiffness, and the solid lines represent the experimental results. It is shown that the theoretical results closely match the experimental results. However, the experimental results exhibit the hysteresis phenomenon owing to such factors as friction, backlash, and internal damping [22]. It is worth noting that when the spring angle $\theta_2 = 120^\circ$, the theoretical stiffness reaches infinity, while the experimental result shows that the actual stiffness is about 3851 Nm/rad. This deviation is mainly due to the structural flexibility of the intermediate plates and the thrust plates, and higher stiffness could be achieved by strengthening internal structures of the VSA prototype.

To verify the stiffness regulation performance, the output frame is fixed and spring angular displacement $\theta_2$ is used to track the desired trajectory $\theta_{2d}$. Figs. 9(a)-(b) show the step response of $\theta_2$ for the load-free case and the load case, respectively. It is worth noting that $\theta_2$ varies from $10^\circ$ to $110^\circ$, which is almost the full range of stiffness regulation ($0^\circ$–$120^\circ$). The response time in the load-free case is about 70 ms, and that in the load case is about 100 ms. Therefore, the stiffness of the proposed VSA can be regulated quickly.

B. Collision Safety Experiment

Fast collisions between robots with a stiff structure will result in extremely short and high force peaks [2]. Although peak forces can be reduced by specially designed controllers that enhance collision safety for agile robots such as DLR Lightweight Robot [42], joint actuators should withstand the impact by themselves, which may cause undue damages. In contrast, springs in VSAs helps to reduce the peak torque in the drive train between motors, and if the springs are flexible enough, VSAs can cushion the actuator from overload. In this section, a collision safety experiment is conducted to verify the advantages of the proposed VSA. In general, two cases are considered. In the first case, the proposed VSA collides with a fixed obstacle using a fixed stiffness, while safety enhancement strategies are used in the second case. The impact force is measured with a force sensor.

The collision detection method proposed in [21] is used to judge whether the collision occurs, i.e., a threshold $\gamma_{th} = 1.62^\circ$ is set for the deflection $\gamma$. If the actual deflection $\gamma$ varies out of the threshold $\gamma_{th}$, the collision is considered to be occurred. Then, the main drive motor stops immediately, and the stiffness regulation motor immediately regulates the stiffness to the minimum. Fig. 10(a) shows the output position of the actuator, and Fig. 10(b) shows the impact force in both cases. It can be seen that the actual collision occurs at 2.18 s, the collision is detected at 2.22 s. The maximum impact force is significantly reduced from 47.2 N in the first case to 24.9 N in the second case. Therefore, the actuator with variable stiffness mechanism helps to achieve a small impact force during the collision due to its adjustable stiffness, which is valuable for shock absorption and agile robot-environment interaction.

C. Trajectory Tracking Results

Trajectory tracking studies for the output frame are carried out to verify the performance of the proposed controller. To provide a fair comparison, two controllers are chosen:

- The proposed disturbance observer based nonlinear control (DNC). The values of the control parameters are chosen as $k_o = 100$, $k_{q1} = 50$, $k_{q2} = 0.09$, $k_{q3} = 0.4$, $k_{q1} = 50$, $k_{q2} = 1.5$, and $Q = diag(1e-4, 1e-5, 1e-5, 1e-5)$.
- Model based controller (MBC). The disturbance observer is not used. $k_o$ is set to be zero, other parameters are the same as corresponding parameters in the DNC [38],[43].

Step response results of $q$ for the load-free case at a fixed actuator stiffness ($K = 50$ Nm/rad) are shown in Figs. 11(a), (c) and (e). It is clear that both controllers behave similarly in terms of the trajectory tracking performance because the load disturbance is rather small is this case.

Fig. 11. Step response results of $q$ for the load-free (left) and load case (right) at fixed actuator stiffness ($K = 50$ Nm/rad). (a)-(b) Tracking performance comparisons of the two controllers. (c)-(d) Tracking error comparisons of the two controllers. (e)-(f) Disturbance estimate $\hat{\tau}_q$ of the proposed DNC.
(see Fig. 11(e)). In contrast, Figs. 11(b), (d) and (f) show the step response results for the load case at the same actuator stiffness setting. By comparing the two cases, it can be seen that the torque disturbance is larger in the load case than in the load-free case. Furthermore, the tracking performance of the MBC is more sensitive to load torque variations compared with the proposed DNC. In contrast, the proposed DNC exhibits consistent tracking performance with the help of the disturbance observer, implying robust control performance.

To further verify the effectiveness of the proposed DNC, the sinusoidal response results of \( q \) for the load-free case at the same actuator stiffness (\( K = 50 \) Nm/rad) are shown in Figs. 12(a), (c) and (e), and those in the load case are shown in Figs. 12(b), (d) and (f). Both controllers exhibit similar control performances when the load torque is small. However, as it increases, the influence of load torque on the tracking performance is more prominent for the MBC compared with the DNC, implying the robustness of the proposed DNC.

To further verify the performance of the proposed DNC when the actuator stiffness is varying, the sinusoidal output frame trajectory \( q_d \) shown in Fig. 12 is adopted, and three different stiffness trajectories, i.e., sinusoidal, ramp, and step trajectories as shown in Figs. 13(a)–(c), are used. Fig. 13(a) shows the tracking results of DNC and MBC under sinusoidally varying actuator stiffness for the load-free case and the load case. It is shown that both controllers behave similarly in terms of the tracking error for the load-free case because the load disturbance is small. However, MBC exhibits larger tracking error than the proposed DNC when the load torque increases, which verifies the robustness of the proposed DNC when the actuator stiffness is sinusoidally varying. From Figs. 13(b)–(c), similar conclusions can be drawn when the actuator stiffness is regulated following ramp and step trajectories. Fig. 14 shows the maximum absolute error (MAE) and the root mean square error (RMSE) of the two controllers under different stiffness trajectories. It is shown that the proposed DNC helps to achieve better control performance than MBC under different working points when the stiffness is varying, which demonstrates the superiority of the proposed DNC.

D. Cyclic Movements

Cyclic movements consist of repetitive accelerations and decelerations of actuators. When the eigenfrequency of the actuator output side resonates with the frequency of the cyclic movements, energy consumption can be minimized [2]. VSAs can regulate the actuator stiffness to achieve the resonance, which leads to reduced energy consumption compared with SEAs of fixed stiffness [18]. To demonstrate the superiority of VSAs over SEAs for cyclic movements, the method proposed in [18] is applied. A load with mass \( m = 1 \) kg, and a link with length \( L = 0.5 \) m and moment of inertia \( M = 0.1 \) kgm\(^2\) are used. The eigenfrequency \( f_n \) of the actuator output side can be approximated by \( f_n = \frac{1}{2\pi} \sqrt{\frac{K}{mL^2 + mL}} \). A sinusoidal trajectory \( q_d \) of which the frequency changes linearly from 1 Hz to 2 Hz within 10 s is adopted. Fig. 15(a) shows the tracking results at a fixed actuator stiffness 31.1 Nm/rad, which corresponds to an eigenfrequency \( f_n = 1.5 \) Hz. Fig. 15(b) shows the tracking results at varying stiffness from 13.8 Nm/rad to 55.3 Nm/rad, which closely follows the varying trajectory frequency. By comparing Fig. 15(a) and (b), it is shown that the power of the main drive motor is significantly reduced when the stiffness is varying. Besides, the energy consumption of the main drive motor is 15.7 J when the stiffness is varying, which is lower than that when the stiffness is fixed (19.5 J). Note that the energy consumption of the stiffness regulation mechanism is negligible compared with the main drive mechanism, which will be shown in Section VI. Therefore, the superiority of VSAs over SEAs for cyclic movements is verified.

VI. Discussions

To further verify the validity of the proposed design, comparisons with ten well-known VSAs of similar output power are given in Table II. As shown, the proposed S\(^3\)VSA ranks the second among all the designs in terms of the power-to-weight ratio and the power-to-volume ratio. As a prominent representative design of the antagonistic controlled stiffness principle, BAVS [8] ranks highest by this measure. However, it is generally challenging to achieve near-infinite stiffness for designs based on this principle. Therefore, the compactness of the proposed design is verified. Note that the proposed design does not rank high in terms of output torque. This is mainly because that the gear ratio for the main drive in the proposed design is relatively smaller compared with previous designs. The output torque can be easily increased by using a high gear ratio if a higher output torque is needed.

In terms of the stiffness regulation, the proposed design enables a capacity of wide-ranging stiffness regulation from
Fig. 13. Tracking performance of the MBC and the proposed DNC for the load-free case and the load case when the VSA stiffness is varying (15 Nm/rad ↔ 375 Nm/rad) following $\theta_{\text{dd}}$ trajectories of: (a) sinusoid, (b) ramp, and (c) step.

Fig. 14. Tracking performance of the MBC and the proposed DNC when the actuator stiffness is varying (15 Nm/rad ↔ 375 Nm/rad) following sinusoid, ramp, and step trajectories. (a)–(b) MAE and RMSE for the load-free case. (c)–(d) MAE and RMSE for the load case.

Fig. 15. Energy saving performance for cyclic movement of which the frequency changes linearly from 1 Hz to 2 Hz under: (a) constant stiffness (31.1 Nm/rad), and (b) varying stiffness (13.8 Nm/rad → 55.3 Nm/rad).

12.12 Nm/rad to near-infinite. Although the minimum stiffness is slightly higher than designs such as HVSA (4 Nm/rad) and TSA (6.2 Nm/rad), the minimum stiffness of the proposed design can be further reduced by modifying design variables of the spring (e.g., material, geometric shape of the beam, and spring number) if a lower stiffness is needed in certain applications. Besides, the stiffness regulation speed of the proposed design also ranks high among all the designs. Consequently, the actuator can be used for fast stiffness regulation, which is valuable for applications such as shock absorption and agile robot-environment interaction [2].

Fig. 16 shows the experimental result on the energy consumption for the stiffness regulation mechanism of the proposed design obtained using the method proposed in [18]. It is shown that if the deflection angle $\gamma$ is small, the energy cost of stiffness regulation is low, which verifies the theoretical analysis of the energy consumption.

VII. Conclusions

In this paper, a novel rotational variable stiffness actuator that uses S-shaped springs as elastic elements is proposed. Wide-ranging and easily customizable stiffness regulation from a minimum value to near-infinite can be achieved with the help of the S-shaped spring. The proposed actuator ranks high among designs of the similar output power in terms of the power-to-weight ratio and the power-to-volume ratio; therefore the compactness of the proposed design is verified. Besides, the stiffness regulation speed of the proposed design also ranks high among the designs. Consequently, the actuator can be used for fast stiffness regulation. To achieve robust position tracking under different operating conditions, a disturbance observer is used to estimate the lumped disturbance. The stability of the system is proved using the Lyapunov method. The proposed design has potential usages in
TABLE II
Comparisons of different VSAs

<table>
<thead>
<tr>
<th>Name</th>
<th>Stiffness (Nm/rad)</th>
<th>SRT (s)</th>
<th>Deflection (°)</th>
<th>NS (rad/s)</th>
<th>NT (Nm)</th>
<th>PT (W)</th>
<th>Volume (10²m³)</th>
<th>Power (W)</th>
<th>Mass (Kg)</th>
<th>Power/Volume (W/Kg)</th>
<th>Power/Mass (W/Kg)</th>
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<tbody>
<tr>
<td>BAVS [8]</td>
<td>3.9-146.6</td>
<td>0.014</td>
<td>18.2</td>
<td>12.6</td>
<td>4.8</td>
<td>8</td>
<td>134</td>
<td>0.56</td>
<td>0.75</td>
<td>239.29</td>
<td>178.67</td>
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<td>CompAct-VSA [20]</td>
<td>0-∞</td>
<td>0.1</td>
<td>±20</td>
<td>4.25</td>
<td>10.75</td>
<td>117</td>
<td>2.88</td>
<td>2</td>
<td>19.44</td>
<td>28.00</td>
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<tr>
<td>VSI [27]</td>
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<td>0.3</td>
<td>±10</td>
<td>/</td>
<td>/</td>
<td>30</td>
<td>200</td>
<td>3.07</td>
<td>4.95</td>
<td>65.16</td>
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<td>HVSA [30]</td>
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<td>±30</td>
<td></td>
<td>8.5</td>
<td>50</td>
<td>60</td>
<td>2.58</td>
<td>2.36</td>
<td>23.26</td>
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<td>AwAS-II [14]</td>
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<td>2.5</td>
<td>±17</td>
<td>10.2</td>
<td>10.75</td>
<td>80</td>
<td>56</td>
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<tr>
<td>vsaUT-II [19]</td>
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<td>0.9</td>
<td>±20</td>
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<td>15</td>
<td>60</td>
<td>150</td>
<td>13.50*</td>
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<td>MACCEPA [32]</td>
<td>5-110</td>
<td>0.16</td>
<td>±30</td>
<td></td>
<td>3.52</td>
<td>40</td>
<td>70</td>
<td>200</td>
<td>4.80*</td>
<td>24.67</td>
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<tr>
<td>TSA [21]</td>
<td>6.2-∞</td>
<td>0.25</td>
<td>±21.22</td>
<td>3.88</td>
<td>29.2</td>
<td>58</td>
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<td>5.65</td>
<td>±2.87</td>
<td>/</td>
<td>90</td>
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<td>24.32</td>
<td>29.03</td>
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<td>SVSA [22]</td>
<td>1.7-150.56</td>
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<td>9.46</td>
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<td>±12</td>
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<td>7.6</td>
<td>22.7</td>
<td>200</td>
<td>2.24</td>
<td>1.54</td>
<td>89.21</td>
<td>129.87</td>
</tr>
</tbody>
</table>

* Values estimated from literatures, SRT: Stiffness Regulation Time, NS: Nominal Speed, NT: Nominal Torque, PT: Peak Torque.

numerous applications where adaptable compliance is critical, e.g., legged robots, and human-robot interaction.

One limitation of the proposed design is that the range of rotation is from 0° to 360°, however, it can be alleviated in the following studies by using slip rings to transmit signals if continuous rotation is needed. Besides, the volume and the weight of the prototype can be further reduced by reasonable structural optimization. Modularization of the proposed design is another notable research direction. Future research could be dedicated to the force control and human-robot interaction control of the variable stiffness actuator.

References
