Numerical and Experimental Investigations of Motion-induced Eddy Current for Contactless Speed Estimation based on Distributed Current Source Model

Zhengya Guo, Kok-Meng Lee*, Life Fellow, IEEE/ASME, and Zhenhua Xiong*, Member, IEEE

Abstract— This paper presents a distributed current source (DCS) model for calculating the motion-induced eddy-current (EC) in a conductor with uniform rectilinear motion and its generated magnetic flux density. By discretizing the conductor into elemental current sources, EC is formulated in state space as an optimization problem for investigating the effects of motion-induced EC on measurements in manufacturing. Along with a parametric study on the effects of motion velocity on the choice of sensing points, the results are numerically verified by comparing with finite element analysis which show excellent agreements. Mechanical scanning has some influence on detection results but can be alleviated by carefully selecting the sensing points. This study has led to the assembling of a prototype sensor for contactless speed estimation. Two sets of experiments were conducted on a CNC to evaluate the accuracy and response speed of the sensor which demonstrates high accuracy when measuring a stationary movement and fast response when dealing with a transient movement.

I. INTRODUCTION

Eddy current (EC) effects, which can be induced in conductive materials without contact, have been employed in many applications such as post-fabrication quality inspection, parameter estimation and real-time process monitoring of metal additive-manufacturing. Because EC-based sensors are widely available in small-footprint at low-cost and enjoy several inherent advantages (including high sensitivity, fast response, and harsh environment workability) as compared to their counterparts, they are also used in closed-loop controlled system applications such as lathe-turning [1] where an array of EC sensors offers the displacement field perception ability. In practice, mechanical EC scanning is often indispensable but the accompanied motion-induced EC effects on testing results are often neglected because a lack of effective methods to isolate them for elimination. Motivated by the needs to differentiate motion-induced EC effects from measurements, the paper presents a method to model, calculate and analyze the EC field induced by the scanning motion.

In conventional EC testing with mechanical scanning, electromotive force (emf) is generated along with EC induced in the conductor by an electromagnet (EM) with alternating current (AC) excitation; the former results in a change on the EM-induced EC distribution, and hence in the testing results. In closed-loop rotational motion-controlled systems, motion-induced EC may introduce some errors that could potentially degrade system performance since the sensors are usually calibrated in static conditions while the specimen quickly rotates during actual measurements. For achieving high accuracy, most EC testing is implemented with low scanning velocity (usually lower than tens of millimeters per second) [2] so that emf effects can be neglected or measurements are obtained quasi-statically [3], both of which are less than desired. Different from EM-based EC testing, in Lorentz forces evaluation method where the force acting on the excitation source (usually a permanent magnet, PM) is employed as a factor to realize defect detection [4], the EC distribution is directly determined by the relative motion between the conductor and the PM. Thus, for both EM-based EC testing and Lorentz force evaluation method, analyzing the motion-induced EC will be beneficial for elevating testing accuracy and efficiency.

On the other hand, because emf has a close connection with the conductor movements, motion-induced EC provides an opportunity to realize contactless speed estimation in some special areas. In many control systems like machining and robotics, speed estimation is necessary in the feedback step and its sensing accuracy and response speed directly affects the system output. Optical speed sensors are commonly used as traditional methods [5] while their sensing results can be easily disturbed by illumination conditions thus cannot be applied in harsh environment. Speed sensors designed based on electromagnetic systems have been developed [6-8], while these sensors need a spatial variation on electromagnetic properties of the testing target, which is inappropriate for workpiece with homogenous materials and smooth surfaces. A sensor based on EC effects was presented in [9] while this sensor is essentially a mechanical system, resulting in an unsatisfactory transient response. Another EC-based sensor was proposed in [10] while it needs to measure the coil voltage whose signal to noise ratio will decrease with low frequency excitation. But if high excitation frequency is used, the motion-induced part in EC will be small; thus, the sensitivity of this sensor is limited when dealing with a low-speed conductor. Developing a speed sensor based on magnetic flux density (MFD) measurement will be a promising method to overcome these disadvantages.

Analysis of motion-induced EC and the design of a contactless speed sensor call for high-fidelity and yet computationally efficient models. Analytical methods are attractive because of their closed form solutions and high efficiency [11] [12] while these models can only deal with...
problems with simple geometries. The mutual induction is usually omitted, and the small displacement assumption is adopted for facilitating the calculation, which may bring some errors. Different numerical methods were proposed [13][14], which theoretically can solve three-dimensional problems with complex geometries, but these methods are error-prone and computationally demanding as a large air space is usually incorporated into the solving domain. More recently, a general and physically intuitive approach, referred to here as distributed current source (DCS) method, was developed for calculating the magnetic fields of an electromagnetic actuator [15]. The DCS method was extended to investigate the motion-induced EC for damping system [16] where a 2D axisymmetric system was considered. By discretizing the conductor into elemental current sources, the DCS method solves an EC problem in state-space representation with no air space required in the modeling process [17][18].

Built upon the DCS method, a forward and accurate model for simulating the motion-induced EC is established, based on which the effects of motion velocity on EC testing are investigated and the feasibility of contactless speed estimation is studied. The remainder of this paper offers the following:
- A normalized numerical model based on the DCS method is formulated for simulating the motion-induced EC in a conductor that moves with a constant velocity near the excitation source. The obtained results are compared with FEA for verifying its accuracy and efficiency.
- Parametric effects of the motion velocity on MFD and the transient response of a contactless speed sensor prototype.

Fig. 1 An EC system with the conductor moving along its length direction.

II. FORMULATION AND VERIFICATION OF DCS MODEL

Figure 1 schematically depicts a typical motion-induced EC system, where the conductor translates (constant velocity \( v_0 \)) along its length direction near a stationary excitation source. Without loss of generality, the excitation source is an electromagnet (EM) with inner and outer radii \( (a_o, a_i) \) and height \( (2a) \); and the conductor possesses an infinite length with an invariant cross section. In Fig. 1 where the conductor-fixed moving coordinate frame \( (\omega, \Xi, \Sigma) \) and the EM-fixed global frame \( (O, XYZ) \) obey the right-hand rule, both \( \gamma \) and \( Y \) axes coincide and point away from the moving direction. The \( x_oz \) and \( XOZ \) planes coincide with each other at the initial moment. Then, any point \( K \) in \( XYZ \) can be expressed in the moving \( x\Sigma yz \) using (1):

\[
R_K = \begin{bmatrix} x_K & y_K & z_K \end{bmatrix}^T = [x_K, y_K - v_0t, z_K]^T = r_K - [0, v_0t, 0]^T
\]  

(1)

A. The DCS Model for Solving the Motion-Induced EC

The following assumptions are made in formulating the motion-induced EC problem using a DCS method: 1) the EM is excited by a direct current (DC) or a single frequency alternating current (AC) in the frequency range that the displacement current effects are negligible. 2) The conductor material is homogeneous, isotropic, and non-ferromagnetic.

When the periodic system reaches a stationary state, physical fields expressed in the global frame must be invariant in frequency domain, making it more convenient to solve this problem in O-XYZ. Since EC mainly exists in the region near the excitation, we divide the conductor into three regions by two truncation planes \( (\gamma = \pm \delta_0) \) and select the middle area (non-transparent part in Fig. 1) as the solution domain of interest with the EC in other regions neglected. Without considering the electrical potential, ECD at any point \( K \) (fixed in O-XYZ) in the solving domain can be written as

\[
\frac{\mathbf{J}(R_K, t)}{-\sigma} = \frac{\partial}{\partial t} \mathbf{a}_r (R_K, t) + \frac{\partial}{\partial r} \mathbf{A}_r (R_K, t) - v \times \mathbf{B}_r (R_K, t)
\]

(2)

where \( \mathbf{J} = [J_K, J_Y, J_Z]^T \) denotes the ECD expressed in O-XYZ; \( \mathbf{B}_r = [B_{rX}, B_{rY}, B_{rZ}]^T \) and \( \mathbf{A}_r = [A_{rX}, A_{rY}, A_{rZ}]^T \) respectively mean the magnetic flux density (MFD) and the magnetic vector potential (MVP) generated by the external excitation source and they are expressed in O-XYZ; \( \mathbf{a}_r \) is the MVP generated by EC and it is expressed in o-\( xyz \); \( \sigma \) is the conductivity of the conductor; \( v = -\omega \mathbf{a}_r \) and \( \mathbf{v} \) means the unit vector along \( Y \) direction; \( t \) denotes time.

In the DCS model, the solution domain is discretized into \( N \) volume elemental current sources \( \mathbf{J}_j \) with their centers denoted by displacement vectors \( \mathbf{R}_j \) (\( j = 1 \cdots N \)) which are constant in O-XYZ, as illustrated in Fig. 1. According to (2), ECD in the \( j \)th current source can be calculated by (3):

\[
\mathbf{J}_j = -\frac{\mu_0}{4\pi} \left\{ \sum_{i=1}^N \gamma_{ji} v_j \mathbf{J}_i - v_0 \sum_{i=1}^N \gamma_{ji} v_i \mathbf{J}_i + \mathbf{A}_{ji} + v_0 \mathbf{v} \times \mathbf{B}_{ji} \right\} = \mathbf{J}_j
\]

(3)

where \( \gamma_{ji} = \frac{|\mathbf{R}_j - \mathbf{R}_i|}{2.061/b_j} \) and \( b_j \) is the radius of a minimum bounding sphere that enclose the \( j \)th current source [18]; \( \mu_0 \) is the magnetic permeability in free space; \( v_0 \) is the volume of the \( j \)th current source; \( \mathbf{A}_{ji} = \mathbf{A}_r (\mathbf{R}_j, t) - \mathbf{A}_r (\mathbf{R}_i, t) \); the second term on the right side of (3) comes from the partial derivative of \( \mathbf{a}_r \) with respect to \( t \) in (2) according to (A.4) in Appendix; thus, \( \gamma_{ji} = \frac{\mu_0}{4\pi} \left\{ (Y_j - Y_i)^2 \right\} / |\mathbf{R}_j - \mathbf{R}_i|^3, j \neq i \).

To facilitate design analyses, the parameters in (3) are normalized as (4) where \( I_e \) is the magnitude of the excitation current in the coil:

\[
\frac{4\pi a_o}{\mu_0 I_e} \mathbf{B} = \frac{4\pi a_i}{\mu_0 I_e} \mathbf{A} = \frac{a_o^2}{a_i^2} \mathbf{J} = \frac{1}{a_i} \mathbf{v}, \quad \frac{4\pi a_o}{\mu_0} \frac{\gamma_{ji}}{\gamma_{jj}} = \frac{4\pi a_i}{\mu_0} \frac{X_{ji}}{X_{jj}} = 1
\]

(4)

In terms of (4), (3) is normalized and rewritten in matrix form in (5) where \( f \) is the excitation frequency:

\[
\mathbf{J} = -\frac{1}{4\pi \Delta f} \left\{ \frac{1}{f} \left( \mathbf{G} \mathbf{J} + \mathbf{X} \mathbf{J} \right) + \mathbf{v} \mathbf{B} + \frac{1}{f \mu_0} \left( \mathbf{H} \mathbf{J} + \mathbf{F} \mathbf{J} \right) \right\}
\]

(5)

and where \( \Delta f = a_o \sqrt{f \sigma / \mu_0} \); \( \mathbf{J} \in \mathbb{P} \Rightarrow \mathbf{J} = \begin{bmatrix} \mathbf{J}_1 \cdots \mathbf{J}_N \end{bmatrix}^T \);
In (5), the $1^\text{st}$ term on the right is contributed by the external excitation and mutual inductance between different current sources; and the $2^\text{nd}$ term accounts for the emf effects. The non-dimensional $\psi_0/(f_0 a_0)$ ratio characterizes the relative contribution of the motion-induced EC. Equation (5) can be rewritten as a state-space representation:

$$\dot{[\mathbf{J}]} = [\mathbf{A}][\mathbf{J}] + [\mathbf{u}]$$

(6a)

where

$$[\mathbf{u}] = [\mathbf{G}]^{-1}( -4\pi^2 \omega^2 f[I]_N + \tau^{-1}([\mathbf{H}]))$$

(6b)

$$[\mathbf{u}] = [\mathbf{G}]^{-1}( -\tau^{-1}([\mathbf{A}]_k + \tau^{-1}([\mathbf{F}]))$$

(8a)

where $j = \sqrt{-1}$, and $\delta_{i,j} = \{1,i = k \quad 0,i \neq k \}$.  

In (8a), $[\mathbf{J}]_{k,i}$ denotes the coefficient matrices of the $k^\text{th}$ harmonic component in Fourier series of $[\mathbf{J}]$. As $e^{j\omega t}$ are linearly independent with each other, all coefficients before them in (8a) must be zero. By the aid of the fact that $\{j\omega_0[I]_N - [\mathbf{A}]\}$ is invertible, the solution to (8a) can be written as

$$[\mathbf{J}]_{k,i} = \{\{j\omega_0[I]_N - [\mathbf{A}]\}^{-1}[\mathbf{U}], \quad k = 1 \quad k \neq 1 \}$$

(9)

Due to the omission of the electrical potential term when deriving (7) and (9), a constraint satisfying the conservation law of charge must be imposed on the field [17] for solving a physically relevant ECD solution denoted as the vector $\mathbf{J}(\in R^{3xN}) = [\mathbf{J}_1 \cdots \mathbf{J}_N]^T$. The motion-induced EC problem can be formulated as an optimization problem in (10):

$$\min_{\mathbf{A}} \|\mathbf{J} - \mathbf{J}_0\|^2 \quad \text{s.t.} \quad \{\mathbf{P}\} \mathbf{J} = 0 \quad \text{(10a, b)}$$

In (10a), $\mathbf{J}_0(\in R^{3xN}) = [\mathbf{J}_0, \cdots \mathbf{J}_0]^T$ denotes the vector form of the solution directly calculated with (7) or (9) and the weight matrix $[\mathbf{A}] = \text{diag}([\mathbf{F}_v \mathbf{F}_v \cdots \mathbf{F}_v]) \otimes [111]^T$ accounts for the volume difference among elemental current sources. The definition of constraint matrix $[\mathbf{P}](\in R^{(N-1)x3})$ in (10b) can be found in [18]. Once the ECD is solved, the normalized EC-generated MFD at the sensing point $\mathbf{R}_s$ can be calculated according to Biot-Savart law:

$$\mathbf{B}_s(\mathbf{R}_s) = \sum_{i=1}^{3} \frac{\mathbf{R}_s \times \mathbf{R}_s}{|\mathbf{R}_s|^3} \cdot \mathbf{F}_s \quad \text{and} \quad \mathbf{R}_s = \frac{\mathbf{R}_s - \mathbf{R}_e}{a_0}$$

(11a, b)

B. Numerical Verification of the DCS Model

The DCS model is numerically verified by comparing the simulated ECD field induced in a plate conductor (width $w$, thickness $h$) moving at a specific $v_0$ with that computed in commercial FEA software (COMSOL Multiphysics). The characteristic parameters and the mesh applied in FEA and the DCS model are presented in Fig. 2, along with the related parametric values in Table I. All numerical investigations were conducted on a PC (Intel Core i7-8750H, 2.2GHz CPU, 40GB RAM and 64-bit OS). The optimization problem in the DCS model was solved in MATLAB (function “quadprog”). In the DCS model the MVP and MFD generated by the external excitation were calculated according to the analytical solution derived from the TREE method [20].

![Fig. 2 CAD models used in calculation. (a) FEA. (b) DCS.](image)

**TABLE I PARAMETRIC VALUES USED IN NUMERICAL INVESTIGATION**

<table>
<thead>
<tr>
<th>Coil</th>
<th>Geometry</th>
<th>$\alpha$, $\beta$, $w$ (10, 5, 5) mm</th>
<th># of turns: 260</th>
<th>Excitation</th>
<th>Lift-off</th>
<th>$f$ for DC: 0 Hz, for AC: 500 Hz; $I_e$ = 1 A</th>
<th>$L_e$ = 4 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductor</td>
<td>Size (mm)</td>
<td>Properties</td>
<td>$v_0$ = 15 m/s</td>
<td>Conductor</td>
<td>0.6–8</td>
<td>$I_e$ = 1 A</td>
<td></td>
</tr>
<tr>
<td>Sensing line $(X,Y,Z)$</td>
<td>$S_1$: (0, -20, 1.25) mm; $S_2$: (0, 20, 1.25) mm</td>
<td>Computation</td>
<td>DC DCS: 25 s; FEA: 8 min.</td>
<td>AC</td>
<td>DCS: 65 s; FEA: 12 min.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

![Fig. 3 Simulated normalized EC-generated MFD. (a) DC- (b) AC- excitations.](image)
conductor surface from \(Y = -20\) to 20 mm. The normalized MFDs along \(S_1-S_2\), which are generated by the EC under (DC, AC) excitations, are simulated in Figs. 3(a, b).

As compared in Fig. 3(a, b), while both models simulate the motion-induced EC and agree well with each other, the DCS method takes much less time (last row of Table I) approximately (5%, 9%) of the time required by FEA for the (DC, AC) excitations to calculate the ECD field solutions for a given moving velocity with the related coefficient matrices obtained offline in advance.

### III. Motion-Induced EC Analysis and Application

The motion-induced EC modeled using the DCS model is analyzed and then applied for contactless speed estimation:

- **Numerical investigation**: The parametric effects of conductor velocity on sensing points have been analyzed, which offer guidelines on sensing point selection. The utilized parametric values can be found in Table I. The findings are presented in Table II and Figs. 4 and 5.

- **Experimental investigation**: A prototype sensor for contactless speed estimation has been developed and its effectiveness has been experimentally evaluated on a CNC lathe (Fig. 6); both stationary and transient states were considered. The results are presented in Figs. 7 and 8.

#### TABLE II MFD SENSING POINTS AND MAXIMUM RELATIVE VARIATION

<table>
<thead>
<tr>
<th>Sensing point</th>
<th>((X, Y, Z)) mm</th>
<th>Relative variation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SP-A: (0, 7.5, 1.25); SP-B: (0, -7.5, 1.25);</td>
<td>(\left(\vec{B}<em>{\text{max}}^x, \epsilon</em>{\text{max}}(\vec{B}^x_{\text{max}})\right)) ((\vec{B}<em>{\text{max}}^y, \epsilon</em>{\text{max}}(\vec{B}^y_{\text{max}}))) (\left(\vec{B}<em>{\text{max}}^z, \epsilon</em>{\text{max}}(\vec{B}^z_{\text{max}})\right))</td>
</tr>
<tr>
<td></td>
<td>SP-C: (7.5, 0, 1.25); SP-D: (0, 0, 1.25);</td>
<td>(\text{Re}(\vec{B}^x), \text{Re}(\vec{B}^y), \text{Re}(\vec{B}^z))</td>
</tr>
<tr>
<td></td>
<td>SP-A</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td>SP-B</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td>SP-C</td>
<td>(-116.9, 3.8%)</td>
</tr>
<tr>
<td></td>
<td>SP-D</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td>Im(\vec{B}^x), Im(\vec{B}^y), Im(\vec{B}^z)</td>
<td>(\text{Im}(\vec{B}^x), \text{Im}(\vec{B}^y), \text{Im}(\vec{B}^z))</td>
</tr>
<tr>
<td></td>
<td>SP-A</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td>SP-B</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td>SP-C</td>
<td>(-108.3, 13.9%)</td>
</tr>
<tr>
<td></td>
<td>SP-D</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

A. Parametric effects of Conductor Velocity on Sensing

Figure 4(a, b) present the \((Y, Z)\) MFD components along the sensing line \(S_1-S_2\) (Fig. 2b), which are generated by the EC induced in the plate-conductor moving at different \(v_0\) under DC excitation. Due to symmetry, the \(X\) component is zero and not presented in Fig. 4. Figure 5 shows the MFD variation \((\Delta \vec{B}_e = \vec{B}_e - \vec{B}_{e0}\) where \(\vec{B}_{e0}\) corresponds to \(v_0 = 0\) m/s) under AC excitation with \(v_0/f_{0}\) ranging from 0 to 3 at four sensing points (denoted as SP-A, -B, -C, and -D) in Table II. Along with the sensor coordinates, the largest relative variation \(\epsilon_{\text{max}} = \max(\Delta \vec{B}_e) / \vec{B}_{e0}\) are also listed in Table II where the symbols, (+) and (-), indicate that the MFD is always zero and that \(\epsilon_{\text{max}}\) cannot be calculated due to the zero value of \(\vec{B}_{e0}\). Below are some findings from Figs. 5 and 6 and Table II:

- In Fig. 4, MFD peaks vary non-linearly with \(v_0\) implying that the motion-induced EC system is non-linear with respect to \(v_0\), which can also be inferred from (7). As both \((Y, Z)\) MFD components along \(S_1-S_2\) change with \(v_0\), the associated testing results can be easily affected by the velocity fluctuations if these two components are used. Since EC is stronger near the coil center, \(X\) MFD component at the point under the coil center is more suitable for small defect detection, since theoretically no matter how large the actual moving velocity is, \(\vec{B}_{eX}\) at this point will always be zero if no defect exists.

  - In Fig. 5, nearly all the MFD components at the four sensing points vary with \(v_0\), suggesting that mechanical scanning has some effects on the EC testing results if \(v_0\) is sufficiently large. When EC testing is applied to measure system parameters, the Re-part of \(B_{eX}\) (at SP-C), \(B_{eY}\) (at SP-A) or \(B_{eZ}\) (at SP-C and D) is preferred since their relative variations are small. However, \(B_{eX}\) (at SP-A and B) are more appropriate for sensing small defects.

- For speed estimation, the variations of both \(B_{eY}\) (at SP-D) and \(B_{eZ}\) (at SP-B) are more dominant than the others; the latter has a very large basic part implying that a large measurement range \(B_{eZ}\) (at SP-B) will be necessary in practice considering the superposition of the coil-generated part \(B_{eZ}\). On the other hand, \(B_{eY}\) (at SP-D) senses only the portion of EC induced by motion; thus, highly accurate and sensitive MFD measurements of motion-induced EC can be more easily achieved.

**B. Contactless Speed Estimation**

The MFD-based contactless speed estimation was experimentally investigated on a CNC lathe (Fig. 6). Based on the numerical analysis in Section 3.A, the magnitude of the measured \(B_{eY}\) (at SP-D) was used to solve for \(v_0\). According to numerical results for both (DC, AC) excitations, \(|\vec{B}_e|\) and \(v_0\) at
SP-D can be related by the curve-fit (12) where \( p_k \) are the polynomial coefficients for the measurement range \([0, v_{\max}]\)

\[
\vec{B}_y = \sum_{k=0}^{n} p_k v_y^k, \quad v_y \in [0, v_{\max}] \text{ and } k = 0,1,2,\ldots \quad (12)
\]

Practical implementation suggests that sufficient accuracy can be achieved with the first 7 terms \((n=6)\) of the polynomial fit. The solution uniqueness of \( v_0 \) is ensured by the monotonicity of the polynomial in its domain of definition.

In this experiment where the coordinate frames are the same with those defined in Fig. 2 a circle aluminum plate that possesses a relatively large radius \( r_1 (=180 \text{ mm}) \) was fixed onto the 3-jaw chuck with its center aligned to the lathe spindle. The prototype velocity sensor consisting of a coil and a MFD sensor (TMR2001, Multi Dimension) was installed on the machine feed table, which can be fine-tuned via three \( \mu \) motion stages. As shown in Fig. 6 (bottom right), the centerline of the coil and that of the plate-conductor were parallel \((r_2 = 130 \text{ mm})\) to the lathe rotating axis. The rectilinear motion was simulated by the plate rotation with the velocity approximated by \( v_0 = \omega_r r_2 \) where \( \omega_r \) defines the conductor angular velocity and was measured by the encoder mounted on the CNC lathe. The excitation signal was generated by a NI-DAQ device (NI-9264, 16-bit DAC) and amplified by a current amplifier before input to the coil. The sensing system outputs were digitized by another NI-DAQ device (NI-9205, 16-bit ADC).

Fig. 6 Experimental setup for validating the contactless speed estimation.

Fig. 7 Speed estimation results for the stationary movements. (a) DC-excited situation. (b) AC-excited situation.

Two experiments were conducted for separately validating the accuracy and response speed of the velocity sensor:

1) The MFDs were measured at several specified velocities to calibrate the prototype speed-sensor, from which the polynomial coefficients \( p_k \) were obtained. Next, additional MFDs were measured at some other velocities to solve \( v_0 \) from (12) for comparison with the true values to assess sensor accuracy.

2) The MFDs were measured in a conductor acceleration process, from which the velocity variation with respect to \( t \) was solved. For DC excitation, the mean value of the latest 50 signal points was used as the MFD magnitude. For AC excitation, the data in the latest sinusoidal response period were employed to calculate the MFD magnitude with FFT.

Fig. 8 Speed estimation results for the acceleration process. (a) DC-excited situation. (b) AC-excited situation.

Parameters utilized in the experiments are the same with those in Table I. Figure 7 plots the speed estimation for the stationary movements; and Fig. 8 shows the test results for the acceleration process. In Figs. (7, 8), the \( 1^{\text{st}}, 2^{\text{nd}} \) columns show the results with the (DC, AC) excitation, respectively. The findings in Figs. 7 and 8 are summarized as follows:

- In Fig. 7, all test-points are located near the fitting curves, suggesting that the speed-sensor can obtain precise results. The maximum relative errors, defined as \( e = |(v_{\text{mea}} - v_0)/v_0| \), where \((v_{\text{mea}}, v_0)\) are the (measured, true) values, are about 4% and 1.6% for the DC and AC excitation, respectively. On the other hand, according to the fitting curves, detection sensitivity gradually decreases with the increase of \( v_0 \) in DC excited situation while it basically keeps invariant with AC excitation; thus, when the target velocity is relatively large, AC excitation may be preferred.

- As shown in Fig. 8, the prototype sensor successfully traces the velocity variation during an acceleration process, which is a natural outcome of the fast response characteristic of the electromagnetic system. For both DC and AC excitation, the acceleration time (about 3.25s) obtained by the prototype sensor agrees well with that measured by the encoder. With the function “roots” in MATLAB, it only takes about 40 \( \mu \)s to solve \( v_0 \) from the polynomial equation (12); thus, the sensor delay mainly depends on the time to achieve the MFD magnitude. For DC excitation, 0.5 ms is necessary to obtain the latest 50 signal points for a sampling frequency of 100 kHz while a complete sinusoidal period lasts 2 ms for the 500 Hz AC excitation. These results suggest that this speed sensor may have the potential for real-time applications.

- Some fluctuations exist (zoom-in views in Fig. 8) when measuring the acceleration process, which were mainly caused by the conductor warpage/misalignment leading to periodic variations of the lift-off distance during rotation. The fluctuation frequency was 16 Hz, which is exactly the revolution rate. The fluctuations in DC excitation are larger, which is mainly caused by two reasons: 1) For DC excitation the MFD at SP-D is more sensitive to lift-off variation; 2) At the velocity of 13.07 m/s the sensitivity of the speed sensor
with DC excitation is small so a faint MFD variation will lead to a large estimation difference. When subjected to AC excitation, the velocity estimation was non-zero before the conductor begins to rotate. This error may also be induced by lift-off variation since it is difficult to guarantee the same initial conductor position in every trial. Another plausible cause may be the slight relative movement between coil and sensor in experiments, which can be mitigated by using more rigid frameworks.

IV. CONCLUSION

A numerical model based on the DCS method has been presented for calculating the motion-induced EC caused by the uniform rectilinear motion of a conductor near an excitation source. During the discretization step, local refined current sources were utilized for acquiring tradeoff between accuracy and efficiency. The MFD obtained by the DCS model was compared with FEA, which shows great agreement with each other, demonstrating the accuracy of the established model. Besides, with geometry-related coefficient matrices calculated offline, the DCS model takes much less time than FEA to accomplish the computation for each velocity.

Parametric study about the motion velocity was conducted and the variation of EC-generated MFD are calculated. The results suggest that mechanical scanning do has some impacts on EC testing, but they can be partially alleviated by carefully selecting sensing points according to application purpose. For a cylindrical coil, the point under the coil center is more suitable for contactless speed estimation. Based on this result, a speed sensor prototype was assembled, and its response was experimentally investigated. Testing results indicate that for stationary movements high accuracy can be obtained with both DC and AC excitation; the maximum relative error is 4%. For an acceleration process, the sensor prototype can successfully catch the speed variation, exhibiting its relatively broad bandwidth. Some fluctuations and disagreement appear in transient testing results while they are mainly induced by manufacturing and assembling errors; thus, they can be eliminated by more precisely establishing the sensing system.

APPENDIX

Relation between the Derivative of a Field in Two Frames

According to (1), for any vector field (A.1) holds when \( \mathbf{F} \) and \( \mathbf{f} \) denote the expression of the same vector field in \( O-XYZ \) and \( o-xyz \), respectively

\[
\mathbf{F}(x, y - v_0 y, z, t) = \mathbf{f}(x, y, z, t) \tag{A.1}
\]

Taking the derivative for both sides of (A.1) with respect to \( t \)

\[
\frac{\partial \mathbf{F}}{\partial t} = \frac{\partial \mathbf{f}}{\partial t} \tag{A.2}
\]

With chain rule the right side of (A.2) can be rewritten as

\[
\frac{\partial \mathbf{F}}{\partial t} = -v_0 \mathbf{F}'(x, y - v_0 t, z, t) + \mathbf{F}'(x, y, z, t) \tag{A.3}
\]

where \( \mathbf{F}'(x_1, x_2, x_3, x_4) \) means taking the derivative of \( \mathbf{F} \) with respect to \( x_i \). Combining (1), (2) and (3) gives

\[
\frac{\partial \mathbf{f}}{\partial t} = -v_0 \frac{\partial \mathbf{F}(X, Y, Z, t)}{\partial Y} + \frac{\partial \mathbf{F}(X, Y, Z, t)}{\partial t} \tag{A.4}
\]

Equation (A.4) describes the relationship between derivatives of a field in different coordinate frames used in Fig. 1.

REFERENCES


