2023 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM)
June 28-30, 2023. Seattle, Washington, USA

Prediction-Based Control for Uncertain Systems With Input Time Delay and Disturbance

Seong-Min Lee, *Member, IEEE* and Hungsun Son, *Member, IEEE*

***Abstract*—This paper presents a new concept of a predictive controller utilizing future reference trajectory to reduce the delay effect. Time delay critically affects mechatronics systems, and it is still a remaining issue that must be solved. However, existing controllers have a limitation of input type as a constant desired trajectory, and it may limit implementation into various applications. The controller in the paper overcomes the limitation and is capable of tracking time-varying trajectories. The simulation results show that the controller can be applied to more general trajectories for practical applications.**

I. Introduction

The time delay problem has frequently occurred in various industrial applications, including mechatronics and robotic systems, due to latencies by wireless communication and heavy computation to process complex algorithms. The delay may cause the control system to be unstable with operation failure. To overcome the time delay problem and ensure desired control performance, several types of research have been intensively explored. The main challenge for developing delay-tolerant control is to guarantee control stability and improve tracking control performance despite time delay and disturbance.

An overview of the existing strategies is given in the sequel.

1) *Prediction-based controller:* The main concept of the method is to predict the system state to obtain delay-free dynamics in the class of linear systems. The advantage of the control is to deal with a large time delay, where the future state prediction can improve the tracking performance. The standard prediction method using finite spectrum assignment (FSA) is first introduced in [1], and further, the reduction method is developed [2]. However, the methods do not show robustness to uncertainties and external disturbances. Several prediction strategies have been studied [3]–[7] to ensure robust tracking control. The researchers developed a new prediction method [3], [4] robust to the disturbance without disturbance observer (DOB). In the research [5], [6], the future information of the disturbance is predicted to more precisely obtain the future state. They improve state prediction accuracy under disturbance as well as the time delay. In addition, robust controllers have been developed based on the sliding mode control [8], [9] to compensate for the disturbance robustly. However, the existing prediction-based controllers ensure the tracking control for only constant reference trajectory, and eventually, it will result in delayed tracking control.
2) *Predictor-free feedback controller:* Several methods have been explored to deal with input delay. The control methods [10], [11] are predictor-free feedback handling nonlinear systems. However, the controller is limited in dealing with large delays due to the absence of state prediction. Otherwise, predictor-based feedback controllers [12], [13] can handle large delays due to the advantage of allowing exact state prediction under time delay and disturbance. However, the methods have been developed without a future trajectory, resulting in delayed convergence for time-varying reference trajectories. The performance of the proposed controller in this paper shows fast and simple expandability to existing predictor-based controllers ensuring the convergence of the state to time-varying trajectories.

In this paper, a new predictive control is proposed for uncertain linear systems under time delay and disturbance. The proposed controller utilizes an enhanced state prediction method using disturbance prediction and future reference trajectory. Following the controller, it is ensured that the state converges to the desired trajectory when the predicted state converges to the future reference. As a result, it can compensate for the time delay effect and achieve the desired tracking.

The remainder of the paper is organized as follows. The problem is first formulated in Section II. The uncertain linear system with input time delay is represented. In addition, the existing predictive controller is analyzed. In Section III, the proposed controller is presented, and its effectiveness is proved. It is also verified from numerical simulations that the proposed controller can improve the tracking performance despite time delay and external disturbance in Section IV. Section V concludes this paper.

II. Problem Formulation

A. System Modeling With Input Delay

Consider an open-loop LTI system with an input delay, parametric uncertainties ΔA, ΔB, and unknown external disturbance d as

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A)x(t)+(B+\Delta B)u(t-h)+d \\ &= Ax(t)+Bu(t-h)+d_1(t)\end{aligned} \tag{1}$$

This work was supported in part by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2020R1A6A1A03040570), the Ministry of Trade, Industry and Energy (MOTIE) and Korea Evaluation Institute of Industrial Technology (KEIT) through the Helicopter Electric Multiple Tail Rotor Technology R&D program under Grant (RS-2022-00155776), and, Future Innovation Research Funds of the Ulsan National Institute of Science and Technology (1.230014.01), respectively.

Seong-min Lee is a research assistant professor in Department of Mechanical Engineering, Ulsan National Institute of Science and Technology, Ulsan, South Korea. (e-mail: leesm128@unist.ac.kr)

Hungsun Son (corresponding author) is a professor in Department of Mechanical Engineering, Ulsan National Institute of Science and Technology, Ulsan, South Korea (email: hson@unist.ac.kr)

TABLE I
COMPARISON OF PREDICTIVE CONTROLLERS

	[4]	[6]	Proposed control
Predictive scheme	$X_p(t) = x_p(t) + x(t) - x_p(t-h)$, (see (4) for $x_p(t)$)	$X_{p3}(t) = X_{p2}(t) + x(t) - X_{p2}(t-h)$	$X_{p2}(t) = e^{Ah}x(t) + \int_{t-h}^{t} e^{A(t-s)}[Bu(s) + \hat{d}_1(s+h)]\mathrm{d}s$
Tracking error	$\tilde{X}_p(t) = X_p(t) - x_r(t)$	$X_{p3}(t) = X_{p3}(t) - x_r(t)$	$\tilde{X}_{p2}(t) = X_{p2}(t) - x_r(t+h)$
Control input	$u(t) = u_{PI}(\tilde{X}(t)) + u_r$ $u_r(t) = -ab^{-1}x_r$	$u(t) = u_{PI}(X_{p3}(t)) + u_r(t) - b^{-1}\hat{d}_1(t+h)$ $u_r(t) = b^{-1}(\dot{x}_r(t) - ax_r(t))$	$u(t) = u_{PI}(\tilde{X}_{p2}(t)) + u_r(t) - b^{-1}\hat{d}_1(t+h)$ $u_r(t) = b^{-1}(\dot{x}_r(t+h) - ax_r(t+h))$

where the subscript *PI* represents the PI controller.

where $d_1(t) = \Delta Ax(t) + \Delta Bu(t-h) + d$. $x(t) \in \mathbb{R}^n$ is the state vector of the system and is fully measurable, $u(t-h) \in \mathbb{R}^m$ is the control input with a time delay $h \in \mathbb{R}$, and $d_1(t) \in \mathbb{R}^n$ is the disturbance. The disturbance d_1 includes parametric uncertainties $\Delta A \in \mathbb{R}^{n\times n}$ and $\Delta B \in \mathbb{R}^{n\times n}$ and a constant disturbance $d \in \mathbb{R}$. $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times n}$ are system and input matrices, respectively.

Assumption 1: The pair (A,B) is controllable, where the matrices A and B are known.

Assumption 2 ([4]): The constant and unknown parametric uncertainties ΔA and ΔB do not affect the controllability of the dynamic system.

Assumption 3 ([5]): The unknown disturbance d is bounded by $\|d_1(t)\| \le D_0$ and it is $(r+1)$-times continuously differentiable with $\|d_1^{(r+1)}(t)\| \le D_{r+1},\ \forall t \ge 0$.

Assumption 4: The time delay h is constant and known.

The dynamics of the error $e(t) = x(t) - x_r(t)$ without delay and uncertainties can be expressed as follows:

$$\begin{aligned}\dot{e}(t) &= Ae(t) + Bu'(t) + Ax_r(t) - \dot{x}_r(t) + Bu_r(t) \\ &= Ae(t) + Bu'(t)\end{aligned} \tag{2}$$

$$0 = Ax_r(t) - \dot{x}_r(t) + Bu_r(t) \tag{3}$$

where $u(t) = u'(t) + u_r(t)$ and $u'(t)$ is a feedback controller to stabilize the state $x(t)$. $x_r(t)$ is continuously differentiable and it is noted that the control input $u_r(t)$ to maintain x at the equilibrium point x_r is represented as (3) on the contrary to [1, (2)]. The following relation can be obtained from (3) as $u_r(t) = (A^TA)^{-1}A^T[Ax_r(t) - \dot{x}_r(t)]$.

B. Analysis of Existing Predictive Controller

In the existing research [4], a predictive control strategy is presented by utilizing a modified prediction method [3] as follows:

$$X_p(t) = x_p(t) + x(t) - x_p(t-h) \tag{4}$$

where $x_p(t) = \hat{x}(t+h) = e^{Ah}x(t) + \int_{t-h}^{t} e^{A(t-s)}Bu(s)\mathrm{d}s.$

The predictive control scheme based on [4, Proposition 2] with the prediction $X_p(t)$ can achieve desired tracking control, ensuring that the system state $x(t)$ eventually converges to the reference x_r. However, it should be assumed that x_r is constant to verify the convergence. Unfortunately, the limitation leads to a decrease in the effectiveness of the controller for practical applications, including time-varying trajectories. Moreover, [4, Proposition 2] is verified from the experimental results [4, Figs. 2, 3]. However, the results represented in [4, Figs. 6, 7] cannot fully demonstrate the convergence of $x(t)$ to $x_r(t)$ due to the assumption that x_r is constant. This paper intends to explore the limitation of the previous control in [4] for more general trajectories.

Differentiating (4) and applying (1) result in the dynamics of $X_p(t)$ as

$$\dot{X}_p(t) = AX_p(t) + Bu(t) + d_2(t) \tag{5}$$

$$d_2(t) = d_1(t) + e^{Ah}\left[d_1(t) - d_1(t-h)\right] \tag{6}$$

where (5) is equivalent to [4, (14)]. In (5), as referred to in [4], too large parametric uncertainties ΔA and ΔB will incur a decrease in the prediction performance due to d_2 in (6). The tracking error dynamics of $\tilde{X}_p(t) = X_p(t) - x_r(t)$ is represented as follows:

$$\dot{\tilde{X}}_p(t) = A\tilde{X}_p(t) + Bu(t) + Ax_r(t) - \dot{x}_r(t) + d_2(t) \tag{7}$$

Lemma 1 ([4]): Consider uncertain dynamic system (1) satisfying Assumptions 1, 2, 4, and reference trajectory x_r is constant, $\dot{x}_r(t) = 0, \forall t > 0$. The disturbance d is sufficiently slowly varying with respect to the delay size [1]. Suppose that there exists a predictive controller $u(\tilde{X}_p(t))$ continuous at the origin such that $\tilde{X}_p(t)$ and its time derivative $\dot{\tilde{X}}_p(t)$ tend to zero. Then, the state $x(t)$ converges to the reference x_r.

Proof: See Appendix A.

Remark 1: Notice that in Lemma 1, the convergence for the constant x_r was only demonstrated, and it can decrease the effectiveness of the controller for practical applications.

III. PREDICTIVE CONTROL WITH FUTURE REFERENCE TRAJECTORY

In this section, the new predictive control strategy is proposed by using the future reference trajectory to compensate for the delay effect. First, the following Proposition verifies that the existing predictive controller cannot guarantee the desired tracking performance for time-varying trajectories.

Proposition 1: Consider the system (1) satisfying Assumption 1–4 and a controller $u(\tilde{X}_p)$ continuous at the origin such that $\tilde{X}_p(t)$ and $\dot{\tilde{X}}_p(t)$ tend to zero. Then, the convergence of $x(t)$ to the time-varying reference $x_r(t)$ is not ensured.

Proof: Consider that $\tilde{X}_p(t)$ converges to zero by applying the controller $u(\tilde{X}_p(t))$. Then, from (7), it is noted that when the reference $x_r(t)$ is time-varying, [4, (18)] is not satisfied, but the following relation would hold

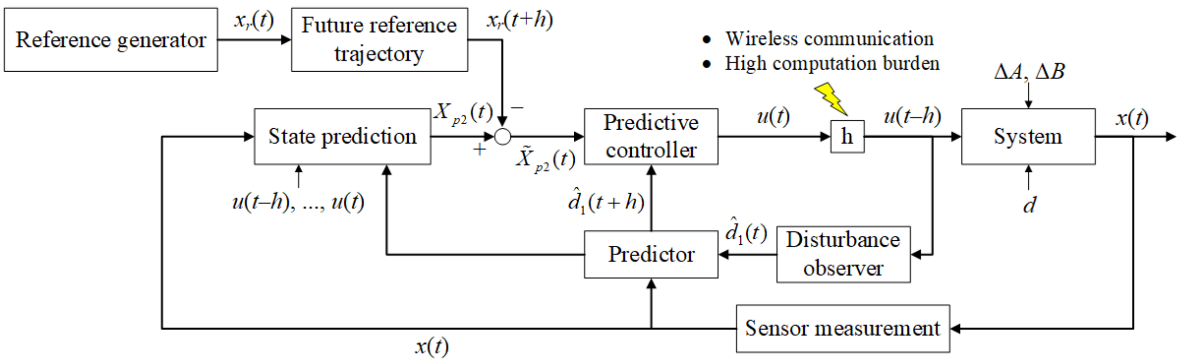


Fig. 1. Structure of predictive control with future reference trajectory.

$$\begin{aligned}\lim_{t\to+\infty} Bu\left(\tilde{X}_p(t)\right) &= Bu\left(\tilde{X}_{p,\infty}=0\right) = Bu_\infty \\ &= \lim_{t\to+\infty}\left(\dot{x}_r(t) - Ax_r(t) - d_2(t)\right)\end{aligned} \tag{8}$$

From [1, (13)] and (8), it follows that

$$\begin{aligned}\lim_{t\to+\infty} X_p(t) &= \lim_{t\to+\infty}\left(e^{Ah}x(t) + \int_{t-h}^{t} e^{A(t-s)}\left[Bu(s)+d_1(s)\right]\mathrm{d}s\right) \\ &= e^{Ah}x_\infty + \lim_{t\to+\infty}\int_{t-h}^{t} e^{A(t-s)}\left[\dot{x}_r(s) - Ax_r(s)\right]\mathrm{d}s \\ &\quad - \lim_{t\to+\infty}\int_{t-h}^{t} e^{A(t-s+h)}\left[d_1(s) - d_1(s-h)\right]\mathrm{d}s\end{aligned} \tag{9}$$

where $\lim_{t\to+\infty}x(t) = x_\infty$. Noting that A and $e^{A(t-s)}$ are commutative and $e^{A(t-s)}\left[\dot{x}_r(s) - Ax_r(s)\right] = \frac{\mathrm{d}\left(e^{A(t-s)}x_r(s)\right)}{\mathrm{d}s}$, and $\tilde{X}_\infty = 0$, (9) becomes that

$$0 = e^{Ah}\left(x_\infty - v_{r,\infty}\right) - \underbrace{\lim_{t\to+\infty}\int_{t-h}^{t} e^{A(t-s+h)}\left[d_1(s) - d_1(s-h)\right]\mathrm{d}s}_{=\varepsilon_{d1}} \tag{10}$$

where $v_r(t) = x_r(t-h)$ and $\lim_{t\to+\infty}v_r(t) = v_{r,\infty}$. It should be noted that $\lim_{t\to+\infty}\|x_r(t) - v_r(t)\| \neq 0$. Then, since $e^{Ah} \neq 0$, one has $x_\infty = v_{r,\infty}$ despite the convergence of ε_{d1} to zero. The proof is complete. ■

Remark 2: The prediction $X_p(t)$ is limited to attenuating ε_{d1} for time-varying reference. In addition, suppose that ε_{d1} is perfectly removed. Then, when $\lim_{t\to+\infty}\tilde{X}_p(t) = 0$, the state x tends to v_r, showing delayed convergence. As a result, the predictive controller [4] is limited to only constant reference trajectories.

Proposition 2: Consider the system (1) satisfying Assumptions 1–4. Suppose that the reference $x_r(t+h)$ at time $t+h$ can be predicted. In addition, there exists a controller $u(\tilde{X}_{p2})$ continuous at the origin such that $\tilde{X}_{p2}(t)$ and $\dot{\tilde{X}}_{p2}(t)$ tend to zero, where the tracking error is defined as $\tilde{X}_{p2}(t) = X_{p2}(t) - x_r(t+h)$ and $X_{p2}(t) = e^{Ah}x(t) + \int_{t-h}^{t} e^{A(t-s)}[Bu(s) + \hat{d}_1(s+h)]\mathrm{d}s$ presented in [5]. The future information of disturbance $\hat{d}_1(t+h)$ can be predicted from exponential disturbance predictors [5], [6], [17] or a finite-time disturbance predictor [16]. Then, the convergence of the state $x(t)$ to the time-varying reference $x_r(t)$ is ensured without delay effect. Table I shows the comparison of the proposed controller with [4] and [6].

Proof: Similar to (5), the dynamics $X_{p2}(t)$ can be rewritten as

$$\dot{X}_{p2}(t) = AX_{p2}(t) + Bu(t) + \hat{d}_1(t+h) - e^{Ah}\varepsilon_{d2}(t-h) \tag{11}$$

where $\varepsilon_{d2}(t) = \hat{d}_1(t+h) - d_1(t+h)$ and error dynamics of $\tilde{X}_{p2}(t)$ is represented as

$$\begin{aligned}\dot{\tilde{X}}_{p2}(t) &= A\tilde{X}_{p2}(t) + Bu(t) + \hat{d}_1(t+h) - e^{Ah}\varepsilon_{d2}(t) \\ &\quad - \dot{x}_r(t+h) + Ax_r(t+h)\end{aligned} \tag{12}$$

Considering the controller $u(\tilde{X}_{p2})$, which can make $\tilde{X}_{p2}$ tend to zero, it is deduced that

$$\begin{aligned}&Bu\left(\tilde{X}_{p2,\infty}=0\right) = Bu_\infty \\ &= \lim_{t\to+\infty}\left(\dot{x}_r(t+h) - Ax_r(t+h) - \hat{d}_1(t+h) + e^{Ah}\varepsilon_{d2}(t)\right)\end{aligned} \tag{13}$$

From (13) and the definition of $\tilde{X}_{p2}(t)$, one has the following equation:

$$\begin{aligned}&\lim_{t\to+\infty}\tilde{X}_{p2}(t) \\ &= \lim_{t\to+\infty}\left(e^{Ah}x(t) + \int_{t-h}^{t} e^{A(t-s)}\left[Bu(s) + \hat{d}_1(s+h)\right]\mathrm{d}s - x_r(t+h)\right) \\ &= \lim_{t\to+\infty}\left(\begin{aligned}&\int_{t-h}^{t} e^{A(t-s)}\left[\dot{x}_r(s+h) - Ax_r(s+h)\right]\mathrm{d}s \\ &+e^{Ah}x(t) - x_r(t+h)\end{aligned}\right) + \varepsilon_{d3} \\ &= e^{Ah}\left(x_\infty - x_{r,\infty}\right) + \varepsilon_{d3}\end{aligned} \tag{14}$$

where $\varepsilon_{d3} = \lim_{t\to+\infty}\int_{t-h}^{t} e^{A(t-s+h)}\varepsilon_{d2}(t)\mathrm{d}s$. The error ε_{d3} can be converged by the exponentially stable predictor [5] with Assumption 3 and minimized by accurate prediction of $d_1(t+h)$. Thus, when $\lim_{t\to+\infty}\tilde{X}_{p2}(t) = 0$, since $e^{Ah} \neq 0$, x eventually tends to x_r with sufficiently small bounds. The proof is complete. ■

Remark 3: Note that the tracking error $e(t)$ depends on the prediction error for $x_r(t+h)$ and $d_1(t+h)$ as well as the tracking error for $\tilde{X}_{p2}(t)$ against disturbance and parametric uncertainties.

IV. Numerical Simulations

In the numerical simulation, Propositions 1 and 2 are verified with a dc motor and an unmanned aerial vehicle (UAV).

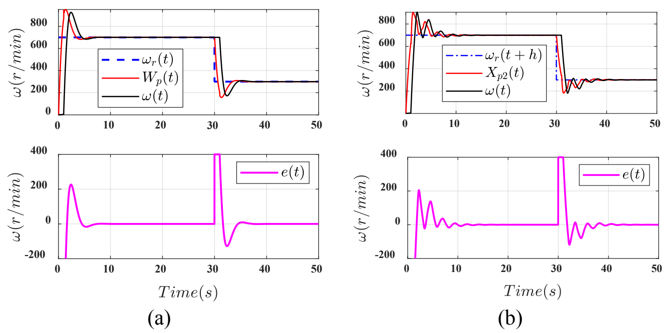


Fig. 2. Simulation results to prove Lemma 1. (a) Trajectory tracking of $W_p(t)$. (b) Trajectory tracking of X_{p2}. ($e(t) = \omega(t) - \omega_r(t)$)

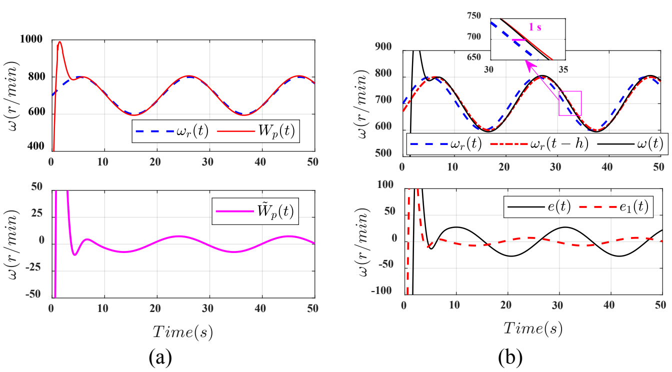


Fig. 3. Simulation results to prove Proposition 1. (a) Trajectory tracking of $W_p(t)$. (b) Trajectory tracking of ω. ($\tilde{W}_p(t) = W_p(t) - \omega_r(t)$ represents tracking error, $e(t) = \omega(t) - \omega_r(t)$, and $e_1(t) = \omega(t) - \omega_r(t-h)$)

A. Application to dc motor

The first-order dc motor dynamics [1] can be defined as

$$\begin{aligned}\dot{\omega}(t) &= (a+\Delta a)\omega(t) + (b+\Delta b)u(t-h) + d \\ &= a\omega(t) + bu(t-h) + d_1\end{aligned} \tag{15}$$

where $d_1(t) = \Delta a\omega(t) + \Delta bu(t-h) + d$. $\omega(t)$ and $u(t)$ represent the angular velocity of the motor and input voltage, respectively. The mechanical parameters are listed in Table II. It is assumed that the input time delay h = 1sec is known. Two cases are simulated to verify the tracking control for constant and time-varying reference trajectories. The constant reference trajectory is set as $\omega_r(t)$ = 700 ($t < 30$), 300 ($30 \le t < 60$), 500 ($60 \le t$) r/min. The time-varying reference velocity is set as $\omega_r(t)$ = 700 + 100sin(0.3t) r/min, and the system starts from $\omega(0) = 0$.

The PI controller similar to [4, (51)] is utilized to compare the performance. In [4], $W_p(t)$ is utilized for the predictive controller ($W_p(t) = \omega_p(t) + \omega(t) - \omega_p(t-h)$, where ω_p is the standard predictive scheme in [1, (4)]). The tracking error is represented as $\tilde{W}_p(t) = W_p(t) - \omega_r(t)$. On the contrary, the proposed controller uses the prediction method expressed as $X_{p2}(t) = e^{ah}\omega(t) + \int_{t-h}^{t} e^{a(t-s)}[bu(s) + \hat{d}_1(s+h)]\mathrm{d}s$ with the prediction of future disturbance information. An exponentially stable predictor (see [5, Lemma 6]) can be applied to compute $\hat{d}_1(t+h)$, where the parameters r and p_0 are set as $r = 2$ and $p_0 = 10$ [5]. The tracking error is defined as $\tilde{X}_{p2}(t) = X_{p2}(t) - \omega_r(t+h)$. Then, the proposed control input is computed as

$$u(t) = -k_p\tilde{X}_{p2}(t) - k_I\int_0^t \tilde{X}_{p2}(t)dt + u_r(t) - b^{-1}\hat{d}_1(t+h) \tag{16}$$

$$u_r(t) = b^{-1}(\dot{\omega}_r(t+h) - a\omega_r(t+h)) \tag{17}$$

Note that the control input u_r is determined as (17) instead of $u_r = -ab^{-1}\omega_r$.

TABLE II
List of Specifications of Mechanical Parameters

Nomial parameters: a = –1/1.1 s^{-1}, b = 894/1.1 $min^{-1}s^{-1}$
Parametric uncertainties: $\Delta a = -0.2a$, $\Delta b = 0.2b$
Constant disturbance: d = 5 r•$min^{-1}s^{-1}$
Time delay: h = 1sec

1) Case 1: Simulation With Constant Reference Trajectory

In Lemma 1, the proposed controller can guarantee the convergence of the state to the reference despite input time delay and disturbance. Fig. 2 shows the tracking results for constant reference trajectory. For the predictive controller [4], when the predicted state $W_p(t)$ tends to the reference ω_r, the system state $\omega(t)$ tracks ω_r without delay effect. The proposed controller shows a similar result to demonstrate Lemma 1. Fig. 2(b) shows the chattering due to the derivative of disturbance to obtain future information of the disturbance.

2) Case 2: Simulation With Time-Varying Reference Trajectory

Fig. 3 shows that the controller [4, (51)] can make the predicted state W_p track the reference ω_r, and then, the state ω tends to $\omega_r(t-h)$ instead of $\omega_r(t)$, verifying Proposition 1.

The tracking error $\tilde{W}_p(t)$ shown in Fig. 3(a) is caused by the prediction error in $W_p(t)$, but it can be reduced by applying a robust controller and enhanced prediction strategy. In addition, the result in Fig. 3(b) implies the predictive controller [4] ensures delayed convergence showing that $e_1(t)$ only tends to zero with bounds, but it does not guarantee the convergence of $e(t)$, where $e(t) = \omega(t) - \omega_r(t)$ and $e_1(t) = \omega(t) - \omega_r(t-h)$. Thus, the results show that the controller is limited to the time-varying trajectory.

Fig. 4 also demonstrates Proposition 2, showing that when $X_{p2}(t)$ tends to $\omega_r(t+h)$, $\omega(t)$ eventually tracks $\omega_r(t)$ in spite of time delay, parameter uncertainties, and disturbance. It implies that the proposed predictive controller can achieve desired tracking control without delay effect by applying predicted future reference $\omega_r(t+h)$. In addition, reminding Remark 3, the bounded error in Fig. 5 can be reduced by accurate prediction of $d_1(t+h)$ and robust tracking control to converge $\tilde{X}_{p2}(t)$ to zero. Fig. 5 shows the prediction results of the future information of

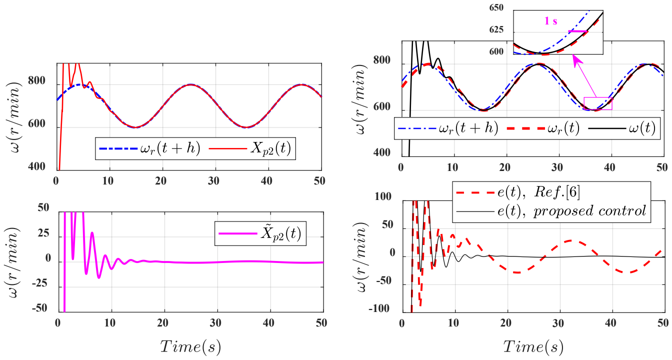


Fig. 4. Tracking results for the proposed control. (a) Trajectory tracking of $X_{p2}(t)$. (b) Performance comparison of tracking error $e(t)$. ($\tilde{X}_{p2}(t) = X_{p2}(t) - \omega_r(t)$ represents tracking error and $e(t) = \omega(t) - \omega_r(t)$)

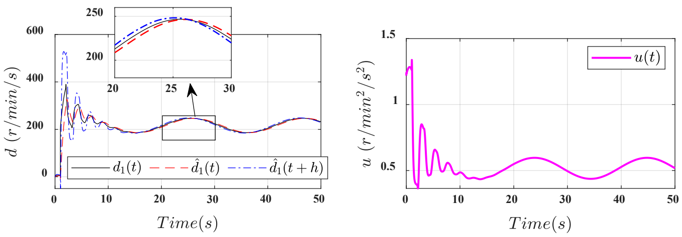


Fig. 5. External disturbance $d_1(t)$ and control input $u(t)$.

the disturbance $\hat{d}_1(t+h)$ and control input $u(t)$ for the proposed control.

Remark 4: In [4, Fig. 6–7], the experimental results only showed the convergence of the predicted states $\omega_1(W_p)$ and $\omega_2(\omega_p)$ to ω_r, but not the real state ω. Then, $\omega(t)$ will tend to $\omega_r(t\text{–}h)$, noting Proposition 1 and Fig. 3. Thus, the results and Lemma 1 cannot guarantee the convergence of the motor velocity ω to the time-varying desired velocity ω_r.

3) Simulation Results and Discussions

The performance of the proposed controller is compared with other predictive controllers in [6] shown in Table I. The controller in [6] showed better performance than the existing controller [4]. Similarly, the control inputs are compared in Table I, where the existing controllers utilized the current trajectory. The result in Fig. 4(b) shows that the controller in [6] is still limited to the time-varying reference, requiring the future reference trajectory. As expected, the proposed controller performs better and effectively converges the error to zero.

In practice, the delay could be measured by using time-stamped packets [14] and estimated by utilizing control inputs $u(t)$ and $u(t\text{–}h)$ [4], [15].

B. Application to multirotor UAV

The performance of the proposed control system is investigated with a multirotor UAV. The dynamic model, including input time delay for the multirotor UAV [16], is represented as follows:

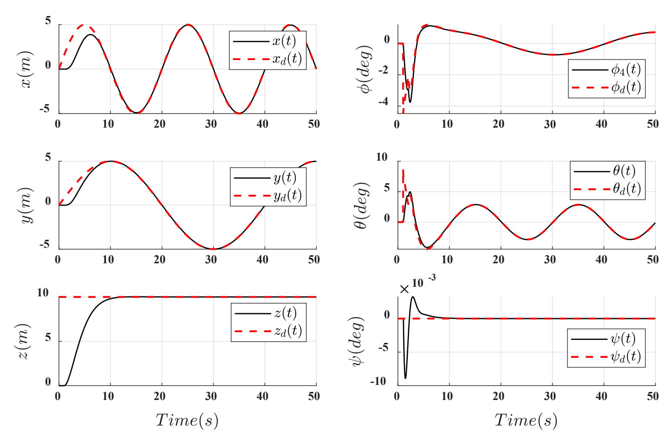
$$\dot{x}(t) = f(x, u_1(t-h), u_2(t)) + d(t) \tag{18}$$

where $x = [x_1^T \ x_2^T]^T = [[x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z}] \ [\phi \ \theta \ \psi \ \dot{\phi} \ \dot{\theta} \ \dot{\psi}]]^T$, $f_1(x,u_1,u_2) - \Delta f(x,u_1,u_2) = f(x,u_1,u_2) = [u_1 - c_1\dot{x} \ \ u_2 - c_2\dot{y} \ \ u_3 - c_3\dot{z} \ \ U_2 - c_4\dot{\phi} + (I_{yy} - I_{zz})\dot{\theta}\dot{\psi}/I_{xx} \ \ U_2 - c_4\dot{\theta} + (I_{zz} - I_{xx})\dot{\phi}\dot{\psi}/I_{yy} \ \ U_4 - c_6\dot{\psi} + (I_{xx} - I_{yy})\dot{\phi}\dot{\theta}/I_{zz}]^T$, and $d = [d_1^T \ d_2^T]^T - \Delta f(x,u_1,u_2)$. The nonlinear dynamics (18) can be simply linearized as $\dot{x}_1(t) = A_1x_1(t) + B_1[u_1(t) + d_1(t)]$ and $\dot{x}_2(t) = A_2x_2(t) + B_2[u_2(t\text{–}h) + d_2(t)]$, where A_1 and A_2 are system matrices, B_1 and B_2 are input matrices. $u_1 = [u_x \ u_y \ u_z]^T = [(c_\phi s_\theta c_\psi + s_\phi s_\psi)U_1 \ \ (c_\phi s_\theta s_\psi - s_\phi c_\psi)U_1 \ \ c_\phi c_\theta U_1 - g]^T$, where and c indicate sine and cosine functions of the subscript angle, respectively. $u_2 = [U_2 \ U_3 \ U_4]^T$ and $[U_1 \ U_2 \ U_3 \ U_4]^T = \Lambda F$ (see [16]), where $F = [F_1 \ F_2 \ F_3 \ F_4]^T$ is a motor input; d_1 and d_2 are external disturbances. The mechanical parameters for simulations are $I_{xx} = I_{yy} = 0.032$ kgm², $I_{zz} = 0.064$ kgm², c_i (i=1,2,…,6) = 0.01s⁻¹, and $\Lambda = [1.11I_{4\times1} \ 5.16I_{4\times1} \ 5.16I_{4\times1} \ 2.34I_{4\times1}]^T$. The time delay is set to be as 1 sec.

The control input is defined by utilizing the prediction method as follows:

$$\begin{cases} u_1(t) = -K_1e_1(t) + u_{d1}(t) - \hat{d}_1(t+h) + G \\ u_2(t) = -K_2e_2(t) + u_{d2}(t) - \hat{d}_2(t+h) \end{cases} \tag{19}$$

where $G = [0 \ 0 \ 1]^T g$, $u_{d1} = [\ddot{x}_d(t+h) \ \ddot{y}_d(t+h) \ \ddot{z}_d(t+h)]^T$, and $u_{d2} = [\ddot{\phi}_d(t+h) \ \ \ddot{\theta}_d(t+h) \ \ \ddot{\psi}_d(t+h)]^T$. The control tracking error can be defined as $e_1(t) = X_{p2,\mathrm{pos}}(t) - x_{d1}(t+h)$ and $e_2(t) = X_{p2,\mathrm{att}}(t) - x_{d2}(t+h)$, where $x_{d1} = [x_d \ y_d \ z_d \ \dot{x}_d \ \dot{y}_d \ \dot{z}_d]^T$ and $x_{d2} = [\phi_d \ \theta_d \ \psi_d \ \dot{\phi}_d \ \dot{\theta}_d \ \dot{\psi}_d]^T$. Note the the attitude setpoint can be calculated by the following equations as $\phi_d = \sin^{-1}((u_x\sin\psi_d - u_y\cos\psi_d)/U_1)$ and $\theta_d = \tan^{-1}((u_x\cos\psi_d + u_y\sin\psi_d)/(u_z + g))$, where $U_1 = (u_x^2 + u_y^2 + (u_z+g)^2)^{0.5}$. The control parameters are chosen as $K_1 = [0.3I_{3\times3} \ 1I_{3\times3}]$ and $K_2 = [6I_{3\times3} \ 3I_{3\times3}]$. The reference trajectory is defined as $x_d = 5\sin(0.1\pi t)$ m, $y_d = 5\sin(0.05\pi t)$ m, and $z_d = 10$ m. $d_1 = -[0.3+c_{v,1}\dot{x} \ \ 0.3+c_{v,2}\dot{y} \ \ 0.3+c_{v,3}\dot{z}]^T$ and $d_2 = -[0.2+c_{v,4}\dot{\phi} + \dot{\theta}\dot{\psi}(I_{zz} - I_{yy})/I_{xx} \ \ 0.2+c_{v,5}\dot{\theta} + \dot{\phi}\dot{\psi}(I_{xx} - I_{zz})/I_{yy} \ \ 0.2+c_{v,6}\dot{\psi} + \dot{\phi}\dot{\theta}(I_{yy} - I_{xx})/I_{zz}]^T$.



(a)

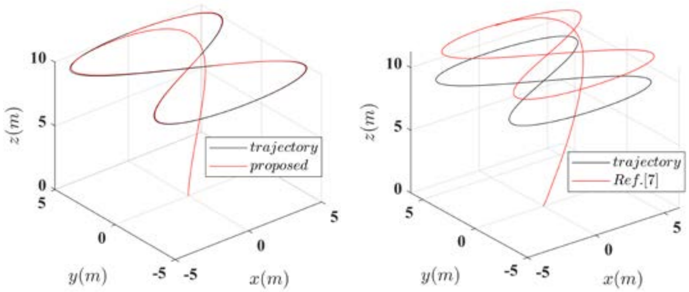


Fig. 6. Tracking results for a multirotor UAV. (a), (b) Control results for proposed control. (c) Control results for [7].

Fig. 6 shows the control results of the proposed controller with robustness to disturbances. In addition, the controller can compensate for the input delay. The existing controller [7] can stabilize the state of the UAV but with a large tracking error. The simulation results validate the proposed controller can be implemented in various applications utilizing time-varying reference trajectories in the presence of time delays and disturbances.

V. Conclusion

In this paper, it was shown that the existing predictive controllers have a limitation in the case of time-varying reference trajectories. Numerical simulation results demonstrate Proposition 1 and 2, verifying that the controller can be improved by predicted future reference to make the system state track the desired state without delay effect.

For future works, the time delay can be considered unknown and time-varying for practical implementation. In addition, it should be extended to a nonlinear system. The developed controller will be demonstrated by the actual platform to verify its effectiveness.

Appendix A
Proof of Lemma 1 [4]

The dynamics of $X_{\hat{p}}(t)$ in (5) can be rewritten as

$$\dot{X}_p(t) = (A+\Delta A)X_p(t) + Bu(t) + d_3(t) \qquad \text{(A1)}$$

where $d_3(t) = d_2(t) - \Delta Ax(t) - e^{Ah}\Delta A[x(t) - x(t-h)] - \Delta A\int_{t-h}^{t} e^{A(t-s)}B[u(s)-u(s-h)]\mathrm{d}s$ and $d_2(t) = \Delta Ax(t) + \Delta Bu(t-h) + d + e^{Ah}[\Delta Ax(t) + \Delta Bu(t-h) - \Delta Ax(t-h) + \Delta Bu(t-2h)]$. Consider that $\tilde{X}_p(t)$ $(= X_p(t) - x_r(t))$ converges to zero by applying the controller $u(\tilde{X}_p(t))$. Then, from (A1), the following relation holds

$$\begin{aligned}\lim_{t\to+\infty}(B+\Delta B)u\left(\tilde{X}_p(t)\right) &= \lim_{t\to+\infty}\left(\dot{x}_r - Ax_r - d_3(t)\right)\\ &= Bu_r - \Delta Ax_r - d \qquad \text{(A2)}\\ &= -(A+\Delta A)x_r - d\end{aligned}$$

From [1, (20)] and (A2), it follows that

$$\begin{aligned}\tilde{X}_p(\infty) &= e^{(A+\Delta A)h}x(\infty) - x_r + \int_{-h}^{0} e^{-(A+\Delta A)s}\mathrm{d}s\left[(B+\Delta B)u(\infty)+d\right]\\ &= e^{(A+\Delta A)h}x(\infty) - x_r - x_r\int_{-h}^{0} e^{-(A+\Delta A)s}(A+\Delta A)\mathrm{d}s\\ &= e^{(A+\Delta A)h}[x(\infty) - x_r]\end{aligned} \qquad \text{(A3)}$$

Then, since $e^{(A+\Delta A)h} \neq 0$, it is proved that $x_\infty = x_r$. The proof is complete. ■

References

[1] A. Manitius and A. W. Olbrot, “Finite spectrum assignment problem for systems with delays,” *IEEE Trans. Autom. Control*, vol. AC-24, no. 4, pp. 541–552, Aug. 1979.

[2] Z. Artstein, “Linear systems with delayed controls: a reduction,” *IEEE Trans. Autom. Control*, vol. AC-27, no. 4, pp. 869–879, Aug. 1982.

[3] V. Léchappé, E. Moulay, F. Plestan, A. Glumineau, and A. Chriette, “New predictive scheme for the control of LTI systems with input delay and unknown disturbances,” *Automatica*, vol. 52, pp. 179–184, 2015.

[4] V. Léchappé *et al*., “Delay estimation and predictive control of uncertain systems with input delay: Application to a dc motor,” *IEEE Trans. Ind. Electron*., vol. 63, no. 9, pp. 5849–5857, Sep. 2016.

[5] R. Sanz, P. Garcia, and P. Albertos, “Enhanced disturbance rejection for a predictor-based control of LTI systems with input delay,” *Automatica*, vol. 72, no. 10, pp. 205–208, Oct. 2016.

[6] Y. Du *et al*., “Disturbance rejection for input-delay system using observer-predictor-based output feedback control,” *IEEE Trans. Ind. Inform*., vol. 16, no. 7, pp. 4489–4497, Jul. 2020.

[7] R. Sanz, P. Garcia, Q. C. Zhong, and P. Albertos, “Predictor-based control of a class of time-delay systems and its application to quadrotors,” *IEEE Trans. Ind. Electron.*, vol. 64, no. 1, pp. 459–469, Jan. 2017.

[8] Y.-H. Roh and J.-H. Oh, “Robust stabilization of uncertain input-delay systems by sliding mode control with delay compensation,” *Automatica*, vol. 35, no. 11, pp. 1861–1865, 1999.

[9] H. Caballero-Barragán, L. P. Osuna-Ibarra, A. G. Loukianov, and F. Plestan, “Sliding mode predictive control of linear uncertain systems with delays,” *Automatica*, vol. 94, pp. 409–415, 2018.

[10] S. Obuz, J. R. Klotz, R. Kamalapurkar, W. Dixon, “Unknown time-varying input delay compensation for uncertain nonlinear systems,” *Automatica*, vol. 76, pp. 222–229, 2017.

[11] B. C. Allen, C. A. Cousin, C. A. Rouse, and W. E. Dixon, “Robust cadence tracking for switched FES-cycling with an unknown time-varying input delay,” *IEEE Trans. Control Syst. Technol.*, vol. 30, no. 2, pp. 827–834, Mar. 2022.

[12] X. Xu, L. Liu, M. Krstic, and G. Feng, “Predictor feedback and integrator backstepping of linear systems with distributed unbounded delays,” *Int. J. Robust Nonlinear Control* vol. 32, no. 6, pp. 3281–3291, Apr. 2022.

[13] R. Sanz, P. Garcia, E. Fridman, and P. Albertos, “Robust predictive extended state observer for a class of nonlinear systems with time-varying input delay,” *Int. J. Control*, vol. 93, no. 2, pp. 217–225, 2020.

[14] C.-L. Lai and P.-L. Hsu, “Design the remote control system with the time-delay estimator and the adaptive smith predictor,” *IEEE Trans. Ind. Inform*., vol. 6, no. 1, pp. 73–80, Feb. 2010.

[15] Y. Deng, V. Léchappé, S. Rouquet, E. Moulay, and F. Plestan, “Super-twisting algorithm-based time-varying delay estimation with external signal,” *IEEE Trans. Ind. Electron*., vol. 67, no. 12, pp. 10663–10671, Oct. 2020.

[16] S.-M. Lee, M. Shin, and H. Son, “Robust predictor-based control for multirotor UAV with various time delays,” *IEEE Trans. Ind. Electron*., vol. 70, no. 8, pp. 8151–8162, Aug. 2023.

[17] S.-M. Lee and H. Son, “Prediction-based preview control of motion platform with time delay,” *IEEE Trans. Intell. Transp*., vol. 23, no. 12, pp. 23346–23357, Dec. 2022.