Human Cooperation Control of Two-Wheeled Robot for Stairs Riding Up*

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Abstract—In recent years, population aging has progressed worldwide. It is predicted that labor shortage due to population aging will become more serious. Therefore, it is expected that luggage transport robots be introduced into human living environments. However, stairs is an issue that must be addressed when using robots in the human environment. Typically, robots are large and mechanical, and their movement speed tends to decrease. Therefore, in this study, we propose a stair-ride-up method using a two-wheeled robot. The proposed method achieves the ride-up of stairs via a pushing force through cooperation with humans. Robots that use the proposed method can be manufactured in a small size, incurs a low cost, and offers a relatively high climbing speed. Experiments are conducted for cases of ride-up with and without control. Results show that only half of the work is required by the control. Furthermore, it is confirmed that the proposed method can achieve a ride-up stair speed of 0.3 step/s.

I. INTRODUCTION

In recent years, population aging has progressed worldwide [1]. Fertility rates are declining in many countries, in particular developed countries. Hence, chronic labor shortage is expected in the future. Therefore, robots are being introduced to human living environments. However, to use robots in the human environment, issues pertaining to stairs must be addressed. Various methods and mechanisms for stair-climbing robots have been proposed previously, and they can be primarily classified into five methods, as follows [2], [3], [4], [5], [6], [7], [8], [9], [10]:

- Planetary wheel
- Mechanically Movable Wheel
- Crawler
- Joints and legs
- Jumping

Each of these five methods has their own advantages and disadvantages. In general, they are mechanical approaches, and most of them tend to exhibit low climbing speeds. In addition, the mechanism size must be increased to overcome a difference of approximately 18 to 20 cm, such as in stairs. Therefore, aiming to the development of a compact and fast stair climbing capable robot. In [4], a planetary gear mechanism was used to ascend and descend steps. However, the height assumed in the experiment was 5 cm. Therefore, it was assumed that the wheel enlarged when the height of the general stairs was approximately 18 cm. In [9], a double pendulum mechanism was used to increase and decrease the number of steps in stairs. However, the motion was complex and a significant amount of time was required to ride up the stairs. In [2], a robot with a stair-climbing function was developed using a two-wheeled robot, where a pulley-and-belt mechanism was used in the robot body. Tires with a diameter of 508 mm were used based on the assumption that the height of the stairs was 120–130 mm. Meanwhile, the wheel diameter of the robot used in this study was 360 mm based on the assumption that the height of the stairs was 180 mm. Therefore, it can be assumed that the height of the stairs was higher than those in similar studies, and the robot was more compact.

It can be inferred that many problems exist in previous studies that used robots in the real environment. Herein, we propose a new control method for a two-wheeled robot. The proposed robot was designed such that the wheel radius and height of the stairs were similar to prevent a large robot size. Therefore, it was difficult to ride the stairs even when drive wheels were used [11]. Accordingly, the proposed method utilizes the pushing force through cooperative control with humans. By utilizing the pushing force and increasing the friction force, the robot can ride up the stairs and is expected to achieve the same or a higher ride-up speed compared with the conventional system.

II. TWO WHEELED ROBOT

A. ROBOT

Fig. 1(a) shows a photograph of the robot used in this study. A three-axis acceleration sensor and a three-axis gyro sensor were installed in the center of the robot body to measure the motion of the robot. The robot comprised six 2 L plastic bottles, and the wheels were pneumatic tires. The robot had a width of 54 cm, wheel radius of 18 cm, and height of 100 cm. The wheel radius and height of the stairs were similar. The motor used was a MAXON RE40 (150 W) with a 26:1 reduction gear. In addition, six load cells were used to measure the human operating force, and six load cells were embedded in the handle. Therefore, the forces and torques of the six axes can be measured using coordinate transformation. For the motor driver, no-limit drivers capable of supplying up to 30 A were used. This driver was used to generate the impulse torque required when riding the stairs.
\[ f_s = [f_1 \ f_2 \ f_3 \ f_4 \ f_5 \ f_6]^T \]
The matrices transforming the measured force vector \( f_s \) to the force vector \( f_h^H \) and the torque vector \( \tau_h^H \) in the handlebar coordinate space are denoted as \( T_F \) and \( T_T \), respectively.

\[
f_h^H = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} f_s = T_F f_s \tag{5}
\]

\[
\tau_h^H = \begin{bmatrix} -\frac{W}{2} & 0 & 0 & 0 & \frac{W}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{D}{2} & 0 \end{bmatrix} f_s = T_T f_s \tag{6}
\]

However, in the handle coordinate space, the torsional force around the Y-axis of the handle cannot be measured. In addition, we derived a matrix to transform the force from the handle coordinate space to the robot coordinate space. The position vector \( p_h \) of the handle seen from the robot coordinate space is expressed as shown in Eq. (7).

\[
p_h = \begin{bmatrix} D/2 \\ 0 \\ 0 \end{bmatrix}^T \tag{7}
\]

The rotation matrix \( R_P(\theta_P) \) around the Y-axis and the rotation matrix \( R_R(\theta_R) \) around the X-axis are defined as follows:

\[
R_P(\theta_P) = \begin{bmatrix} \cos \theta_P & 0 & \sin \theta_P \\ 0 & 1 & 0 \\ -\sin \theta_P & 0 & \cos \theta_P \end{bmatrix} \tag{8}
\]

\[
R_R(\theta_R) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_R & -\sin \theta_R \\ 0 & \sin \theta_R & \cos \theta_R \end{bmatrix} \tag{9}
\]

Based on the definition shown in Eq. (7), Eq. (8), and Eq. (9), the operating force \( f_h^R \) and operating torque \( \tau_h^R \) in the robot coordinate space were obtained using Eq. (10) and Eq. (11), respectively.

\[
f_h^R = \begin{bmatrix} \bar{F}_x \\ \bar{F}_y \\ \bar{F}_z \end{bmatrix} = R_P(\theta_P)R_R(\theta_R)T_F f_s \tag{10}
\]

\[
\begin{align*}
\tau_h^R &= \begin{bmatrix} \bar{\tau}_x \\ \bar{\tau}_y \\ \bar{\tau}_z \end{bmatrix} = p_h \times f_h^R \\
&\quad + R_P(\theta_P)R_R(\theta_R)T_T f_s \tag{11}
\end{align*}
\]

where \( \times \) denotes the outer product.

III. ANALYSIS OF STAIR RIDE-UP FOR SINGLE WHEEL

A riding-up analysis on a single wheel was performed in this study. Fig. 4 shows the model diagram for the analysis of a single wheel, where \( N_x, N_z, F_x, f_x, f_z \) represent the reaction force of the stair riser surface, reaction force of the floor, pressing force, friction force in the \( x \)-axis direction, and friction force in the \( z \)-axis direction, respectively. Under the condition that the wheel does not slip, the equations of motion for the \( x \)-axis, \( z \)-axis, and rotation are expressed as shown in Eq. (12), Eq. (13), and Eq. (14), respectively, where \( M, J = \frac{1}{2} MR^2, g, R, \theta, \tau_m \) represent the weight of the wheel, moment of inertia of the wheel, gravitational acceleration, wheel radius, wheel angle, and wheel torque, respectively.

\[
N_x - F_x - f_x = M\ddot{x} \tag{12}
\]

\[
N_z + f_z - Mg = M\ddot{z} \tag{13}
\]

\[
\tau_m - (f_x + f_z)R = J\ddot{\theta} \tag{14}
\]

The friction forces \( f_x \) and \( f_z \) can be expressed as follows using the static friction coefficient \( \mu_s \):

\[
f_x = \mu_s N_z \tag{15}
\]

\[
f_z = \mu_s N_x \tag{16}
\]

In addition, if the frictional force is higher than the wheel torque, the wheel will not slip. The slip condition can be defined using Eq. (17).

\[
\mu_s(N_x + N_z)R > \tau_m \tag{17}
\]

The reaction force is eliminated from the equation of motion and then substituted into Eq. (17). The final nonslip conditional equation is shown in Eq. (18). However, the coefficient of static friction, \( \mu_s \), is assumed to be smaller than 1.

\[
\frac{\mu_s}{1 + \mu_s^2}((1 - \mu_s)F_x + (1 + \mu_s)MG) > \tau_m \tag{18}
\]

With reference to Reference [13], the coefficient of static friction \( \mu_s = 0.8 \), wheel radius \( R = 0.18 \) cm, step height \( h_z = 23 \) cm, and mass \( M = 14 \) kg; additionally, the maximum non-slip torque is 22.1 Nm based on Eq. (18). This torque is equivalent to 4.68 times the torque generated when a rated current of 6 A is applied to the motor. Therefore, the probability of slipping is considered to be low, even when two motors are used to generate the pushing force.

A ride-up stair simulation was performed for a single wheel. The coefficient of dynamic friction was set to \( \mu_d = 0.6 \), and the friction force was calculated using the Coulomb friction model [14]. The reaction force was approximated and calculated using the penalty method using the spring-damper model [15]. The reaction force was calculated using the spring constant \( K = 60000 \) N/m and damper constant \( D_f = 2000 \) Ns/m. To generate the impulse torque, it was controlled by the force control law expressed in Eq. (19). However, \( K = -10 \) was the gain, and \( g = 1 \) rad/s was the cutoff frequency of the high-pass filter. Pushing force \( F_x = -165 \) N and pulling force \( F_z = 40 \) N were applied 5 s after the start of the simulation.

\[
\tau_m = K\frac{s}{s + g} F_x \tag{19}
\]

The locus of the ride-up is shown in Fig. 5. This result shows that the wheel successfully performed a ride-up even when the pull-up force was 40 N. This indicates that ride-up is possible even when the wheel radius is lower than the step height.

IV. CONTROL SYSTEM DESIGN

Two stair-riding motions are possible for a two-wheeled robot. The first motion is to ride up on both wheels, and the second motion is to ride up for each wheel. Fig. 7 shows an
identification experiment.

\[	f_d^\text{react} = \frac{g_{\text{react}}}{s + g_{\text{react}}} \left[ f_d^\text{ref} + g_{\text{react}} n_{22} d - f^*(d) \right] \]

\[
f^*(d) = F^* \tanh(\gamma d) + F_d d \quad (23)
\]

The control law for push control using the estimated value of the reaction force \(f_d^\text{react}\) is expressed as shown in Eq. (24).

\[
f_p^\text{ref} = K_F \left[ f_p^\text{cmd} - f_d^\text{react} \right] \quad (24)
\]

where \(K_F\) represents the force gain, and \(f_p^\text{cmd}\) the force command. The force reference in Eq. (20) and the push force reference of Eq. (24) are combined using the variable gain \(\alpha\).

\[
f^\text{ref} = \alpha f_p^\text{ref} + (1 - \alpha) f_d^\text{ref} + \hat{f}^\text{dis} \quad (25)
\]

Here, \(\hat{f}^\text{dis}\) represents the disturbance force/torque, including the external force/torque, a friction term, and the torque resulting from variations in the parameters [17]. The variable gain \(\alpha\) is a variable that depends on the user's push-in force \(F_x\), as shown in Eq. (26), where \(g_\alpha\) is the cutoff frequency of the low-pass filter. It is used to reduce the time response of the variable gain. Meanwhile, \(\beta\) is a parameter that determines the rate of change owing to the pressing force.

\[
\alpha = \frac{g_\alpha}{s + g_\alpha} \left[ \tanh (\beta (f_p^\text{cmd} - F_x) + 1) \right] \quad (26)
\]

Furthermore, using the Jacobi matrix, the torque reference value in the wheel space can be expressed as shown in Eq. (27).

\[
T^\text{ref} = J_T^\text{aco} f^\text{ref} \quad (27)
\]

Stair ride-up can be realized using the control law expressed in Eq. (25). A block diagram of the proposed method is presented in Fig. 6.

V. EXPERIMENT

The experimental environment was a flight of stairs made of stone with a height of 18 cm and a depth of 31 cm. We conducted an experiment by riding up to three steps of the stairs while measuring the work performed by humans and the time response of the external force. The parameters used in the experiment are listed in Table II. The work \(J_W\) [J] was calculated using Eq. (28), where \(T\) represents the end of the experiment. The definitions of \(v^\text{res}\) and \(\omega^\text{res}\) are shown in Eq. (29) and Eq. (30), respectively. These velocity vectors were calculated based on measurements from the accelerometer and gyroscope sensors inside the robot. Fig. 8, Fig. 9, Fig. 10, and Fig. 11 show photographs of a two-wheeled robot riding up, the time variation of work, force response with control, and force response without control, respectively.

\[
J_W = \int_0^T \left[ f_h^R \right]^T v^\text{res} + \left[ \tau_h^R \right]^T \omega^\text{res} \, dt \quad (28)
\]

\[
v^\text{res} = \begin{bmatrix} \dot{X}_L & \dot{Y}_L & \dot{Z}_L \end{bmatrix}^T \quad (29)
\]

\[
\omega^\text{res} = \begin{bmatrix} \dot{\theta}_R & \dot{\theta}_P & \dot{\theta}_Y \end{bmatrix}^T \quad (30)
\]
We validated the results shown in Fig. 9. Without and with the control, the final values were 74.9 and 30.4 J, respectively. Therefore, the energy reduced by 43.9 J. The result implies that half of the energy can be reduced using the control. By contrast, the results shown in Fig. 11 indicate that the force was imposed for a longer time and required more effort. Fig. 8 shows the state of the ride up the stairs at each time step. From Fig. 8, it can be seen that humans and robots work together to successfully ride up the stairs.

VI. CONCLUSION

In this study, we proposed a method for stair ride-up using a two-wheeled robot. A pushing force through cooperation with humans was utilized in the proposed method. It was demonstrated that the proposed method enabled the robot to ride up the stairs faster and with a lighter and smaller size compared with conventional robots. Furthermore, the effectiveness of the proposed method was experimentally confirmed. It was demonstrated that the proposed method required only half of the energy of the robot without control. These results validated the effectiveness of the proposed method. In addition, the proposed method yielded a speed of 0.3 steps/s.

However, the average human climbing speed is approximately 2 step/s. To achieve a more effective use of the system, faster operations must be realized. Hence, intelligent control methods such as machine learning should be introduced. In addition, there is a problem with the large peak external force required for stair rides up. It is necessary to consider how to deal with this problem in the future. Also, experiments need to be conducted by changing the step height of the stairs and the road surface conditions.

REFERENCES

Fig. 8. Two-wheeled robot stair riding up motion

(a) t=0s  (b) t=0.2s  (c) t=0.4s  (d) t=0.6s  (e) t=0.8s

Fig. 9. Work response

![Work response graph](image)

Without control Work [J]  
With control Work [J]

0 20 40 60 80 100
0 2 4 6 8 10 12
43.9J

Fig. 10. External force response with control

![External force response graph](image)

X-axis Force  
Y-axis Force  
Z-axis Force

0 50 100 150 200
0 2 4 6 8 10 12

Fig. 11. External torque response without control

![External torque response graph](image)

X-axis Force  
Y-axis Force  
Z-axis Force

0 50 100 150 200
0 2 4 6 8 10 12


