A Parametric Design Method of Functionally Graded Lattice Structure Based on the Stress Distribution

Cun Zhao, Guoxi Li, Meng Zhang, Dong Wang, Rihuang Du

Abstract—Lattice structural materials with lightweight, multifunctional and excellent designability provide rich possibilities for intelligent materials and advanced structural design. To solve the problems of uneven stress distribution, stress concentration, and local failure, a parametric design method of functionally graded lattice structure based on stress distribution law is proposed. According to the stress distribution, the density gradient function and gradient lattice mathematical model is established, and the gradient penalty calculation of lattice cell bar size is carried out so that the relative density of cell and bar size change continuously along the height direction. The numerical simulation results show that the stress distribution of functionally graded lattice structure obtained by this method tends to be uniform under the impact load, which effectively alleviates the stress concentration effect and reduces the maximum stress by about 19%. Under the same weight reduction ratio, the material distribution is more reasonable, and the reliability and impact resistance of the structure are improved.

Keywords—graded lattice structure, stress distribution, impact, mathematical model, numerical simulation

I. INTRODUCTION

In recent years, with the increasing maturity of additive manufacturing technology, the rapid development of lattice structure material technology has been promoted [1]. Lattice structural materials have been recognized as high-performance materials at home and abroad because of their lightweight, cyclical characteristics and multi-functional potential of vibration and shock resistance [2,3]. The multi-function and excellent designability of lattice structures provide abundant possibilities for the design of intelligent materials and advanced structures. Based on the traditional uniform lattice structure, the concept of gradient is introduced into the gradient lattice structure, which makes the material distribution and the dimension of cellular members continuously change along the gradient direction. Under certain working conditions, it is expected to perform better in mechanical properties and functional characteristics [4].

With the deepening of the research, the research work on graded lattice structure is expanding day by day. In 2013, Shen et al. [5] gave an analytical expression for the dynamic compressive response of gradient porous structure according to one-dimensional shock wave theory. It suggested that the weakest part of the gradient structure was placed at the impact side to obtain the maximum energy absorption effect. In 2013, Shen et al. [6] established a chain-type porous metal structures with hexagonal or circular cells, and a finite element model of porous metal material with gradient strength distribution. It was found that when the material was compressed at high speed along the positive gradient direction, only one wavefront was generated at the loading end, while when it was compressed at high speed along the negative gradient direction, one wavefront was generated at each end. In 2013, Wang et al. [7] established a two-dimensional density gradient Voronoi finite element models and found that the gradient foam metal had a higher energy absorption characteristic under low energy impact. In 2013, Fan et al. [8] considered that the dynamic platform stress and dense strain energy at the impact end increased with the increase of impact velocity, and pointed out that the energy absorption would be maximized if the large density layers were placed at the loading end (that is, the small density layer was placed at the static end). In 2017, Sing et al. [9] designed and fabricated cubic lattice structures with different diameters and densities. The study found that structural collapse always extends from the densest layer to the most compact layer. In 2017, Liu et al. [10] studied the dynamic response of nonlinear density gradient foam under impact. The results show that the appropriate nonlinear density gradient distribution can improve the impact resistance of foam rods. In 2018, Alsaedi et al. [11] used compression tests and finite element analysis to study the mechanical properties of aluminum alloy functionally graded lattice structure, which showed that the energy absorption per unit volume of functionally graded lattice structure was higher than that of the uniform lattice structure. A large number of studies have shown that under the condition of impact, vibration, and other dynamic loads, the energy absorption, frequency response, and damping characteristics of the graded lattice structure are obviously better than those of the uniform lattice structure, which has a wider engineering application value and applicability in engineering field [12].

However, most of the researches focus on the mechanical properties of the graded lattice structure, and the research on the design method of graded lattice structure is less. Therefore, this paper attempts to put forward a parametric design method...
of functionally graded lattice structure based on the stress distribution law, to realize the size parameter calculation and weight reduction ratio matching design of graded lattice structure.

II. MATHEMATICAL MODEL

A. Parametric Model of Graded Lattice Structure

Compared with the conventional uniform lattice structure, the graded lattice structure is characterized by the continuous change of the relative density of lattice cells and the size of bars along the gradient direction. The envelope space of cubic lattice cells (such as EC, FCC, and BCC.) is a standard cube, and the shape of the cell is regular, which ensures the elastic isotropy of the material. It is easy to arrange and map the lattice, and very suitable for computing and designing graded lattice structures.

The BCC lattice cell is described as an example. The dimension of lattice cell side length is defined as \( a \) and the dimension of cell bar section side length is defined as \( t \). The relative density of cell can be changed by changing the dimension of a bar. The gradient factor \( f_k \) is introduced to change the relative density of the lattice cell by calculating the gradient penalty of the dimension \( t \) of the lattice cell. The parametric model of the graded lattice structure is shown in Figure 1. Figure (a) is the BCC lattice cell model, and figure (b) is the gradient BCC lattice structure model. The \( z \)-direction is the gradient direction, and the size of the cell member decreases gradually along the gradient direction.

![Fig. 1. A parametric model of the graded lattice structure](image)

According to the parametric model of the graded lattice structure, the mathematical model of gradient lattice is established by theoretical analysis and calculation as follows:

\[
\begin{align*}
D_k &= (a_0, t_k, f_k) \\
t_k &= t_0 - f_k \\
\gamma_k &= \left( \frac{V_{ak}}{V_{hk}} \right) = \frac{36a_0^2t_k^2 - 36\sqrt[3]{b_k^3}}{a_0^3} \\
\beta &= (1 - \sum_{k=1}^{n} \gamma_k) \times 100\%
\end{align*}
\]

In (1), \( D_k \) is the design parameter of the graded lattice structure, \( a_0 \) and \( t_0 \) are the initial side length and member size of the cell, \( t_k \) is the member size of the \( k \)th layer cell, \( k \) corresponds to the number of layers, \( f_k \) is the gradient factor, which is determined by the density gradient function, \( \gamma_k \) is the relative density of the \( k \)th layer cell, \( V_{hk} \) is the member solid volume of the \( k \)th layer cell, \( V_{ak} \) is the envelope volume of the \( k \)th layer cell \( \beta \) is the total weight loss ratio of the graded lattice structure.

When the initial lattice cell size parameters, gradient factor, and scale factor are determined, the structure and parameters of the whole gradient lattice can be determined by the above mathematical model, and the relative density and total weight loss ratio of the graded lattice structure can be calculated through the mathematical relationship.

B. Gradient Factor and Scale Factor

By changing the gradient factor, the relative density of lattice cells and the size of bars can be controlled to change continuously along the gradient direction. The gradient factor is determined by the gradient function

The gradient factor formula is as follows:

\[
\begin{align*}
\gamma_k &= \frac{V_{ak}}{V_{hk}} = \frac{36a_0^2t_k^2 - 36\sqrt[3]{b_k^3}}{a_0^3} \\
\beta &= (1 - \sum_{k=1}^{n} \gamma_k) \times 100\%
\end{align*}
\]

In (2), \( f_k \) is the gradient factor, \( F(h) \) is the density gradient function, \( \sigma(h) \) is the stress distribution function, \( k \) corresponds to the number of layers, and \( \lambda \) is the scale factor, which is determined by the weight loss ratio.

According to the stress area formula, the equal proportion relation is defined, and the density gradient distribution function \( F(h) \) based on the stress distribution function \( \sigma(h) \) is established as follows:

\[
\frac{\sigma(h_k) - \sigma_{\text{max}}}{\sigma_{\text{min}} - \sigma_{\text{max}}} = \frac{\sigma_{\text{min}} - \sigma_{\text{max}}}{t_k^2 - t_0^2} \quad \text{(3)}
\]

In (3), \( \sigma_{\text{min}} \), \( \sigma_{\text{max}} \) and \( \sigma(h_k) \) are the minimum stress, maximum stress, and variable stress; \( t_{\text{min}} \), \( t_{\text{max}} \), \( t_0 \) and \( t_k \) are the minimum member size, maximum member size, initial member size, and variable member size of the lattice cell.

The expression of initial density gradient distribution function \( F(h) \) is established as follows:

\[
F(h) = \frac{1}{t_0} \sqrt{\frac{\sigma_{\text{min}} - \sigma_{\text{max}}}{\sigma_{\text{min}} - \sigma_{\text{max}}}} \left( t_k^2 - t_0^2 \right) + t_0 \quad \text{(4)}
\]
According to the relationship among volume, relative density, and weight loss ratio, the formula of magnification factor of initial density gradient distribution function is derived under the condition of the same weight loss ratio as follows:

$$\lambda = \frac{n \cdot V_{b0}}{\sum_{k=1}^{n} V_{bk}}$$  \hspace{1cm} (5)

In (5), $\lambda$ is the scale factor, $t_0$ is the initial member size of the cell, $t_k$ is the member size of the $k$th layer cell, $n$ corresponds to the number of layers, $V_{b0}$ is the member solid volume of the uniform lattice cell, $V_{bk}$ is the member solid volume of the graded lattice structure. Equation (5) ensures that the uniform lattice structure and graded lattice structure meet the same weight loss ratio.

The density gradient function can also be established by function fitting according to the stress distribution, and the gradient factor is obtained by discretization calculation according to the gradient function. By setting different density gradient functions, the gradient factor can be changed, so that the structure and size parameters of gradient lattice can be changed, so that the structure has different mechanical and functional properties to meet the requirements of different working conditions.

III. DESIGN FLOW

The parametric design process of graded lattice structure is shown in Figure 2.

1. For a part to design, the range of design domain $(w, l, h)$ is determined first. According to the size of the design domain, the initial cell side length size $a_0$ and the bar size $t_0$ are selected, the number $(nw, nl, nh)$ of cell arrangement, and the layers are calculated.

   - The number of cells in x-direction, $nw = w/a_0$;
   - The number of cells in y-direction, $nl = l/a_0$;
   - The layers of cell in z-direction, $nh = h/a_0$.

2. The stress distribution law is obtained through the impact finite element simulation, and the density gradient function $F(h)$ is established based on the stress distribution function $\sigma(h)$.

3. The weight reduction ratio is taken as the design index, and the gradient factor $f_i$ and the scale factor $\lambda$ are determined by combining the density gradient function, and the gradient function model of the gradient lattice density is determined.

4. According to the mathematical model of gradient lattice, the gradient penalty calculation is carried out for the lattice cell size by gradient factor, and all the lattice cell members are obtained by continuous cycle calculation. The dimensions of the cell members of the same lattice are the same, and the calculation formula of the cell member size layer by layer is as follows:

$$t(i, j, k) = t_k = t_0 \cdot f(h_k)$$
$$t(i, j + 1, k + 1) = t(i, j, k + 1)$$
$$t(i + 1, j, k + 1) = t(i, j, k + 1)$$  \hspace{1cm} (6)

In (6), $t(i, j, k)$ represents the dimension of the cell member whose coordinates are $(i, j, k)$ in the geometry of the continuum, $i$ is the cell number in the $x$-direction, $j$ is the cell number in the $y$-direction, and $k$ is the cell number in the $z$-direction.

Fig. 2. The parametric design process of the graded lattice structure.
(5) Based on the user-defined auxiliary modeling program of 3D modeling software Creo, lattice cell design and cyclic assembly are carried out. The steps are as follows:

a) initial lattice cell size parameters \((a_0, t_0)\) are initialized to generate initial cubic lattice cell model elements;

b) the initial origin and datum of line assembly are set and matched with the datum of lattice cell element;

c) take \(k=1\), \(t_i=t_0 \cdot f_i\) to assemble the first lattice cell element in the first layer, and complete the regeneration and assembly of all lattice cell elements in the first layer according to \(t(1, 1, 1) = t(i, 1, 1)\);

d) Take \(k=k+1\), \(t_{k+1}=t_0 \cdot f_{k+1}\) to regenerate lattice cell elements, and complete the assembly of all lattice cell elements in the layer \(k+1\).

(6) According to the above steps, the lattice cells are repeatedly assembled until \(k > nh\), all lattice cells are assembled, and the final graded lattice structure model is output.

IV. SIMULATION

According to the weight reduction ratio index, the graded lattice structure of the given parts is optimized by using the above-mentioned graded lattice structure parametric design method. According to the design process in Figure 2, the gradient lattice structure is designed and modeled. The lattice cell is established in Creo, a three-dimensional modeling software. According to the mathematical model of the graded lattice structure, the gradient lattice cell size calculation algorithm is established, and the algorithm is imported into the user-defined program of the modeling software. Finally, the 3D model of lattice structure is obtained. The obtained gradient lattice structure model is imported into the finite element software. The impact simulation analysis of lattice structure is carried out in the explicit dynamics module of ANSYS Workbench to compare and verify the dynamic performance of graded lattice structures.

A. Simulation model of the graded lattice structure

The given part is shown in Figure 3, the geometric dimension is \(l=20\) mm, \(w=20\) mm, \(h=60\) mm, the bottom end of the cuboid structure is fixed, the upper end bears impact load \((m=20\) g, \(v=12\) m/s). The material is aluminum, the elastic modulus \(E=70\) GPa, and the weight reduction index \(\beta\) of the given structure is about 60%. By the designing of a graded lattice structure, the parts are improved to meet the lightweight index, optimize the material distribution and improve the impact resistance of the structure.

The cell type is BCC lattice, and the cell side length is selected according to the design domain size, \(a_0=5\) mm; the initial cell member size is calculated according to the weight reduction ratio index, \(t_0=0.6\) mm.

The stress distribution of uniform lattice structure under impact load is analyzed by impact simulation. According to the simulated stress distribution program, the effective stress parameters are extracted layer by layer for analysis, and the stress distribution function \(\sigma(h)\) is fitted. Referring to equations (2), (3), and (4), the density gradient function is established with the height of the geometric structure along the gradient direction as the independent variable as follows:

\[
F(h) = \left( 0.00011 h^2 - 0.016 h + 0.99 \right)^{\frac{1}{2}}
\]

In combination with the weight reduction ratio, the scale factor is calculated according to equation (5), \(\lambda=0.72\).

The mathematical model of gradient lattice is obtained by substituting the specific parameters and functions as follows:

\[
D_k = (5, t_k, f_k)
\]

\[
t_k = 5 f_k
\]

\[
f_k = 0.72 F(h)_{h=k}
\]

\[
F(h) = \left( 0.00011 h^2 - 0.016 h + 0.99 \right)^{\frac{1}{2}}
\]

\[
\gamma_k = \frac{V_{h_k}}{V_{ak}} \frac{180 T_k^2 - 36 \sqrt{6} h_k^3}{125}
\]

\[
\beta = (1 - \sum_{n=1}^{n} \gamma_k) \times 100\% = 60\%
\]

According to the function relation, the dimension function \(t_k\) of gradient lattice cell is determined as follows:

\[
t_k = 0.72 \left( 0.00011 h^2 - 0.016 h + 0.99 \right)^{\frac{1}{2}}
\]

Draw the curve of the member size of the graded lattice cell with the number of layers as shown in Figure 3.

![Fig. 3. The curve of the member size of graded lattice cell.](image)

According to formula (8), the dimension of the cell member of the graded lattice structure can be calculated by layer, and the structural parameters of uniform lattice and gradient lattice are shown in Table I. And the maximum and minimum cell member dimensions of the gradient lattice are calculated, \(T_{\min}=0.45\) mm, \(T_{\max}=0.7\) mm.

For the uniform lattice structure, the lattice cell envelope size is \(a\) and the lattice cell bar size is \(t\), keeping the cell
envelope size and the cell bar size unchanged. For the graded lattice structure, keeping the cell envelope size unchanged and changing the cell bar size, the lattice cell gradient layout design is realized on the premise of the same weight reduction ratio. Compared with the uniform lattice structure, the cell size of the maximum stress concentration area at the bottom of the gradient lattice structure is increased, \( t_{\text{max}} = 0.72 \) mm, \( t_{\text{max}} > t_0 \), while the cell size of the smaller stress region at the top is decreased, \( t_{\text{min}} = 0.48 \) mm, \( t_{\text{min}} < t_0 \). The overall structure presents the characteristics of gradient layered arrangement, which makes the material distribution tend to the larger stress area and improves the mechanical properties of the structure.

**TABLE I. PARAMETERS OF LATTICE STRUCTURE**

<table>
<thead>
<tr>
<th>Names</th>
<th>Symbols</th>
<th>Uniform lattice</th>
<th>Gradient lattice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell size</td>
<td>( a )</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Initial member size</td>
<td>( l_0 )</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Max-member size</td>
<td>( l_{\text{max}} )</td>
<td>-</td>
<td>0.72</td>
</tr>
<tr>
<td>Min-member size</td>
<td>( l_{\text{min}} )</td>
<td>-</td>
<td>0.48</td>
</tr>
<tr>
<td>Lattice layers</td>
<td>( n )</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>Graded factor</td>
<td>( f )</td>
<td>-</td>
<td>( f_s )</td>
</tr>
</tbody>
</table>

According to the given geometric part size and lattice structure design parameters, the uniform lattice structure and graded lattice structure are used for geometric filling, and 3D simulation models of the uniform lattice and gradient lattice are established, as shown in Figure 4. Figure (b) is the uniform lattice structure, and figure (c) is the graded lattice structure.

![Fig. 4. 3D models of lattice structures.](image)

**B. Simulation Result**

The stress distribution of two kinds of lattice structures under impact load is shown in Figure 5. Figure (a) shows the stress distribution of the homogenized lattice structure, and figure (b) shows the stress distribution of the graded lattice structure.

As shown in figure (a), the average stress value of lattice cells in each layer is extracted according to the stress distribution results as the effective stress data for analysis of the uniform lattice structure. The data show that the maximum stress \( \sigma_{1\text{max}} \) is 1293.2 MPa, and the minimum stress \( \sigma_{1\text{min}} \) is 4.6 MPa. The stress value of the first four layers of lattice near the lower support bundle is larger, and from the fifth layer, the stress value decreases significantly, and the uniform point is smaller. The stress drop of the array structure along the height direction is large. It can be seen from the figure that when the uniform lattice structure is subjected to impact load, the stress is mainly concentrated in the bottom region of the uniform lattice structure, and the stress changes non-uniformly along the height direction. The bottom end of the uniform lattice structure is prone to local deformation, crack and collapse due to stress concentration, which leads to the failure of parts.

![Fig. 5. The stress distribution of lattice structures.](image)

As shown in figure (b), the average stress value of each layer lattice cell is extracted according to the stress distribution results as the effective stress data for analysis of graded lattice structure. The data shows that the maximum stress \( \sigma_{2\text{max}} \) is 1045.3 MPa, and the minimum stress \( \sigma_{2\text{min}} \) is 6.6 Mpa. The drop of stress value along the height direction of the graded lattice structure is small. It can be seen from the figure that the stress distribution tends to be uniform when the graded lattice structure is subjected to impact load. Compared with the uniform lattice structure, the maximum stress of the graded lattice structure under the impact load is significantly reduced by about 19% under the same weight reduction ratio.

The results show that the graded lattice structure design can significantly improve the state of stress concentration, reduce the local deformation, cracks, and parts failure caused by stress concentration, and improve the reliability and impact resistance of the structure.

**V. CONCLUSION**

In this paper, based on the existing research work of graded lattice structure, a parametric design method of functionally graded lattice structure based on stress distribution law is
proposed. In this method, the stress distribution law in the structure is obtained through the finite element mechanical analysis of a given part. According to the stress distribution law, the corresponding density gradient function and graded lattice structure mathematical model is established. Based on this model, the parametric design method and process of functionally graded lattice structure is constructed. The model of functionally graded lattice structure is designed by combining with an example, and its performance is verified by finite element simulation. It is proved that the advantages of the design method proposed are as follows:

(1) According to the parameterized design method of the functionally graded lattice structure, the relative density of cells and the size of bars is continuously changed along the height direction by gradient penalty calculation of the size of lattice cells. The calculation of cell sizes parameters and the matching design of weight reduction ratio can be realized, and the functionally graded lattice structure models meeting the design requirements can be obtained.

(2) The finite element simulation results show that compared with the uniform lattice structure, the functionally graded lattice structure obtained by this design method significantly improves the state of stress concentration, the stress distribution tends to be uniform, and the maximum stress is reduced by about 19%.

(3) The material distribution of the functionally graded lattice structure obtained by the method is more reasonable under the condition of the same weight reduction ratio, and the reliability and impact resistance of the structure is improved while the structure is lightweight.

(4) The geometry and functional grading of the graded lattice structure are limited by the accuracy of additive manufacturing technology. By adjusting the density gradient function and mathematical model, the direction and functional grading can be changed. The graded lattice structure is expected to show more structural variability and functional potential, so as to further improve the mechanical properties.

The research work of this paper extends the design theory and method of the graded lattice structure. The research work has a certain engineering guiding significance for the design and application of graded lattice structure materials. The method also has the possibility of popularization.

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REFERENCES