An Efficient Inverse Kinematic Method for SSRMS-type Manipulator with Conformal Geometric Algebra

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Abstract—SSRMS-type Manipulator is a typical type of space manipulator featuring 7-degree-of-freedom offset configuration with redundancy characteristic. It is challenging to solve inverse kinematics in high efficiency, make the upmost of redundancy characteristic, and select optimal redundant parameter. This paper proposes an analytical inverse kinematic method, named three-continuous-parallel-Link Direction Vector Parameterization (LDVP), in the theory of Conformal Geometric Algebra. This method employs the direction vector of the axis of parallel Joint 3, Joint 4, and Joint 5 as the parameter, and selects the optimal parameter according to context plan-points in a given trajectory. The process of solving inverse kinematics is divided into two steps, including solving joint positions and joint variables. The average solution time of forward and inverse kinematics is 9.79 us and 0.25 ms respectively. The continuous path tracking experiment verifies that all seven joints changing to pursue the optimal configuration, which makes full use of redundancy characteristic.

Index Terms—inverse kinematics, SSRMS-type Manipulator, conformal geometric algebra

I. INTRODUCTION

Space manipulators are commonly utilized in orbit services [1]. Most of them share the same configuration with Space Station Remote Manipulator System (SSRMS), featuring a 7-degree-of-freedom (7-DOF) offset manipulator with redundancy characteristic, and the Experimental Module Manipulator (EMM) [2], as shown in Fig. 1, is a typical case. Redundancy characteristic contributes to advantages such as obstacle avoidance and a flexible working space [4], but also brings challenges to the solution of inverse kinematics (IK) [5]. Moreover, SSRMS-type manipulator has offset at shoulder, wrist and elbow. Compared with the non-offset SRS (Spherical-Roll-Spherical) configuration commonly used in 7-DOF humanoid arms, the SSRMS-type with offset further increases the difficulty of solving the inverse kinematics [6].

In recent decades, scholars have carried out a lot of research on the IK solution of SSRMS-type manipulator, and methods are divided into velocity-level methods and position-level methods. The velocity-level methods mainly includes Jacobian transpose and pseudoinverse [7], which give numerical solutions and have the advantage of real-time efficiency, but may produce several numerical errors [6]. Other numerical methods such as KDL [8] and TRAC-IK [9] are applied to obtaining the



Fig. 1. The Experimental Module Manipulator. [3]

single IK solution, but there exists problems of low efficiency and uncertainty caused by iteration. The position-level method such as arm angle parameterization method [10] [11] [12] solved the IK solution of the SRS manipulator and generalized the results to SSRMS-type manipulator in accordance with the relationship of joint angles between two types of manipulator. However, this method contributes to uncontrollable errors [6]. Another method named joint angle parameterization (JAP) method [13] is broadly used to yield closed-form solutions and has the advantage of high accuracy. However, it is difficult to determine the optimized parameter. Meanwhile, the redundant characteristic is limited because one of the seven joint angles is fixed, meaning the manipulator operates as a 6-DOF robot.

The contributions of this paper are:

(1) Applying the theory of Conformal Geometric Algebra (CGA) into the kinematic analysis of SSRMS-type manipulator, and the efficiency of both Forward Kinematics (FK) and IK is increased.

(2) Proposing an efficient IK strategy, named threecontinuous-parallel-Link Direction Vector Parameterization (LDVP), which provides an analytical solution in high efficiency, and fully utilize the redundancy characteristic of SSRMS-type manipulator.

The remainder of this paper is organized as follows. In Section II, the basic theory of CGA is introduced to establish the kinematic model of SSRMS-type manipulator, and the parameter of LDVP is defined. In Section III, the procedure of

TABLE I GEOMETRY REPRESENTATIONS IN CGA

Geometry	Representation	Remark
Point Sphere Plane	$P = e_0 + p + 0.5p^2 e_{\infty}$ $S = P - 0.5R^2 e_{\infty}$ $\pi = *(P \cdot n)$	P : point in the conformal space p : point in the Euclidean space P : circle center, R : radius π passes through P and takes n as the normal vector.

inverse kinematic solution using LDVP is described in detail. In Section IV, the validity and efficiency of this method is verified in simulation by cases. In Section V, this paper is concluded with final remarks.

II. PRELIMINARIES

In this section, the basic theory of CGA is introduced, the parameter vector of LDVP is defined, and the kinematic model of SSRMS-type manipulator is established.

A. Basic Theory of Conformal Geometric Algebra

Conformal Geometric Algebra extends Euclidean space to the conformal space by introducing two additional basis vectors e_0 and e_∞ based on the basis vectors of Euclidean space [14]. This extension allows for the representation of various geometric objects in the conformal space as shown in I. Additionally, information like relative positional relationships and distances of geometric objects can be obtained by performing positional or mathematical operations [15] as shown in TABLE II.

Geometric operators in conformal space can represent rotation, translation, and general motion. Robot kinematic equations can be obtained by multiplying geometric operators. The rotor R, translator T, motor M are represented as:

$$R = \cos(\theta/2) - \sin(\theta/2)\boldsymbol{I}$$
(1)

$$T = 1 - 0.5 te_{\infty} \tag{2}$$

$$M = TR \tag{3}$$

where I denotes the direction vector of rotation axis, θ denotes the rotation angle, and t denotes the direction vector of translation, with its value is the translation distance.

The geometry after transformation represents as:

$$\boldsymbol{O}_{i+1} = M \boldsymbol{O}_i M \tag{4}$$

where O_i and O_{i+1} respectively denotes the presentation of the geometry before and after transformation.

TABLE II Formula of Calculations in CGA

Calculation	Inner Product	Outer Product	Intersection	Dual	Reverse
Formula	$A \cdot B$	$A \wedge B$	A&B	*A	Ã



Fig. 2. CGA model of SSRMS-type manipulator.



Fig. 3. The projection of P_1P_8 on *para_vec* is constant.

B. Establishing the kinematic model of SSRMS manipulator

Establishing the world coordinate with e_1 , e_2 , e_3 , and establishing the geometric algebra model of SSRMS-type manipulator in the conformal space, as depicted in Fig. 2.

To establish the kinematic equation of SSRMS-type manipulator, information should be given as shown in TA-BLE III.Bring the information into (1), (2), (3) and (4), and the forward kinematic equation of the SSRMS-type manipulator can be expressed as (5).

C. Definition of parameter vector

The typical characteristic of SSRMS-type manipulator is that the elbow joint is always parallel with Joint 3 and Joint 5, and we name them as three-continuous-parallel-link. Define the direction vector of these three joints as the parameter vector *para_vec*, which is the core parameter of LDVP method.

The SSRMS-type manipulator exhibits two additional characteristics: the base position is fixed, and the adjacent links



Fig. 4. Definition of the parameter vector para_vec.

TABLE III						
INFORMATION TO BE USED IN THE FORWARD KINEMATICS.						

Information	Label			
Link lengths	$a_i (i=0,1,\ldots,8)$			
Joint variables	$ heta_i (i=1,\ldots,7)$			
Initial axis	$I_i(i = 1,, 7) =$ $e_2, -e_1, e_2, e_2, e_2, -e_1, e_2$			
Initial direction vectors	$t_j (j = 0,, 8) =$ $e_2, -e_1, e_2, -e_1, e_2, -e_1, e_2, -e_1, e_2, -e_1$			

$$\begin{cases} \boldsymbol{p}_{\boldsymbol{e}} = \prod_{i=0}^{8} \left(1 - \frac{a_{i}}{2} \boldsymbol{t}_{i} \boldsymbol{e}_{\infty}\right) \prod_{j=1}^{7} \left(\cos \frac{\theta_{j}}{2} - \sin \frac{\theta_{j}}{2} \boldsymbol{I}_{i}\right) \boldsymbol{P}_{0} \prod_{j=1}^{7} \left(\cos \frac{\theta_{j}}{2} - \sin \frac{\theta_{j}}{2} \boldsymbol{I}_{i}\right) \prod_{i=0}^{8} \left(1 - \frac{a_{i}}{2} \boldsymbol{t}_{i} \boldsymbol{e}_{\infty}\right) \\ \boldsymbol{a} = \prod_{i=0}^{8} \left(1 - \frac{a_{i}}{2} \boldsymbol{t}_{i} \boldsymbol{e}_{\infty}\right) \prod_{j=1}^{7} \left(\cos \frac{\theta_{j}}{2} - \sin \frac{\theta_{j}}{2} \boldsymbol{I}_{i}\right) \boldsymbol{e}_{2} \prod_{j=1}^{7} \left(\cos \frac{\theta_{j}}{2} - \sin \frac{\theta_{j}}{2} \boldsymbol{I}_{i}\right) \prod_{i=0}^{8} \left(1 - \frac{a_{i}}{2} \boldsymbol{t}_{i} \boldsymbol{e}_{\infty}\right) \\ \boldsymbol{o} = \prod_{i=0}^{8} \left(1 - \frac{a_{i}}{2} \boldsymbol{t}_{i} \boldsymbol{e}_{\infty}\right) \prod_{j=1}^{7} \left(\cos \frac{\theta_{j}}{2} - \sin \frac{\theta_{j}}{2} \boldsymbol{I}_{i}\right) \left(-\boldsymbol{e}_{1}\right) \prod_{j=1}^{7} \left(\cos \frac{\theta_{j}}{2} - \sin \frac{\theta_{j}}{2} \boldsymbol{I}_{i}\right) \prod_{i=0}^{8} \left(1 - \frac{a_{i}}{2} \boldsymbol{t}_{i} \boldsymbol{e}_{\infty}\right) \end{cases}$$
(5)



Fig. 5. Algorithm diagram of efficient inverse kinematic method LDVP.



Fig. 6. Solving for the optimized parameter vector.

are always perpendicular to each other. Based on above three characteristics, joint positions can be solved in theory of CGA.

First of all, fixing the base position of the manipulator at the origin of the conformal space, as shown in Fig. 2. Constructing sphere $S_{P1} = \mathbf{P}_1 - 0.5R_1^2 \mathbf{e}_{\infty}$ $(R_1 = a_2 + a_4 + a_6)$ and $S_{P8} = \mathbf{P}_8 - 0.5R_8^2 \mathbf{e}_{\infty}$ $(R_8 = \sqrt{|\mathbf{P}_1\mathbf{P}_8|^2 - R_1^2})$, which intersect to get a circle $Cir_{1.8} = S_{P1}\&S_{P8}$. Selecting a point \mathbf{P}_{para_p} randomly on the circle $Cir_{1.8}$. The unit vector in the direction of the line connecting Joint 1 and \mathbf{P}_{para_p} is the defined parameter vector *para_vec*. The process of this description is shown in Fig. 3 and Fig. 4.

III. EFFICIENT INVERSE KINEMATIC METHOD

In this section, an efficient inverse kinematic method is proposed to rapidly solve the exact closed-form solution while making the upmost of the redundancy characteristic of the SSRMS-type manipulator. The algorithm diagram of this strategy is shown in Fig. 5.

A. Select appropriate parameter point P_{para_p}

Section II-C has discussed that in order to determine the parameter vector *para_vec*, the parameter point P_{para_p} should be determined first.

When the manipulator performs a continuous path tracking task, the redundant parameter vector corresponding to the *i*-th path point can be inferred from the inverse kinematic solving procedure of the (*i*-1)-th path point. As shown in Fig. 6, the parameter point selected during the inverse kinematic solution of the (*i*-1)-th path point is named as P_{para_1} . Project P_{para_1} onto the plane where Cir_{1_8} corresponding to the *i*-th path point is located. Connect the center of circle Cir_{1_8} and the projected point, and the connecting line intersects Cir_{1_8} at P_{para_2} , then P_{para_2} becomes the parameter point selected for the *i*-th path point. The unit vector of $P_1P_{para_2}$ is the vector that changes slightly compared to the parameter vector $P_1P_{para_1}$ corresponding to the (*i*-1)-th path point among the infinite sets of candidate vectors, which is the required redundant parameter vector named by *para_vec_2*.

In cases where there is no adjacent path point to determine current parameter point P_{para_p} , such as solving IK of the first path point of the continuous path when there is only a set of end effector pose given, setting every joint variable to zero, and taking the vector of parallel joints as *para_vec*. Then project P_{para_p} on the plane where Cir_{1_8} is located, as depicted in the previous paragraph, to determine the responding redundant parameter vector *para_vec*₂.

B. Solution of Joint Positions

This section primarily analyzes the process of solving for joint positions using given information, including the position of the end effector p_e , approaching vector a, orientation vector o, and the determined parameter point $P_{para p}$.

According to the conformal geography, the position of Joint 7 denoted by P_9 , Joint 6 denoted by P_8 , and Joint 1 denoted by P_1 can be calculated respectively as:

$$\boldsymbol{P}_9 = \boldsymbol{e}_0 + \boldsymbol{p}_e + 0.5 \boldsymbol{p}_e^2 \boldsymbol{e}_\infty. \tag{6}$$

$$P_8 = e_0 + (P_9 - a_8 a) + 0.5 (P_9 - a_8 a)^2 e_{\infty}.$$
 (7)

$$\boldsymbol{P}_1 = \boldsymbol{e}_0 + (a_0 \boldsymbol{e}_2) + 0.5(a_0 \boldsymbol{e}_2)^2 \boldsymbol{e}_{\infty}.$$
 (8)

With the obtained redundant parameter vector, we can continue to solve positions of the remaining joint points.

Constructing a sphere $S_{P7} = \mathbf{P}_8 - 0.5a_7^2 \mathbf{e}_{\infty}$, and a plane $plane_P_8 P_9 = *(\mathbf{P}_8 \cdot \mathbf{a})$, which interact at the circle $Cir_P_7 = S_{P7} \& plane_P_8 P_9$. Similarly, construct the plane $plane_P_7 P_8 = *(\mathbf{P}_8 \cdot \mathbf{para_vec}_2)$, which intersects with Cir_P_7 at point pair P_{p7} . The two points of the point pair P_{p7} represent the positions of Joint 7 \mathbf{P}_7 with different manipulator



Fig. 8. Illustration of no solution for P_7 .

configurations. The solution process is shown in Fig. 7. P_7 can be calculated as:

$$\boldsymbol{P}_{7} = \frac{P_{p7} \pm \sqrt{P_{p7}^{2}}}{-\boldsymbol{e}_{\infty} \cdot P_{p7}} = \frac{P_{p7} + k_{7} \sqrt{P_{p7}^{2}}}{-\boldsymbol{e}_{\infty} \cdot P_{p7}}$$
(9)

It should be noticed that, if θ_6 to be solved occurs to be zero, Cir_P_7 will lie on $plane_P_8P_9$ as shown in Fig. 8, thus there will be no solution of P_7 . The same is true for P_2 if θ_2 to be solved occurs to be zero.

Similarly, P_2 can be obtained by geometric calculations of corresponding geometries including spheres and planes, and P_2 can be calculated as:

$$\boldsymbol{P}_{2} = \frac{P_{p2} \pm \sqrt{P_{p2}^{2}}}{-\boldsymbol{e}_{\infty} \cdot P_{p2}} = \frac{P_{p2} + k_{2} \sqrt{P_{p2}^{2}}}{-\boldsymbol{e}_{\infty} \cdot P_{p2}}$$
(10)

Constructing a sphere $S_{P3} = \mathbf{P}_3 - 0.5a_3^2 \mathbf{e}_{\infty}$, and another sphere $S_{P6} = P_6 - 0.5(a_4^2 + a_5^2)\mathbf{e}_{\infty}$. S_{P3} and S_{P6} interact at circle Cir_P4 . Construct the plane $plane_P4P_5 = *(\mathbf{P}_3 \cdot \mathbf{para_vec}_2)$. $plane_P4P_5$ and Cir_P4 intersect at point pair P_{p4} . Two points of point pair P_{p4} , which represents the positions of Joint 4 P_4 with different manipulator configurations. The solution process is shown in Fig. 9. \mathbf{P}_4 can be calculated as:

$$\boldsymbol{P}_{4} = \frac{P_{p4} \pm \sqrt{P_{p4}}^{2}}{-\boldsymbol{e}_{\infty} \cdot P_{p4}} = \frac{P_{p4} + k_{2}\sqrt{P_{p4}}^{2}}{-\boldsymbol{e}_{\infty} \cdot P_{p4}}$$
(11)

The position P_3 , P_6 , and P_4 can be obtained by respectively translating P_2 , P_7 , and P_5 along the direction of parameter vector, and the moving distance is respectively a_2 , a_6 , and a_4 .

When the method above is used to solve for joint point positions, 8 sets of solutions will be generated. This is because in formula (9) (10) (11), the points P_2 , P_4 and P_7 are presented by point pairs, and $k_2 = \pm 1$, $k_4 = \pm 1$, $k_7 = \pm 1$ corresponding



Fig. 9. The solution process of Joint-4 position.

to different robot arm poses will generate different joint point positions.

C. Solution of Joint Variables

The obtained joint point positions are connected in sequence to obtain the link information, and the rotation angle of each joint is calculated based on the changes in these links.

Define x and y as two vectors respectively, and the rotation plane represented by the binary unit vector is expressed as:

$$\hat{N} = \frac{\hat{\boldsymbol{x}} \wedge \hat{\boldsymbol{y}}}{\|\hat{\boldsymbol{x}} \wedge \hat{\boldsymbol{y}}\|}$$
(12)

 \hat{x} and \hat{y} respectively represents the unit vector of x and y. The angle between x and y is expressed as:

$$\theta(\mathbf{x}, \mathbf{y}) = \operatorname{atan2}(\hat{\mathbf{x}} \cdot \hat{\mathbf{y}}, (\hat{\mathbf{x}} \wedge \hat{\mathbf{y}})\hat{N}^{-1})$$
(13)

The rotation axes of seven joints of the SSRMS-type manipulator are P_0P_1 , P_1P_2 , P_2P_3 , P_3P_4 , P_4P_5 , P_5P_6 , P_6P_7 in sequence, and the rotation angle is determined by the rotation reference vectors P_1P_2 , P_2P_3 , P_3P_4 , P_5P_6 , P_7P_8 , P_8P_9 , o.

When none of the joints rotates, the configuration of the robotic arm in the conformal space is shown in Fig. 2. The initial rotation reference vectors of Joint 1 to Joint 7 are e_2 , $-e_1$, e_2 , e_2 , e_2 , $-e_1$, e_2 in sequence. After the movement occurs, the joint variable θ_1 can be calculated with the joint point position P_1 and P_2 as:

$$\theta_1 = \theta(-\boldsymbol{e}_1, \boldsymbol{P}_1 \boldsymbol{P}_2) \tag{14}$$

Then the rotation reference vector of Joint 2 is $l_2 = M_1 M_0 \boldsymbol{e}_2 \tilde{M}_0 \tilde{M}_1$, the joint variable can be calculated as:

$$\theta_2 = \theta(\boldsymbol{l}_2, \boldsymbol{P}_2 \boldsymbol{P}_3) \tag{15}$$

By analogy, the values of each joint variable θ_3 , θ_4 , θ_5 , θ_6 , θ_7 can be obtained.

If the rotor in (1) can be represented as:

$$R\boldsymbol{O}_{object}R = \boldsymbol{O}_{object}.rotate(\boldsymbol{I}, \theta)$$
(16)

the procedure of solving for joint variables can be represented by the following algorithm 1.

As described in III-B, eight sets of inverse kinematic solutions are obtained for each given pose. In order to select the optimal solution, solutions that do not meet the joint angle restrictions will be first screened, and then the principle of minimizing joint motion is used, thus the absolute sum of joint rotations is minimized.

Algorithm 1 Calculating joint variables.

Require: $ref_axis: \{-e_1, e_2, -e_1, -e_1, -e_1, e_2, -e_1\},$ $current_vector: \{P_0P_1, P_1P_2, P_2P_3, P_3P_4, P_4P_5, P_5P_6, P_6P_7\}, rot_axis: \{P_1P_2, P_2P_3, P_3P_4, P_5P_6, P_7P_8, P_8P_9, o\}.$ 1: $\theta_1 = \theta(-e_1, P_1P_2)$ 2: **for** i = 1 to 7 **do** 3: **for** j = 1 to i **do** 4: $ref_axis[j] \leftarrow ref_axis[j].rotate(rot_axis[j], \theta_j)$ 5: **end for** 6: $\theta_i \leftarrow \theta(ref_axis[i], current_vector[i])$ 7: **end for**

IV. CASE ANALYSIS

This section takes EMM, as shown in Fig. 1, as the example to verify the validity and efficiency of the LDVP method. All trials are implemented in C++, utilizing a computer (Intel(R) Core(TM) i3-4160 CPU @ 3.60GHz RAM8.00GB Ubuntu 20.04) to execute them.

A. FK simulation of 10,000 random poses

In the D-H method, each link is fixedly connected to a coordinate system and different individuals may establish different coordinate systems and provide different D-H parameter tables when solving for FK of the same manipulator. In contrast, in CGA method, all geometric objects are described in the same world coordinate system, and the kinematic equation is derived with continuous product of geometric operators, avoiding complex matrix operations.

Based on the Monte Carlo method, use Stantard D-H (SDH) method, Modified D-H (MDH) method and CGA to solve for FK of EMM. When randomly set 10,000 sets of joint variables, the average calculation time for each interpolation point by three methods is shown in Fig. 10.

It can be seen from the results that CGA method has great advantage in the computation speed of forward kinematics, which is about 1.6 times that of D-H method.

B. IK simulation of single random pose in workspace

Set pose of the end effector as $p_e = (-3.67, 1.62, 0.17)$, a = (-0.23, 0.557, 0.798), o = (-0.187, 0.779, -0.598), and the reference point is selected as $P_{para} = (-0.374, 1.19, 0.796)$, then eight sets of inverse solutions are shown in TABLE IV, and corresponding poses are shown in Fig. 11 respectively.



Fig. 10. Solution time for each interpolation point in the workspace.

 TABLE IV

 Eight Inverse Kinematic Solutions a random given pose

n	$\theta_1(^\circ)$	$\theta_2(^\circ)$	$ heta_3(^\circ)$	$ heta_4(^\circ)$	$\theta_5(^\circ)$	$\theta_6(^\circ)$	$ heta_7(^\circ)$
1	154.85	61.68	159.31	69.03	-100.62	9.53	-12.82
2	154.85	61.68	-131.66	-69.03	-31.59	9.53	-12.82
3	154.85	61.68	160.06	44.00	103.66	-9.53	167.18
4	154.85	61.68	-155.94	-44.00	147.66	-9.53	167.18
5	-25.15	-61.68	-32.95	102.61	-121.94	9.53	-12.82
6	-25.15	-61.68	69.66	-102.61	-19.33	9.53	-12.82
7	-25.15	-61.68	-41.24	87.79	81.17	-9.53	167.81
8	-25.15	-61.68	46.55	-87.79	168.96	-9.53	167.81



Fig. 11. The corresponding configurations of eight sets of solutions.

The LDVP method enables the direct solving of both the positions of each joint point and each link, thus enhancing the visibility of geometric characteristics and making this method more effective in obstacle avoidance.

C. IK simulation of 10,000 random poses in workspace

We challenged the proposed LDVP method against other four IK methods [16]. The trials involved 10,000 random poses in the workspace obtained in IV-A utilized as the desired poses, and results are shown in TABLE V.

Results reveals that the LDVP method outperforms all competitors considering the solution rate, which firmly guarantees stability and reliable task execution. Although the average time of LDVP is not the shortest, the computational burden is still competitive given the high solution rate, and it is much more faster than iterative methods. Moreover, there is no solving error since the LDVP method solves analytical solutions rather

 TABLE V

 A Comparison of Performances of Five IK Methods [16].

	Joint Locking	Position Error(m)	Solve Rate(%)	Average Time(ms)	Multiple Solutions
CCD	No	10^{-6}	96.85	17.64	No
JAP	Yes	No Error	92.12	0.05	Yes
KDL	No	10^{-6}	87.74	1.13	No
TRAC-IK	No	10^{-6}	99.26	0.69	No
LDVP	No	No Error	100	0.25	Yes



Fig. 12. Joint Angle Curves of circular and square trajectory respectively.



Fig. 13. End track of manipulator in trajectory tracking tasks.

than numerical solutions. The simulation results indicate that the proposed LDVP method is an efficient IK method.

It should be further noted that although the possibility of no solution is proposed in III-B, it is difficult to precisely meet the condition of no solution when the pose is randomly selected in the workspace, so the solution success rate is usually 100%.

D. Continuous path tracking experiment

We employed the LDVP method to perform the path tracking task. Two paths, each consisted of 1,000 interpolation points, were planned respectively, namely a circle with (-3, 2) as the center and 1 m as the radius on the plane parallel to y=3, and a square on the x-z plane parallel to y=1.62 with x limited in (-4, -2) and y limited in (-2, 0). Change curves of each joint angle are shown in Fig. 12. Use the joint angle obtained from the solution to solve the end point position through forward kinematics and depict the end point trajectories in Fig. 13.

Considering that the LDVP method can dynamically select the optimal parameter vector based on the context plan-point, ensuring minimal changes in the manipulator and minimizing energy waste during movement along a given path, the proposed method guarantees that seven joints all changing to pursue the optimal configuration to make full use of the redundancy characteristic.

V. CONCLUSION

This paper proposes a method named LDVP to solve for analytical IK solution of SSRMS-type manipulator, which takes the direction vector of three-continuous-parallel-link as the parameter. The efficiency of kinematic solution was obviously enhanced by applying conformal geometric theory to avoid complex matrix calculation as well as iterative process. The optimization problem of redundant parameter was solved by selecting parameter vector according to context plan-point in a given trajectory. Redundancy characteristic was fully utilized by adjusting all seven joints to pursue the optimal solution without locking the configuration of the manipulator.

It should be noted that although singular configurations do not appear in most cases, it will cause great danger when it is encountered in orbit service, so future work will optimize the LDVP to avoid singular configurations. Moreover, the obstacle avoidance function is not considered in the process of selecting the parameter vector, thus the superiority of CGA in geometric calculation will be utilized to reflect the obstacle avoidance advantages of redundant manipulators in future work.

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