Discrete-Time Predictive Controller with Time Delay for Unmanned Helicopter

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Abstract—This paper presents a discrete predictive controller for a class of discrete time-delayed systems. The most time-delayed system, especially a highly unstable unmanned helicopter, has severe difficulties in stability and performance. Furthermore, the presence of disturbance more critically affects the stability of time-delayed systems since the delayed control input may lose the ability to stabilize the state. The proposed controller employs precise prediction of the future state, incorporating exponential stability for predicting future disturbance. In particular, the proposed state prediction can effectively compensate for disturbance effects without any robust control terms. The performance of the proposed controller is validated by numerical simulations. The results can verify the feasibility and performance of the proposed controller in the presence of significant time delay for the unmanned helicopter.

I. INTRODUCTION

For decades, several strategies have been widely explored to deal with time delay. Among them, a prediction-based controller (PC) shows powerful performance to compensate for the time delay. The main concept of PC is to directly predict the system state in the prediction horizon within the size of the delay to deal with large size of delays. The time delay has become more significant in critical safety challenges as interest in autonomous aerial mobility increases. In fact, aerial systems are highly sensitive to time delays due to their unstable nature.

Several strategies have been explored for a multirotor unmanned aerial vehicle (MUAV) to compensate for the time delay effect. The MUAV is necessarily operated in a remote control system but it can incur a delay effect due to wireless communication. In addition, many unmanned aerial systems utilize an additional companion controller for heavy computation within limited computation resources. The

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companion controller is used to apply additional machine vision and artificial intelligence techniques for autonomous navigation, including position control. Then, the position control loop may suffer from delay problems. It is noticed that most MUAVs have a hierarchical control structure between position and attitude controllers since they are underactuated systems. As a result, the delay between the companion board and the main flight controller for attitude control affects control stability as input delay. The input delay can also be incurred between the flight controller and motors, and a robust prediction-based sliding mode control [1] has been developed to deal with various time delays robustly. Furthermore, the MUAV still suffers from measurement delay caused by the sensor and state estimation such as the Kalman filter, so prediction-based control [2] with uncertainty and disturbance estimator has been introduced to compensate for actuator and measurement delays. A predictive descriptor observer [3] is also developed to deal with long time delays.

In general, the MUAVs have a symmetric geometry with uniformly distributed rotors. Otherwise, an unmanned helicopter has more maneuverability and agility but an unstable nature due to asymmetric propulsion systems so that it is more sensitive to small time delays compared with multirotor UAVs. In addition, the helicopter model includes highly nonlinear and complex aerodynamics with model uncertainties [4]. As a result, the time delay is exceptionally critical to ensuring control stability for unmanned helicopters in the presence of uncertainties and disturbances. An active model-based predictive control in the discrete-time domain [5] has been explored to deal with input time delay, and experiments verify the control performance in situations such as sudden mode change and aggressive flight.

Several strategies have been also introduced to develop discrete-time prediction-based controllers [5]–[8]. The predictive scheme [6] has been proposed to attenuate the disturbance effect without a disturbance observer (DOB), verifying the control performance with real-time application to control MUAV along the yaw-axis. However, the roll and pitch-axes response of the unmanned helicopter is much more unstable than the yaw-axis. Thus, a full attitude controller should be further developed. In addition, the recursive form in state prediction for a discrete-time system can improve robustness against disturbance [7], [8], but it cannot guarantee robustness against large disturbances without any robust terms.

In this paper, a new discrete predictive control is proposed for discrete-time systems under time delay, uncertainties, and

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disturbances. The main contributions are summarized as follows:

- 1) A discrete-time exponentially stable predictor is developed to predict the future information of disturbance. The discrete controller based on the state prediction is proposed to ensure the state convergence to the desired trajectory.
- 2) The performance of the proposed controller can be directly applied to a discrete-time system compared to existing PCs, ensuring exponential stability and the convergence of the state to time-varying trajectories.

The remainder of the paper is organized as follows. The problem is first formulated by representing an unmanned helicopter system with input time delay in Sec. II. The proposed controller is analyzed, and its stability and boundedness are analyzed in Sec. III. Numerical simulations verify that the proposed controller can improve the tracking performance despite time delay and external disturbance in Sec. IV and V, respectively. Sec. V concludes this paper.

II. PROBLEM FORMULATION

A. Helicopter Dynamics With Input Delay

A

The nonlinear attitude dynamics of an agile unmanned aerial system as unmanned helicopters [9], including input delay, can be linearized as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c \mathbf{x}(t) + \mathbf{B}_c \left[\mathbf{u}(t - h_c) + \mathbf{M}_d(t) + \mathbf{d}(t) \right]$$
(1)

$$\mathbf{A}_{c} = \frac{\kappa}{-1} \mathbf{I}_{H}^{-1} \begin{bmatrix} \mathbf{0}_{3\times3} & (\Omega \gamma_{\beta} / \kappa) \mathbf{I}_{H} \\ 0 & \gamma_{\beta} S_{\beta} + 16 & \gamma_{\beta} - 16 S_{\beta} \end{bmatrix}$$
(2)

$$\mathbf{B}_{c} = \kappa \mathbf{I}_{H}^{-1} \begin{bmatrix} \mathbf{0}_{3\times3} & 0 & 16S_{\beta} - \gamma_{\beta} & \gamma_{\beta}S_{\beta} + 16 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_{c} = \kappa \mathbf{I}_{H}^{-1} \begin{bmatrix} \mathbf{0}_{3\times3} \\ 1 & -S_{\beta} & 0 \\ S_{a} & 1 & 0 \end{bmatrix}$$

$$(3)$$

$$\begin{bmatrix} \mathbf{S}_{\rho} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\kappa}^{-1} \boldsymbol{l}_{T} \end{bmatrix}$$

where $\mathbf{x} = [\phi \ \theta \ \psi \ p \ q \ r]^T$, \mathbf{I}_H represents an inertia matrix, Ω is the angular velocity of the main rotor, and l_T is the distance between the center of gravity and the rotation center of the tail rotor. $\mathbf{M}_d = [0 \ 0 \ C_{mr}]^T$ represents a counter torque C_{mr} along the yaw axis by the main rotor rotation. The input time delay h_c in continuous time can be incurred in generating roll, pitch, and yaw moments by main and tail rotors, decreasing control performance. S_β is a parameter explaining the ratio of hub stiffness to aerodynamic moments, γ_β is the lock number giving the ratio of aerodynamic to inertia forces, and $\kappa = (K_\beta - T_{mr}h_R)/S_{\beta 0}$ with $S_{\beta 0} = 1+S_\beta^2$.

B. Discrete-Time Dynamics Representation

From (1), the discretized model can be obtained with $\mathbf{A} \approx \mathbf{I}_{6\times 6}$ + $\mathbf{A}_c T$ and $\mathbf{B} \approx \mathbf{B}_c T$ in (2) and (3), where *T* is a sampling period, as follows:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k-h) + \mathbf{B}_d\mathbf{d}(k), \tag{4}$$

where $\mathbf{x}(k) \in \mathbb{R}^{2n}$ is a state vector of the system and is fully measurable. $\mathbf{u}(k-h) \in \mathbb{R}^n$ is a control input with a known and constant time delay $h \in \mathbb{R}$ and the time delay in the discrete-time domain can be computed by h_c . $\mathbf{d}(k) \in \mathbb{R}^n$ represents unknown time-varying matched disturbance, including uncertain aerodynamics and gyroscopic precession, and $\mathbf{d}(k)$ is bounded by $\|\mathbf{d}(k)\| \leq D_0$, $\forall k \geq 1$ and satisfies the following condition as $\|\Delta \mathbf{d}^{(r+1)}(k)\| \leq TD_{r+2}, \forall k \geq 1$, where $\Delta \mathbf{d}^{(r)}(k) = \mathbf{d}^{(r)}(k+1) - \mathbf{d}^{(r)}(k)$ and $\Delta \mathbf{d}^{(0)}(k) = \Delta \mathbf{d}(k)$. $\mathbf{A} \in \mathbb{R}^{2n \times 2n}$ and $\mathbf{B} = [\mathbf{0}_{n \times n} \quad \mathbf{B}_1^T]^T \in \mathbb{R}^{2n \times n}$ are known system and input matrices, respectively, where $\mathbf{B}_1 \in \mathbb{R}^{n \times n}$. For the discrete system (4), the forward difference is defined as $\Delta_k^q \chi(k) =$ $\Delta_h^{q-1}\chi(k+h) - \Delta_h^{q-1}\chi(k)$ for a given vector $\chi(k) = [\chi_1(k) \cdots$ $\chi_n(k)$]^{*T*} $\in \mathbb{R}^n$ and positive integers *q* and *h*, where $\Delta^0 \chi(k) = \chi(k)$, $\Delta^1 \boldsymbol{\gamma}(k) = \Delta \boldsymbol{\gamma}(k)$, and $\Delta_1^q \boldsymbol{\gamma}(k) = \Delta^q \boldsymbol{\gamma}(k)$. For example, $\Delta \boldsymbol{\gamma}(k) = \Delta^q \boldsymbol{\gamma}(k)$ $\chi(k+1) - \chi(k)$ for q = h = 1, and $\Delta_h \chi(k) = \chi(k+h) - \chi(k)$ for q = 1and h > 1.

III. DISCRETE-TIME PREDICTIVE CONTROL

The discrete-time predictive control is developed to ensure stability and performance for a discrete-time system under the time delay and disturbance so that the system state $\mathbf{x}(k)$ tracks the desired trajectory, maintaining stability and decreasing control error. Consider the input-delayed system of the unmanned helicopter (4), then the discrete-time state prediction based on [10] is first introduced as follows:

$$\mathbf{x}_{\hat{p}}(k) = \hat{\mathbf{x}}(k+h)$$

= $\mathbf{A}^{h}\mathbf{x}(k) + \sum_{j=k-h}^{k-1} \mathbf{A}^{k-1-j} \Big[\mathbf{B}\mathbf{u}(j) + \mathbf{B}_{d}\hat{\mathbf{d}}(j+h) \Big],$ (5)

where note that the future information of disturbance $\hat{\mathbf{d}}(k+j)$ should be predicted to complete (5) since it is impossible to measure $\hat{\mathbf{d}}(k+j)$ in practical implementations. In order to predict the disturbance accurately, the following exponentially stable DOB [11] can be implemented to estimate the disturbance $\mathbf{d}(k)$ in (4). For given a matrix $\mathbf{L} \in \mathbb{R}^{n \times 2n}$, the discrete-time DOB is represented as

$$\mathbf{d}(k) = \mathbf{L}\mathbf{x}(k) - \mathbf{z}(k),$$

$$\mathbf{z}(k+1) = \mathbf{z}(k) + \mathbf{L}\Big[(\mathbf{A} - \mathbf{I}_{2n \times 2n})\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k-h) + \mathbf{B}_{d}\hat{\mathbf{d}}(k)\Big],$$

(6)

where $\hat{\mathbf{d}}(k) \in \mathbb{R}^n$ is the result of disturbance estimation. Then, the dynamics for the estimation error $\tilde{\mathbf{d}}(k) = \mathbf{d}(k) - \hat{\mathbf{d}}(k)$ is exponentially stable, where $\hat{\mathbf{d}}(k) = [\hat{d}_1(k) \ \hat{d}_2(k) \cdots \hat{d}_n(k)]^T$, and the estimation error ultimately converges to a bound such that

$$\left|\tilde{d}_{i}(\infty)\right| \leq \frac{TD_{1}}{1 - \left|\lambda_{i}\right|}, \ i = 1, 2, \dots, n.$$

$$(7)$$

Following the result of the DOB (6), the future information of disturbance can be computed, and the discrete-time exponentially stable predictor based on [10] is expressed as

$$\begin{cases} \hat{\xi}_{0}(k+1) = -a_{0}T\tilde{\xi}_{0}(k) + \hat{\xi}_{0}(k) + T\hat{\xi}_{1}(k) \\ \hat{\xi}_{1}(k+1) = -a_{1}T\tilde{\xi}_{0}(k) + \hat{\xi}_{1}(k) + T\hat{\xi}_{2}(k) \\ \vdots \\ \hat{\xi}_{j}(k+1) = -a_{j}T\tilde{\xi}_{0}(k) + \hat{\xi}_{j}(k) + T\hat{\xi}_{j+1}(k) , \qquad (8) \\ \vdots \\ \hat{\xi}_{r-1}(k+1) = -a_{r-1}T\tilde{\xi}_{0}(k) + \hat{\xi}_{r-1}(k) + T\hat{\xi}_{r}(k) \\ \hat{\xi}_{r}(k+1) = -a_{r}T\tilde{\xi}_{0}(k) + \hat{\xi}_{r}(k) \end{cases}$$

where $\tilde{\xi}_j(k) = \xi_j(k) - \hat{\xi}_j(k)$, $\xi_j(k) = \xi^{(j)}(k)$, $\xi_0(k) = \xi(k)$, $a_j = \frac{(r+1)!}{(j+1)!(r-j)!} \omega_0^{j+1}$, j = 0, 1, ..., r, and $r \ge 0$ is an integer. Then, the system (8) can be expressed in the state-space model as

$$\hat{\boldsymbol{\xi}}(k+1) = \mathbf{A}_{p}\hat{\boldsymbol{\xi}}(k) + \mathbf{B}_{p}\boldsymbol{\xi}(k), \qquad (9)$$

$$\begin{bmatrix} 1 - a_{0}T & T & 0 & \cdots & 0 \end{bmatrix} \qquad \begin{bmatrix} a_{0} \end{bmatrix}$$

$$\mathbf{A}_{p} = \begin{bmatrix} -a_{1}T & 1 & T & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ -a_{r}T & 0 & 0 & \cdots & 1 \end{bmatrix}, \quad \mathbf{B}_{p} = T \begin{bmatrix} a_{1}\\ \vdots\\ a_{r} \end{bmatrix}, \quad (10)$$

where $\hat{\boldsymbol{\xi}}(k) = [\hat{\boldsymbol{\xi}}^{(0)}(k) \hat{\boldsymbol{\xi}}^{(1)}(k) \cdots \hat{\boldsymbol{\xi}}^{(r)}(k)]^T \in \mathbb{R}^{r+1}$ and $\boldsymbol{\xi}^{(0)}(k) = \boldsymbol{\xi}(k) \in \mathbb{R}$ are the state and input for the predictor, respectively. In order to obtain the eigenvalue λ_{p1} of \mathbf{A}_p , $\det(\lambda_{p1}\mathbf{I} - \mathbf{A}_p)$ can be computed as

$$det(\lambda_{p1}\mathbf{I} - \mathbf{A}_{p}) = det(\lambda_{p2}\mathbf{I} - (\mathbf{A}_{p} - \mathbf{I}) / T)$$

$$= (\lambda_{p2} + a_{0})\lambda_{p2}^{r} + a_{1}\lambda_{p2}^{r-1} + \begin{vmatrix} a_{2} & -1 & \cdots & 0 \\ a_{3} & \lambda_{p2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{r} & 0 & \cdots & \lambda_{p2} \end{vmatrix}$$

$$= \lambda_{p2}^{r+1} + a_{0}\lambda_{p2}^{r} + \dots + a_{r-1}\lambda_{p2} + a_{r}$$

$$= (\lambda_{p2} + \omega_{0})^{r+1},$$

where $\lambda_{p2} = (\lambda_{p1} - 1)/T$. In addition, (10) can be rewritten in terms of $\xi_i(k)$ as follows:

$$\begin{cases} \tilde{\xi}_{0}(k+1) = \tilde{\xi}_{0}(k) - a_{0}T\tilde{\xi}_{0}(k) + T\tilde{\xi}_{1}(k) \\ \tilde{\xi}_{1}(k+1) = \tilde{\xi}_{1}(k) - a_{1}T\tilde{\xi}_{0}(k) + T\tilde{\xi}_{2}(k) \\ \vdots \\ \tilde{\xi}_{j}(k+1) = \tilde{\xi}_{j}(k) - a_{j}T\tilde{\xi}_{0}(k) + T\tilde{\xi}_{j+1}(k) \\ \vdots \\ \tilde{\xi}_{r-1}(k+1) = \tilde{\xi}_{r-1}(k) - a_{r-1}T\tilde{\xi}_{0}(k) + T\tilde{\xi}_{r}(k) \\ \tilde{\xi}_{r}(k+1) = \tilde{\xi}_{r}(k) - a_{r}T\tilde{\xi}_{0}(k) + T\xi^{(r+1)}(k) \end{cases}$$
(11)

In (11), the following relation can be obtained as $0 = -a_r T\xi_0(\infty) + T\xi^{(r+1)}(\infty)$. In addition, by considering $|\xi_{r+1}| = |\xi^{(r+1)}| \le A_{r+1}$ it results in that

$$\begin{split} &\lim_{k \to \infty} \left| \tilde{\xi}_{0}(k) \right| \leq \frac{A_{r+1}}{a_{r}}, \\ &\lim_{k \to \infty} \left| \tilde{\xi}_{j}(k) \right| \leq \frac{A_{r+1}a_{j-1}}{a_{r}}, \, j = 1, 2, ..., r, \end{split}$$
(12)

and it shows that the predictor (8) is exponentially stable. In addition, from (9), the error dynamics of $\tilde{\xi}(k) (=\xi(k) - \hat{\xi}(k))$ can be represented as

$$\dot{\boldsymbol{\xi}}(k+1) = \boldsymbol{\xi}(k+1) - \hat{\boldsymbol{\xi}}(k+1)$$

$$= \boldsymbol{\xi}(k+1) - \left(\mathbf{A}_{p}\hat{\boldsymbol{\xi}}(k) + \mathbf{B}_{p}\boldsymbol{\xi}(k)\right) \qquad (13)$$

$$= \mathbf{A}_{p}\tilde{\boldsymbol{\xi}}(k) + \mathbf{\Phi}(k),$$

$$\boldsymbol{\Phi}(k) = \Delta \boldsymbol{\xi}(k) + \left(\mathbf{I} - \mathbf{A}_{p}\right)\boldsymbol{\xi}(k) - \mathbf{B}_{p}\boldsymbol{\xi}(k)$$
$$= \begin{bmatrix} 0 \ 0 \ \cdots \ \Delta \boldsymbol{\xi}^{(r)}(k) \end{bmatrix}^{T}.$$
(14)

In order to predict the future information $\hat{\xi}(k+h)$ of the state $\xi(k)$, the discrete-time Taylor series is introduced as follows:

$$\xi(k+h) = \sum_{j=0}^{h} \Delta^{j} \xi(k) {h \choose j}$$

$$= \sum_{j=0}^{r+1} \Delta^{j} \xi(k) {h \choose j} + R(k)$$

$$R(k) = \sum_{j=r+2}^{h} \Delta^{j} \xi(k) {h \choose j},$$
(15)
(15)
(15)
(16)

where *r* is selected by $0 \le r \le h-2$, and R(k) represents Taylor remainder. In addition, suppose that $|\Delta \xi^{(r+1)}(k)| \le TA_{r+2}, \forall k \ge 1$. Then, in (16), the Taylor remainder R(k) of discrete-time Taylor series is satisfied that

$$|R(k)| \le b_{h,r+2} T^2 A_{r+2}, \forall r$$
, (17)

where $b_{h,r+2} = \frac{h!}{(r+2)!(h-r-2)}$. In order to prove (17), R(k) is rewritten as follows:

$$R(k) = b_{h,r+2} \Delta^{r+2} \xi(k) + b_{h,r+3} \Delta^{r+3} \xi(k) + \dots + \Delta^{h} \xi(k)$$

= $\zeta_0 \Delta^{r+2} \xi(k) + \dots + \zeta_{h-r-2} \Delta^{r+2} \xi(k+h-r-2),$ (18)

where ζ_i is a coefficient, and it is obvious that $\sum_{i=r+2}^{h} \zeta_i = \frac{h!}{(r+2)!(h-r-2)} = b_{h,r+2}$. Then, applying $\Delta^{r+2}\zeta(k) = T\Delta\zeta^{(r+1)}(k)$, the following inequality is obtained as $|B(k)| < \zeta T |\Delta^{r+2}\zeta(k)| + \zeta |T\Delta^{r+2}\zeta(k+1)| + \cdots$

$$\begin{aligned} |R(k)| &\leq \zeta_0 T \left| \Delta^{r+2} \xi(k) \right| + \zeta_1 \left| T \Delta^{r+2} \xi(k+1) \right| + \cdots \\ &+ \zeta_{h-r-2} \left| T \Delta^{r+2} \xi(k+h-r-2) \right| \\ &\leq \zeta_0 T \left| \Delta \xi^{(r+1)}(k) \right| + \zeta_1 T \left| \Delta \xi^{(r+1)}(k+1) \right| + \cdots \end{aligned} \tag{19} \\ &+ \zeta_{h-r-2} T \left| \Delta \xi^{(r+1)}(k+h-r-2) \right| \\ &\leq b_{h,r+2} T^2 A_{r+2}. \end{aligned}$$

By utilizing (9) and (15), $\hat{\xi}(k+h)$ can be predicted as

$$\hat{\xi}(k+h) = \sum_{j=0}^{r+1} b_{h,j} \Delta^{j} \hat{\xi}(k)$$

$$= \mathbf{C}_{p} \Delta \hat{\xi}(k) + \xi(k),$$

$$\mathbf{C}_{p} = \begin{bmatrix} h & Tb_{h,2} & \cdots & Tb_{h,r+1} \end{bmatrix}.$$
(21)

Substitute the disturbance $\hat{d}_i(k)$ into $\xi(k)$ in (23) as

$$\Delta \hat{\boldsymbol{\xi}}(k) = \left[\Delta \hat{d}_i(k) \ \Delta \hat{d}_i^{(1)}(k) \ \cdots \ \Delta \hat{d}_i^{(r)}(k) \right]^T$$
$$= \left[\Delta \hat{d}_i(k) \ \Delta^2 \hat{d}_i(k) \ \cdots \ \Delta^{r+1} \hat{d}_i(k) \right]^T,$$

then the disturbance predictor can be obtained as follows:

$$\hat{d}_i(k+h) = \mathbf{C}_p \Delta \hat{\boldsymbol{\xi}}(k) + \hat{d}_i(k) , \qquad (22)$$

where $\hat{\mathbf{d}}(k+h) = [\hat{d}_1(k+h) \ \hat{d}_2(k+h) \ \cdots \ \hat{d}_n(k+h)]^T$. From (20), the prediction error is defined as $\varepsilon_i(k) = d_i(k+h) - \hat{d}_i(k+h)$ and computed by

$$\varepsilon_{i}(k) = \mathbf{C}_{p} \Delta \tilde{\boldsymbol{\xi}}(k) + \tilde{d}_{i}(k) + R_{i}(k)$$

= $\mathbf{C}_{p} \left[\mathbf{A}_{p} - \mathbf{I} \right] \tilde{\boldsymbol{\xi}}(k) + \Phi_{i}(k),$ (23)

$$\Phi_i(k) = \mathbf{C}_p \Phi(k) + \tilde{d}_i(k) + R_i(k) , \qquad (24)$$

and $R_i(k) = \sum_{j=r+2}^{h} b_{h,j} \Delta^j d_i(k) = T \sum_{j=r+2}^{h} b_{h,j} \Delta d_i^{(j-1)}(k)$. Then, from (7), (12), (17), it can be verified that the prediction error $\varepsilon_i(k)$, i=1,2,...,n, is exponentially confined within a bound. Note that \mathbf{A}_p has a unique eigenvalue so that it guarantees exponential stability. Then, the boundedness of $R_i(k)$ in (23) can be first computed as

$$R_{i}(k) \leq b_{h,r+2} \left| \Delta^{r+2} d_{i}(k) \right| = b_{h,r+2} T \left| \Delta d_{i}^{(r+1)}(k) \right|$$

$$\leq b_{h,r+2} T \left\| \Delta \mathbf{d}^{(r+1)}(k) \right\|$$

$$\leq b_{h,r+2} T^{2} D_{r+2}.$$
 (25)

The input for the predictor (8) becomes that $\xi(k) = \hat{d}_i(k)$ in (20), where the disturbance input is unknown but should be estimated. In (14), $\Phi(k)$ is rewritten by considering the estimation error $\tilde{d}_i(k)$ and represented as

$$\boldsymbol{\Phi}(k) = \Delta \boldsymbol{\xi}(k) + \left(\mathbf{I} - \mathbf{A}_{p}\right) \boldsymbol{\xi}(k) - \mathbf{B}_{p} d_{i}(k) + \mathbf{B}_{p} \tilde{d}_{i}(k)$$

$$= \left[\mathbf{0}_{1 \times r} \ 1\right]^{T} \Delta d_{i}^{(r)}(k) + \mathbf{B}_{p} \tilde{d}_{i}(k).$$
(26)

Then, the boundedness of $\Phi(k)$ is represented as

$$\begin{aligned} \mathbf{C}_{p} \mathbf{\Phi}(k) &| \leq c_{r+1} T \left| \Delta d_{i}^{(r)}(k) \right| + \mathbf{C}_{p} \mathbf{B}_{p} \left| \tilde{d}_{i}(k) \right| \\ &\leq c_{r+1} T^{2} D_{r+1} + \mathbf{C}_{p} \mathbf{B}_{p} \frac{T D_{1}}{1 - \lambda_{i}}. \end{aligned}$$
(27)

Then, the following inequality is obtained as follows:

$$\begin{aligned} \left| \varepsilon_{i}(k) \right| &\leq \left\| \mathbf{C}_{p} \mathbf{A}_{p} - \mathbf{C}_{p} \right\| \left\| \tilde{\boldsymbol{\xi}}(k) \right\| + \left| \mathbf{C}_{p} \boldsymbol{\Phi}(k) \right| \\ &+ \left| \tilde{d}_{i}(k) \right| + \left| R_{i}(k) \right|. \end{aligned}$$

$$(28)$$

As a result, from (28), it can be checked that the prediction error of future information of disturbance $\varepsilon_i(k)$, i=1,2,...,n, is ultimately bounded. In addition, the prediction error $\varepsilon_i(k)$ decreases when λ_i gets close to 0 with the increase in *r* by *h*-1.

The proposed state prediction is developed to guarantee robustness against time delay by utilizing the existing state prediction (5) and the discrete-time disturbance predictor (22) as

$$\boldsymbol{\chi}(k) = \mathbf{x}(k) + \Delta_h \mathbf{x}_{\hat{p}}(k-h)$$

= $\mathbf{A}^h \mathbf{x}(k) + \sum_{j=k-h}^{k-1} \mathbf{A}^{k-1-j} \mathbf{B} \Big[\mathbf{u}(j) + \hat{\mathbf{d}}(j+h) + \boldsymbol{\varepsilon}(j-h) \Big],$
(29)

where it is assumed that $\mathbf{B} = \mathbf{B}_d$ and $\mathbf{\varepsilon}(k) = \mathbf{d}(k+h) - \mathbf{\hat{d}}(k+h) = [\varepsilon_1(k) \ \varepsilon_2(k) \cdots \varepsilon_n(k)]^T$. It should be noted that $\mathbf{\varepsilon}(k-h) \neq \mathbf{\tilde{d}}(k)$ since $\mathbf{\varepsilon}(k-h)$ includes disturbance estimation and prediction errors of the previous step at k-h and the current step at k, respectively. Let us consider the exact state prediction with time delay h as follows:

$$\mathbf{x}_{p}(k) = \mathbf{x}(k+h)$$

= $\mathbf{A}^{h}\mathbf{x}(k) + \sum_{j=k-h}^{k-1} \mathbf{A}^{k-1-j}\mathbf{B}[\mathbf{u}(j) + \mathbf{d}(j+h)].$ (30)

Combining (30) with (4), the dynamics of $\mathbf{x}_p(k)$ can be represented as

$$\mathbf{x}_{p}(k+1) = \mathbf{A}^{h}\mathbf{x}(k+1) + \sum_{j=k-h+1}^{k} \mathbf{A}^{k-j}\mathbf{B}[\mathbf{u}(j) + \mathbf{d}(j+h)]$$

= $\mathbf{A}^{h+1}\mathbf{x}(k) + \sum_{j=k-h}^{k-1} \mathbf{A}^{k-j}[\mathbf{u}(j) + \mathbf{d}(j+h)]$ (31)
+ $\mathbf{B}\mathbf{u}(k) + \mathbf{B}\mathbf{d}(k+h)$
= $\mathbf{A}\mathbf{x}_{p}(k) + \mathbf{B}[\mathbf{u}(k) + \mathbf{d}(k+h)].$

Similarly to (31), the input-delayed system in (1) can be expressed in (32) and (33) by applying (29).

$$\boldsymbol{\chi}(k+1) = \mathbf{A}^{h} \mathbf{x}(k) + \sum_{j=k-h}^{k-1} \mathbf{A}^{k-1-j} \mathbf{B} \Big[\mathbf{u}(j) + \hat{\mathbf{d}}(j+h) + \boldsymbol{\varepsilon}(j-h) \Big],$$

= $\mathbf{A} \boldsymbol{\chi}(k) + \mathbf{B} \Big[\mathbf{u}(k) + \hat{\mathbf{d}}(k+h) \Big] + \boldsymbol{\Phi}_{\boldsymbol{\chi}}(k)$
(32)

$$\mathbf{\Phi}_{\chi}(k) = \mathbf{B}\mathbf{d}_{e}(k) + \mathbf{A}^{h}\mathbf{B}[\boldsymbol{\varepsilon}(k-h) - \mathbf{d}_{e}(k-h)].$$
(33)

Then, the delay-free system can be obtained from (32). Utilizing (32) and (33), the proposed predictive control input can be defined as

$$\mathbf{u}(k) = \mathbf{K}\boldsymbol{\chi}(k) - \hat{\mathbf{d}}(k+h), \qquad (34)$$

where $\mathbf{K} \in \mathbb{R}^{n \times 2n}$ is a state feedback gain matrix, where **K** is chosen such that matrix $\overline{\mathbf{A}}$ is Schur stable. Combining (32) and (33) with (34), it becomes that

$$\boldsymbol{\chi}(k+1) = \mathbf{A}\boldsymbol{\chi}(k) + \mathbf{B}\mathbf{d}_{e}(k) + \mathbf{A}^{h}\mathbf{B}\big[\boldsymbol{\varepsilon}(k-h) - \mathbf{d}_{e}(k-h)\big]$$

= $\overline{\mathbf{A}}\boldsymbol{\chi}(k) + \mathbf{B}\big[\boldsymbol{\varepsilon}(k) - \Delta_{h}\boldsymbol{\varepsilon}_{k-h}\big] + \mathbf{A}^{h}\mathbf{B}\Delta_{h}\boldsymbol{\varepsilon}_{k-2h}$ (35)

where $\overline{\mathbf{A}} = \mathbf{A} + \mathbf{B}\mathbf{K}$ and $\mathbf{\varepsilon}(k) - \mathbf{d}_e(k) = \Delta_h \mathbf{\varepsilon}(k-h)$. Suppose that the prediction error of disturbance $\mathbf{\varepsilon}(k)$ satisfies the boundedness as

$$\|\boldsymbol{\varepsilon}(k)\| \le E_0,$$

$$\|\Delta_h \boldsymbol{\varepsilon}(k)\| \le hE_1.$$

(36)

Then, by utilizing [7, Lemma 1], the following boundedness can be represented as

$$\|\boldsymbol{\chi}(k)\| \le \beta \alpha^k \|\boldsymbol{\chi}(0)\| + \nu, \qquad (37)$$

$$\boldsymbol{\nu} = \boldsymbol{\gamma} \left\| \mathbf{B} \right\| \left(\boldsymbol{E}_0 + \boldsymbol{h} \boldsymbol{E}_1 + \left\| \mathbf{A}^h \right\| \boldsymbol{h} \boldsymbol{E}_1 \right).$$
(38)

where $0 < \alpha < 1$, $\beta > 0$ and $\gamma = \frac{\beta}{1-\alpha} > 0$. From (31) and (35), it is obtained that

$$\mathbf{x}(k) - \boldsymbol{\chi}(k-h) = \sum_{j=k-h}^{k-1} \mathbf{A}^{k-1-j} \mathbf{B} \Delta_h \boldsymbol{\varepsilon}(j-2h) .$$
(39)

From (37), (38), and (39), it results in that

$$\|\mathbf{x}(k)\| \leq \|\boldsymbol{\chi}(k-h)\| + \sum_{j=k-h}^{k-1} \|\mathbf{A}^{k-1-j}\| \|\mathbf{B}\| \|\Delta_h \boldsymbol{\varepsilon}(j-2h)\|$$

$$\leq \|\boldsymbol{\chi}(k-h)\| + \eta \|\mathbf{B}\| h E_1,$$
(40)

where $\eta = \sum_{j=k-h}^{k-1} ||\mathbf{A}^{k-1-j}||$. Then, the following boundedness can be finally obtained as

$$\|\mathbf{x}(k)\| \le \alpha^{-h} \alpha^k \|\mathbf{\chi}(0)\| + \eta \|\mathbf{B}\| h E_1 + \nu.$$
(41)

When the predicted state $\hat{\mathbf{x}}(k+h)$ stabilizes to zero, then $\mathbf{x}(k)$ also converges within a bound. It is noted that for the constant disturbance case, the state $\mathbf{x}(k)$ eventually converges to zero when $\lim_{k\to+\infty} ||\mathbf{x}(k)|| = 0$. In addition, the boundedness of $\mathbf{x}(k)$ only depends on the prediction performance by showing the error bounds E_0 and E_1 . The proposed discrete-time predictive controller shows a simple structure combining disturbance predictor (8), (22), future state prediction (29), and control input (34).

IV. NUMERICAL SIMULATIONS

A. Simulation Setup

For the numerical simulation, the mechanical and aerodynamic parameters [9] are set as Table I. Table II represents control parameters, where the gain of disturbance observer is selected to satisfy exponential convergence in disturbance observation with $[I_{3\times3} - LB] \approx diag(\lambda_1, \lambda_2, \lambda_3)$ and $|\lambda i| < 0$. The initial condition is set as $x(0) = 06 \times 1$. For tracking control, the time-varying trajectory is defined as

 $\mathbf{x}_d(k) = [\mathbf{p}_d^T(k) \ \mathbf{v}_d^T(k)]^T$, where $\mathbf{p}_d(k) = [5\sin(\pi fTk) \ 10\sin(2\pi fTk) \ 10]^T \text{deg}, \mathbf{v}_d(k) = \dot{\mathbf{p}}_d(k) \ \text{deg/s}, f = 1/20 \text{Hz}, \text{and } T = 0.01 \text{s}$. The performance of the proposed controller is compared with the existing discrete-time prediction schemes [6], [7].

B. Simulation Results

Fig. 1 shows the comparison of the control performance for time-varying trajectory tracking between the existing prediction-based controllers and the proposed controller. The existing prediction scheme stabilizes the state of the helicopter despite input delay, but it is limited to compensate for the disturbance effect showing large tracking errors without predicting future information of disturbance. Otherwise, the proposed controller (34) can improve control performance under input delay and disturbance due to the accurate state prediction scheme (29) and robustness to disturbance by predictor (8) and (22) despite time-varying trajectory compared with other controllers. In addition, as shown in Fig. 2, the state $\mathbf{x}(k)$ is confined by stabilizing $\boldsymbol{\chi}(k)$ to the reference trajectory. Fig. 3 shows the results of the disturbance observer (6) and predictor (22). Fig. 4 shows the control input for the tracking control. As a result, the simulation results verify that the attitude of the unmanned helicopter can be stabilized despite delay and disturbance.

TABLE I LIST OF SPECIFICATIONS OF MECHANICAL AND AFRODYNAMIC PARAMETERS

MECHANICAL AND AERODYNAMIC PARAMETERS	
Moment of inertia	$I_{xx} = 0.3173$ kgm ² , $I_{yy} = 1.483$ kgm ² ,
	$I_{zz} = 1.539 \text{kgm}^2$, $I_{xz} = -0.0519 \text{kgm}^2$
Aerodynamic parameters	$K_{\beta} = 3$ Nm/rad, $I_{\beta} = 0.01931$ kgm ² ,
	$\gamma_{eta} = 1.0028, S_{eta} = 0.0229$
Distance	$h_R = 0.21 \mathrm{m}, l_T = 0.7978 \mathrm{m}$
Number of blades	$N_b = 2$
Disturbance	$\mathbf{d} = 0.01 \sin(0.001 \pi k) [1 \ 1 \ 1]^T \text{rad/s}^2$
Time delay	$h_c = 0.3 \mathrm{s}$

TABLE II List of Specifications of Control Parameters	
Disturbance observer (6)	$\mathbf{L} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 \end{bmatrix}$
Disturbance predictor (8), (22)	$r = 2, \omega_0 = 0.35$
State-feedback control (35)	$\mathbf{K}_{p} = \begin{bmatrix} 0.2 & 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0.7 & 0 & 0 & 0.8 & 0 \\ 0 & 0 & 0.45 & 0 & 0 & 1 \end{bmatrix}$

V.CONCLUSION

This paper developed the predictive controller for unmanned helicopters that achieves exponential stability for discrete time-delay systems in the presence of disturbances. The stability and control performance were mathematically proved. Then, the efficiency of the discrete predictive controller was demonstrated through numerical simulations, comparing its performance with existing controllers and showing better tracking performance. For future work, the proposed digital controller can be implemented in real applications using unmanned helicopters.



Fig. 1. Comparison of tracking control performance for time-varying reference trajectory.



Fig. 3. Performance of estimation and prediction for disturbance.

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Fig. 2. Control performance of the proposed controller.



Fig. 4. Control input of the proposed controller.

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