Integrated Rational Feedforward in Frequency-Domain Iterative Learning Control for Highly Task-Flexible Motion Control

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Abstract-Iterative learning control (ILC) yields accurate feedforward input by utilizing experimental data from past iterations. However, typically there exists a trade-off between task-flexibility and tracking-performance. This study aims to develop a learning framework with both high task-flexibility and high tracking-performance by integrating rational basis functions with frequency-domain learning. Rational basis functions enable learning of system zeros, enhancing system representation compared to polynomial basis functions. The developed framework is validated through a two-mass motion system, showing high tracking-performance with high taskflexibility, enhanced by the rational basis functions effectively learning the flexible dynamics.

I. INTEGRATING RATIONAL BASIS FUNCTIONS WITH FREQUENCY-DOMAIN ILC

In this study, the combined ILC (C-ILC) framework where 1) the rational basis function FF input $f_{j+1}^{\rm B}$ ensuring high task-flexibility, and 2) the frequency-domain FF input $f_{j+1}^{\rm F}$ only learning the residual dynamics, are simultaneously learned and combined as

$$f_{j+1} = f_{j+1}^{\mathrm{B}} + f_{j+1}^{\mathrm{F}} = \mathbf{F}(\theta_{j+1})r_{j+1} + f_{j+1}^{\mathrm{F}}.$$
 (1)

The entire scheme is illustrated in Fig. 1.

A. Learning of basis function FF input f_{j+1}^{B}

The performance criterion for f_{i+1}^{B} is given by

$$V(\theta_{j+1}) = \frac{1}{2} \|\underline{\hat{e}}_{j+1}^{\theta}\|^2, \quad \theta_{j+1} = \arg\min_{\theta_{j+1}} V(\theta_{j+1}), \quad (2)$$

where $\underline{\hat{e}}_{j+1}^{\theta} = \underline{\hat{e}}_{j+1} + \widehat{J}\underline{f}_{j+1}^{\mathrm{F}}$ and $\widehat{J} = (I + \widehat{G}K)^{-1}\widehat{G}$. From (2), the optimal parameter update is derived as

$$\theta_{j+1} = Q_j \theta_j + L_j (\underline{e}_j + \widehat{J} \underline{f}_j^{\mathrm{F}}), \qquad (3)$$

where the optimal learning matrix L_j and robustness matrix Q_i are iteratively calculated from (2).

B. Learning of frequency-domain FF input $f_{j+1}^{\rm F}$

Update of $f_{i+1}^{\rm F}$ is given by

$$f_{j+1}^{\rm F} = \mathbf{Q}(f_j^{\rm F} + \mathbf{L}e_j) + f_j^{\rm B} - f_{j+1}^{\rm B},$$
 (4)

with learning filter L and robustness filter Q designed by the user.

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Fig. 1: Updating procedure of combined ILC (C-ILC). The flow of the time-domain update for f_{j+1}^{B} (->) and frequencydomain update for $f_{i+1}^{\rm F}$ (\rightarrow) are illustrated.



Fig. 2: The developed C-ILC (rat) (--) achieves as high task-flexibility as B-ILC (rat) (->-), higher than C-ILC (pol) (---) when task changes at j = 15. In terms of tracking--) achieve high performance far exceeding both B-ILC frameworks (--, --).

II. EXPERIMENTAL RESULTS

The error norms per trial in the developed C-ILC (rat) and previous ILC methods are compared in Fig. 2. The results show that C-ILC (rat) performs high tracking-performance against repetitive tasks; even exceeding that of frequencydomain ILC (F-ILC), while ensuring high task-flexibility against changing tasks; identical to that of rational basis function ILC (B-ILC (rat)).