Modelling Non-Linear Dynamic Disturbances in a Linear Track Positioning System

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Abstract—This paper compares three modelling approaches to capture the motion dynamics of a linear track sliding platform. Non-linear disturbances affecting the platform’s motion were observed. Effective control requires that these disturbances are modelled and can be dealt with in the control strategy. Step responses are measured experimentally to study the relation between motor torque and the speed of the platform. The model parameters are identified using the experimental data. A co-simulation based model (Siemens NX - Simulink) in conjunction with an equation-based resisting torque is developed as a first candidate, and compared with a non-linear autoregressive model with exogenous input (NLARX) and a simple first-order transfer function. The models’ ability to capture the system dynamics are then evaluated. The NLARX model and co-simulation models are found to have significant value for control applications.

Index Terms—System modelling, Experimental validation, Linear positioning, Non-linear ARX neural network, Co-simulation

I. INTRODUCTION

Any control problem requires a model to solve. The level of detail which is needed from the model depends on the application. As industry requires increasingly faster and more efficient machines, control algorithms must evolve. Thus, models must also improve both in the level of detail and in their computational loads. Additionally, testing control algorithms in simulation environments helps cut down the risks of mechanical damages and significantly reduces the design cycle time. This has a significant economic impact since expensive parts are less likely to be broken, additionally less R&D hours are required. In this paper, a model of a linear track platform is developed as part of the larger research effort of obtaining efficient adaptive and collaborative control between this platform and mechatronic systems such as a robot arm.

The model developed has multiple uses: first, it will be used to develop and test control strategies in a simulation environment before being validated on the experimental setup. Secondly, it will serve as a digital twin of the system to detect physical changes in the system. Changes in speed, position, weight and temperature disturb the system. These disturbances should thus be compensated in the control loops. Using the platform movement prediction will allow to adjust the control loops of the robot arm and prevent the torque disturbances from affecting the robot’s motion. Moreover, the model will also be used for disturbance rejection inside the platform’s own control loop. These disturbances include non-linear friction dynamics, i.e. the friction force varies with the platform speed and the gearbox oil temperature. Previous studies have shown that adaptive control and feedback successfully countered non-linear friction behaviour for high precision positioning [1]. In [2], sliding mode feedback control is introduced in the control loop, while [3] introduces a fuzzy uncertainty estimator. In [4], a non-linear compensator for friction dynamics is developed, which in conjunction with a Proportional and Derivative (PD) controller, improved setpoint tracking compared to the PD controller alone. LuGre models have also been used to model friction in motion control problems [5]. Most recently, an online parameter identification and friction compensation control method for servo mechanisms was developed [6]. These methods mainly focus on modelling the friction behaviour of systems. Kalman filter based estimations such as used in [7] and [8] have been used to estimate load torque and moment of inertia but can diverge in case of poor initialisation. Sliding mode observers have been used for the same purpose [9]. The novel approach proposed in this paper allows to model more than one system disturbance while being computationally lighter than Kalman filters and sliding mode observers. In this case friction and the resisting torque caused by a folding cable guide. This torque is described as non-linear because it varies non-linearly depending on the position of the platform along the track.

The first model developed in this paper is a co-simulation model which uses the CAD software Siemens NX and Matlab Simulink. Siemens NX models the physical relation between the moving parts of the system. It sends to Simulink the position, speed and acceleration of the system parts in the CAD model. Simulink is used to send torque signals to the CAD model. Controllers can be implemented in Simulink to control the movement of the CAD model using its motion feedback. A co-simulation approach has already been used in the past in [10] for the development of mechatronic systems using the ADAMS-Simulink co-simulation and a simulation prototype of a space manipulator joint [11].

The second model type used in this paper is a non-linear autoregressive model with exogenous input (NLARX). This
model and its linear counterpart, the ARX model, being data-driven models, have the potential to model a wide variety of systems and relations. In [12] ARX and NLARX models were compared for position prediction of a robot arm based on the tension in its actuating tendons. An ARX model was used to model an ultra precise positioning system, a non-linearity was found which impeded the model performance [13]. An NLARX model has also been successfully used to substitute the complex forward kinematics calculations of a Stewart platform robot, thus permitting real-time use in control applications [14].

The third model used is a simple first order transfer function with delay, also known in some literature as the Broı́da model or a 1st order Strejc model. An example in literature is the identification of the synchronous electrical machine [15]. Since the identification of such a model is simple, it will be used as a benchmark to justify the necessity of employing more complex models such as the co-simulation and the NLARX.

In this paper, the experimental setup and data collection are described in § II, the models and their identifications are described in further detail in § III. Then the results of the model simulations are compared against each other and against the experimental data in § IV. Finally the results are summarized in § V.

II. SETUP DESCRIPTION

The linear positioning platform modelled in this paper, is part of a larger experimental setup. The setup is composed of two linear tracks: a top track on which a robot arm is attached and a bottom track with a platform. The lower track and its platform will serve as the basis for the modelling efforts undertaken in this paper. Fig. 1 shows a 3D CAD representation of this bottom linear track.

The platform moves linearly over two linear guides of about 4.5 meters. A permanent magnet synchronous motor drives the platform through a gearbox and a rack and pinion system. An articulated cable guide is attached to the platform which folds and unfolds as the platform moves on the track. Finally, weight can be added or removed to the platform by means of 24 steel plates of 12 kg each; in this study the total platform weight is fixed at 364 kg, i.e., 16 plates are loaded on the platform. The principal characteristics of the bottom platform and drivetrain are summarized in Table I.

The first step of this experimental model validation is data collection. The Simotion Scout software is used to control the movement of the bottom platform by sending torque inputs to the motor. Encoder data is recorded to obtain the position and the linear speed. The torque signal and the rotational speed of the motor are recorded with a sampling rate of 4 ms.

In addition to the non-linear and speed-dependent friction, it was observed that the speed response to a torque step has a sinusoidal component, especially for smaller torque inputs and thus lower speeds. This sinusoidal wave is determined to originate from the cable guide which folds and unfolds with the platform motion. Indeed, the folding mechanism involves an up and down motion as portrayed by the yellow arrows in Fig. 2. Therefore, the movement of the cable guide is seen as a force resisting the platform’s motion, thus the motor sees this as a disturbance torque. Moreover, when the platform is at the rightmost edge of the track, the cable guide is fully folded in its lower tray and thus stays stationary when the platform moves. Conversely, when it is at the left edge of the track, a greater proportion of the cable guide moves with the platform. This creates a position dependent inertia as the weight displaced varies relative to the location of the platform on the track. Finally, it was observed that the speed response to a torque step resembled that of a 1st order system. A frictionless system would have a ramp response to such an input signal. This implies that the system’s friction forces are speed dependent.

An observation made early on while running the experiments is that gearbox temperature has an influence on the dynamics of the system. More specifically, when the gearbox has not been warmed up, the friction force is significantly greater. To ensure consistency in the results, the gearbox was warmed up prior to each set of the experiment by letting the platform run back and forth on the track to keep the gearbox in regime conditions throughout the data collection process.

III. MODEL DESCRIPTION

As previously mentioned, three methods were selected to model the platform motion on the linear guides; the first is a co-simulation between Siemens NX and Simulink, the second is a non-linear ARX model, and the third is a simple Broı́da model. In the co-simulation method, the 3D CAD model is created in Siemens NX. Material properties and motion constraints are then specified for each component. Finally, the co-simulation between Siemens NX and Simulink is parametrized and the Siemens NX block is built into Simulink.

In the co-simulation modelling method, the disturbance torque seen by the motor (resulting from the cable guide and the friction) is modelled as a position dependent resisting torque using (1):

\[ T_f(x(t)) = A \frac{dx}{dt} + Bx(t) + C \sin(Dx(t)) \exp(Ex(t)) \]  

\[ A, B, C, \text{and } D \text{ are the factors applied to each term of } T_f. \]

\[ D \text{ determines the wavelength of the sine wave. } E \text{ is the exponential decay term present to model the attenuation of the} \]

\[ \text{Table I: Linear Track Specifications} \]

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
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<tbody>
<tr>
<td>Nominal Motor Torque</td>
<td>3.2 Nm</td>
</tr>
<tr>
<td>Nominal Motor Speed</td>
<td>3000 rpm</td>
</tr>
<tr>
<td>Radius of pinion</td>
<td>31.83 mm</td>
</tr>
<tr>
<td>Gearbox ratio</td>
<td>29</td>
</tr>
<tr>
<td>Max platform speed</td>
<td>0.57 m/s</td>
</tr>
<tr>
<td>Track length</td>
<td>4.5 m</td>
</tr>
<tr>
<td>Platform weight</td>
<td>364 kg</td>
</tr>
</tbody>
</table>
sinusoidal effect when the cable guide folds. These parameters are then found using the fminsearch function in Matlab. A set of 5 step responses covering the whole length of the track, with step motor torque inputs (0.43 Nm) is used for the parameter identification. The parameter identification is performed using a least-squares criterion between the experimental and model speed curves. This in effect a hybrid system identification method between first-principle modelling and experimental identification using an author defined model structure. This means that model dynamics coming from the CAD model will be exact regardless of the position and speed working points. However, multiple set of parameters are identified to obtain good model performance at all working points.

A Broïda model is identified for each speed working point, as displayed in (2), where $K$ is the gain, $T$ the time constant and $T_m$ the dead time. The fminsearch function is used to find these parameters, using the same least-squares cost criterion as in the co-simulation method. Each speed working point’s model is identified by minimising the cost of multiple step responses covering the whole length of the track. This yields an average model which does not account for position non-linearities.

$$H(s) = \frac{K}{1 + Ts} \exp(-T_m s)$$

A standard SISO ARX one-step ahead predictor, as presented in (3) would suffer from the same limitations as the co-simulation and the Broïda model methods. More specifically, the ARX is a linear model and would thus poorly model the system at its multiple speed and position working points.

$$\hat{y}(t) = a_1 y(t-1) + ... + a_n a y(t-na) + b_1 u(t-nk) + ... + b_n b u(t-nb-nk+1)$$

With $\hat{y}(t)$ being the predicted output at the current timestep, $y(t-n)$ the system’s nth measured past output, $u(t-n)$ the nth system’s past input, $a_n$ and $b_n$ being the weight coefficients.
respectively associated to the nth past output and nth past input. The coefficients \( n_a, n_b \) are respectively the number of past outputs and past inputs used for the prediction and the coefficient \( n_k \) is the delay with which past inputs are fed to the predictor.

Adding sigmoid functions after the ARX linear weights introduces non-linearities in the ARX model structure, allowing to solve the non-linear problem at hand. For this, the narxnet Matlab function is used. This is in effect a single hidden layer neural network. The selected network is depicted in Fig. 3. There are four delay operators in the hidden layer. This means that the four previous input and output values are required to compute the current output. Consequently, it takes 4 timesteps to initialize this model when running it online. 10 non-linear neurons are included in the hidden layer; this parameter is selected experimentally to obtain good performance and avoid overfitting issues. Too many layers or too many neurons per layer would make the model tend to memorize the training data, thus resulting in poor performance when running with online data and the validation data set. Additionally the network is trained using the Matlab implementation of Bayesian regularization (trainbr in Matlab) to further avoid overfitting and improve the model generalization. Finally the model timestep was increased to 0.2 seconds (compared to the original 4 ms) to avoid convergence to a model of the type: \( \hat{y}(t) = y(t-1) \). Indeed, with too small of a timestep, the learning algorithm would converge to such an equation instead of estimating the real system dynamics. This is a data-driven model and thus will suffer from the same problem in extrapolation conditions as the previous two models if the identification data is not representative of the operating conditions.

### IV. RESULTS AND DISCUSSION

Fig. 4 presents the comparison between the speed step responses of the three models for a torque step input with an amplitude of 0.43 Nm. For this torque amplitude, 5 torque steps were applied to cover the whole length of the track. Fig. 4 shows the 2nd leftmost step, this means that the cable guide is relatively unfolded and thus its effect is relatively strong. From the graph, it can be noticed that for this step response, the NLARX model matches best the experimental curve. Indeed, as indicated in the first line of Table II, the NLARX MSE is 7 times lower than that of the co-simulation model and 1.75 times smaller than that of the Broïda model. The Broïda model captures the gain, delay and time constant of the system as the model response aligns relatively well with the experimental curve at the beginning of the step and has a similar steady-state value. Finally, the co-simulation model initially matches the experimental response but then stays at lower values for the rest of the step response. Despite this steady-state error, the model shows potential as the oscillatory behaviour of the platform movement is accurately captured. Indeed, the oscillations of the two step responses have similar wavelengths, and similar amplitudes, albeit with a slight phase shift. The model anticipates the oscillations compared to the system.

Fig. 4. Comparison of the three modelled speed responses (orange curves) with the experimental speed (black curve) to a torque step of 0.43 Nm (dotted blue curve), left of the track

Towards the center of the track, the co-simulation has a smaller steady-state error than at the left edge, as can be observed on Fig. 5. However, the sinusoidal component of its step response is in opposition of phase compared to that of the experimental curve. At the rightmost edge of the linear track, on Fig. 6, the co-simulation performs well as the steady-state error is small and the oscillation is almost in phase when compared with the experimental speed step response. Both at the center and at the right edge of the track, the NLARX model has high MSE values, as displayed on line 2 and 3.
of Table II, this may suggest poor overall performance of the model. However, a visual analysis of Fig. 5 and Fig. 6 indicates that the NLARX step responses only have an initial spike in speed but then match the experimental data. Finally, the Broïda model has good performance for the whole length of the track as indicated by the low MSE values (lines 1 through 3 of Table II). However, these low error values are achieved solely by modelling first-order dynamics. Indeed, the Broïda model does not account for the oscillatory component of the speed step response.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Co-simulation</th>
<th>Broïda</th>
<th>NLARX</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4.91e−05</td>
<td>1.20e−05</td>
<td>6.84e−06</td>
</tr>
<tr>
<td>5</td>
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<td>1.75e−05</td>
<td>9.62e−05</td>
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<tr>
<td>6</td>
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</tr>
<tr>
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<td>4.23e−04</td>
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</tr>
<tr>
<td>8</td>
<td>1.51e−05</td>
<td>1.30e−04</td>
<td>3.33e−05</td>
</tr>
</tbody>
</table>

NLARX has a single set of parameters. Fig. 7 shows the step responses to torque steps of 0.55 Nm at the leftmost edge of the track. By looking at the speed curves in Fig. 7 more closely, it can be observed that the system displays a much smaller oscillating effect than for the smaller 0.43 Nm torque input. Indeed, for higher torque inputs, friction becomes the dominant disturbance, while the folding cable guide has a weaker relative effect. With the new set of parameters, the co-simulation and Broïda models struggle to fully capture the system dynamics at the left edge of the track. Indeed, in Fig. 7, their step responses have a significant error when compared to the experimental curve between time 3s and 6s, and thus a large MSE value. The NLARX model has one wrong prediction point at the beginning of the step response in Fig. 7 then tracks the experimental speed almost perfectly, this spike explains the relatively high MSE in this case. The NLARX performs well at the rightmost edge of the track as displayed in Fig. 8. Despite its weak performance at the left edge of the track, the co-simulation model has performance on par with that of the NLARX model at the right edge of the track. Indeed, the MSE of the NLARX and of the co-simulation are small, 3 and 10 times more compared to the Broïda model, as shown in line 6 of Table II.

Overall, the NLARX model shows good performance for prediction situations. However, its dependence on the digital measured output data makes it a poor performer in simulation situations. The NLARX model is therefore of value as a digital shadow of the system in real-time applications. Furthermore, its slower sampling rate is a significant advantage for real-time calculations. Finally, since it is a non-linear model, it allows to use the same set of parameters for the different speed and position working points of the system, additionally it also allows to model the braking and decelerations of the platform without updating the model parameters. The co-simulation model relies on the torque inputs and on the mechanical links and constraints specified in the CAD model. Thus, this model has superior performance in a simulation environment when compared to the NLARX model and will therefore allow to test control algorithms in a simulation environment before applying them on the physical system.

V. CONCLUSIONS

This paper exposed three candidate models for a linear track positioning platform exhibiting non-linearities. The non-
linearities observed are a speed dependent friction and a resisting force due to the cable guide folding and unfolding. The platform motion was modelled using a CAD based co-simulation associated with an equation-based resisting torque. An NLARX predictor model was also identified as well as a first-order transfer function. The simple transfer function offered the worst performance as it could not model the non-linearities present in the system. The co-simulation model performed better as it captured the sinusoidal oscillations, however it required a new set of parameters for each speed working point. Finally, the NLARX proved to most accurately model the platform motion. In conclusion, the CAD-based model will serve to test control algorithms in a simulation environment while the NLARX is more suited to be used as a digital shadow of the system during its operation.

Future work will focus on including the effects of the platform weight and those of the gearbox temperature. Additionally, the model will be used for future research on collaborative and adaptive control.

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