Predefined-Time Stabilization of Permanent-Magnet Synchronous Motor System with Deterministic and Stochastic Disturbances

Alison Garza-Alonso, Michael Basin, Pablo Rodriguez-Ramirez

Abstract—This paper designs a predefined-time convergent continuous control algorithm to stabilize a permanent-magnet synchronous motor system. Three cases have been considered: disturbance-free, in presence of a deterministic disturbance satisfying a Lipschitz condition, and in presence of both a stochastic white noise and a deterministic disturbance satisfying a Lipschitz condition. The designed control law is free from the restrictions of exponential control growth and exact initial conditions knowledge. This is the first predefined-time convergent continuous control algorithm applied to stabilizing a permanent-magnet synchronous motor system with both deterministic and stochastic disturbances, which enables one to a priori set the predefined convergence time even in presence of various disturbances of different nature. Numerical simulations are provided for a permanent-magnet synchronous motor system to validate the obtained theoretical results in each of the three considered cases. The simulation results demonstrate that the employed values of the predefined-time convergent control inputs are applicable in practice.

I. INTRODUCTION

Permanent-magnet synchronous motors (PMSM) have received increasing attention from industry and household applications due to their characteristics of high power density, high efficiency, high power factor, compact structure, and excellent control performance. The PMSM has been applied in many areas such as aeronautics and astronautics, electric vehicle technology, domestic appliance, fan and pump applications [1], [2], [3]. The PMSM drive plays an important role in motion-control applications. On the other hand, the control performance of the PMSM servo drive is commonly influenced by uncertainties, such as unpredictable plant parameter variations, external load perturbations, or unmodeled dynamics of the controlled plant.

Recently, many researchers have presented various advanced control strategies to efficiently control the PMSM systems such as linearization control [4], adaptive control [5], [6], robust control [7], [8], sliding mode control [9], fixed-time convergent control [10], fuzzy control [11], neural network control [12], and predictive control [13]. Much attention has been paid to designing finite-time and fixed-time convergent control laws and estimating their convergence (settling) times. A continuous finite-time convergent algorithm for a second-order system was designed in [14]. Continuous fixed-time convergent scalar and multi-variable control laws are proposed [15] for a super-twisting system, whose state and disturbance initial conditions are unknown, and the upper estimates of their convergence times are calculated. A continuous fixed-time convergent control law driving the states of a stochastic super-twisting system at the origin for a fixed-time is designed in [16]. A novel continuous controller ensuring an arbitrary predefined convergence time independent of initial conditions is proposed in [17] for the n-dimensional integrator chain, if no disturbances are present.

It can be observed that in most fixed-time approaches, the settling time cannot be assigned a priori. This drawback is corrected in predefined-time convergent algorithms, where the settling time can be assigned arbitrarily. However, the predefined-time convergent algorithm given in [17] results in an exponentially growing control magnitude for negative state initial values and requires exact knowledge of state initial conditions in presence of disturbances. In this paper, the research focuses on designing novel predefined-time convergent control laws for stabilizing PMSM systems affected by deterministic disturbances and stochastic noises, which are free from the mentioned restrictions of exponential control growth and exact initial conditions knowledge. To the best of our knowledge, this is also the first attempt to design a predefined-time convergent control law for a super-twisting system with stochastic noises.

The contributions of this paper are as follows.

- Designing predefined-time convergent control laws driving the states of a super-twisting system with deterministic disturbances and stochastic noises at the origin for an a priori assigned time, which are free from the restrictions of exponential control growth and exact initial conditions knowledge.
- Applying the developed algorithms to a case study of controlling a PMSM system affected by deterministic disturbances and stochastic noises and validating the obtained theoretical results by numerical simulations.

The standard notation is used: $|x(t)|$ denotes the absolute value of a scalar $x(t)$, $\|x(t)\|$ denotes the Euclidean norm of a vector $x(t)$, $a^p$ corresponds to a value $a$ raised to a power $p$. The function $\text{sign}(x)$ is introduced according to Filippov’s definition [18].

II. PROBLEM STATEMENT

The mathematical model of a permanent magnet synchronous motor (Fig. 1) can be represented by the following
Fig. 1. PMSM electromechanical diagram.


equations [3]:
\[
\begin{align*}
\frac{d\omega}{dt} &= \frac{K_t}{J} i_q - \frac{B}{J} \omega - \frac{F_J}{J}, \\
\frac{di_d}{dt} &= -\frac{R_s}{L_d} i_d + n_p \omega i_q + \frac{u_d}{L_d}, \\
\frac{di_q}{dt} &= -\frac{R_s}{L_q} i_q + n_p \omega i_d - \frac{n_p \phi_v}{L_q} \omega + \frac{u_q}{L_q}.
\end{align*}
\]  

Here, \(L_d\) and \(L_q\) are the inductances of \(d-q\) axes satisfying \(L_d = L_q = L\), \(i_d\) and \(i_q\) the stator currents of \(d-q\) axes, \(R_s\) the stator resistance, \(\omega\) the rotor velocity, \(n_p\) the number of pole pairs, \(\phi_v\) the rotor flux linkage, \(K_t = \frac{3 H_p}{2}\), \(F_J\) the load torque, \(J\) the moment of inertia, and \(B\) the viscous friction coefficient.

In this paper, the strategy of \(i_d = 0\) is adopted [3]. Then, the first equation in (1) can be rewritten as
\[
\frac{d\omega}{dt} = \frac{K_t}{J} i_q - \frac{B}{J} \omega + \zeta, \quad \omega(t_0) = \omega_0,
\]  

where \(i_q\) is the control input and \(\zeta = -\frac{F_J}{J}\) is a disturbance.

The control problem is to design a continuous control law driving the states of the resulting closed-loop system at the origin for a predefined time in the sense of the following definitions.

Consider a \(n\)-dimensional system
\[
\begin{align*}
\dot{x}(t) &= u(t) + \xi(t) + \sigma(t, x(t))dW(t), \\
x(t_0) &= x_0 \in \mathbb{R}^n, t > 0
\end{align*}
\]  

where \(x(t) \in \mathbb{R}^n\) is a state, \(u(t) \in \mathbb{R}^n\) is a control input, \(\xi \in \mathbb{R}^n\) is a disturbance satisfying the Lipschitz condition
\[
||\xi(t_1) - \xi(t_2)|| \leq L|t_1 - t_2|
\]

for any \(t_1, t_2 \leq t_0\), with a certain constant \(L\). \(W(t)\) is a Wiener process defined on the complete probability space \((\Omega, F, P)\), where \(\Omega\) is the sample space, \(F\) is a \(\sigma\)-field with a filtration \(\{F_t\}_{t \geq 0}\), and \(P\) a probability measure. The condition \(\sigma(t, 0) = 0\) is satisfied for all \(t \geq 0\).

Consider two cases:

a) \(\sigma(t, x(t)) = 0\). The system (3) reduces to a deterministic system.

\[
\text{Definition 1:} \quad \text{(Predefined-time convergence for a deterministic system.) The system (3) is called predefined-time convergent to the origin, if}
\]

1. it is fixed-time convergent to the origin, for any initial state \(x_0 \in \mathbb{R}^n\), there exists a positive constant \(T_{\text{max}} > 0\) independent of \(x_0\), such that \(x(t) = 0\) for all \(t \geq T_{\text{max}}\),
2. \(T_{\text{max}}\) is independent of initial conditions and disturbances and can be arbitrarily chosen in advance, and
3. \(T_{\text{max}} \geq T_f\), where \(T_f\) is the true convergence time.

b) \(\sigma(t, x(t)) \neq 0\). The system (3) is a stochastic system.

\[
\text{Definition 2:} \quad \text{(Predefined-time convergence for a stochastic system.) The system (3) is called predefined-time convergent to the origin in \(\rho\)-mean, if}
\]

1. it is fixed time convergent to the origin in \(\rho\)-mean, i.e., for any initial state \(x_0 \in \mathbb{R}^n\), there exists a positive constant \(T_{\text{max}} > 0\) independent of \(x_0\), such that \(E[|x(t)|^\rho] = 0\) for all \(t \geq T_{\text{max}}\),
2. \(T_{\text{max}}\) is independent of initial conditions and disturbances and can be arbitrarily chosen in advance, and
3. \(T_{\text{max}} \geq T_f\), where \(T_f\) is the true convergence time.

III. PREFINDED-TIME STABILIZATION WITHOUT DISTURBANCES

A. Control design
Consider the system (2), where \(\zeta = 0\). Then, the equation (2) reduces to
\[
\begin{align*}
\frac{d\omega}{dt} &= \frac{K_t}{J} i_q - \frac{B}{J} \omega, \\
\omega(t_0) &= \omega_0,
\end{align*}
\]  

where the control law is designed as
\[
i_q = \frac{J}{K_t} (u_1(t) + u_2(t)).
\]

Here, \(u_1(t) = \frac{B}{J} \omega\) is the compensation term and \(u_2(t)\) is defined as
\[
u_2(t) = \begin{cases} 
-\eta \left( \frac{e^{n(\omega(t))}}{\epsilon^{n(\omega(t))}} \right) \text{sign}(\omega(t)), & 0 \leq t < t_f, \\
0, & t_f \leq t,
\end{cases}
\]

where \(\eta > 1\).

\[\text{Theorem 1:} \quad \text{The control law (5) drives the state \(\omega(t)\) of the system (4) and its derivative \(\dot{\omega}(t)\) to the origin for an a priori pre-assigned time \(t_f\) and stays there afterwards for any \(t \geq t_f\). In other words, the closed-loop system (4) is predefined-time convergent to the origin.}
\]

\[\text{Remark 1:} \quad \text{Note that the control law (6) depends only on the absolute values of the state \(\omega(t)\). Therefore, the state trajectories and control inputs corresponding to positive and negative initial values of the same magnitude are symmetrically mirrored with respect to the time axis, so the exponential growth phenomenon does not take place even for negative state initial values.}\]
B. PMSM Simulations

To demonstrate efficiency of the proposed control law, numerical simulations have been performed for the PMSM system (4),(5). The PMSM parameters used in the simulation are assigned as $J = 1.74 \times 10^{-4} \text{kg} \times \text{m}^2$, $B = 7.403 \times 10^{-5} \text{N}$, $n_p = 4$, $\phi_o = 0.1167 \text{wb}$. The control parameters are $\eta = 30$, $t_f = 20$. The treated initial conditions and the corresponding settling times obtained by simulation are given in Table I. Figures 2 and 3 present the time histories of $\omega(t)$ for initial condition $\omega_0 = 100$, which demonstrate that the system (4) is predefined-time convergent to the origin for the desired time $t_f$, as stated in Theorem 1. Finally, Figure 4 shows the control input (5) for the initial condition of $\omega_0 = 100$.

<table>
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<tr>
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![Fig. 2. Time histories of $\omega(t)$ with initial condition $\omega_0 = 100$.](image)

![Fig. 3. Time histories of $\omega(t)$ with initial condition $\omega_0 = 100$ (zoomed).](image)

| ![Fig. 4. Control input (5).](image) |

IV. PREDEFINED-TIME STABILIZATION WITH DETERMINISTIC DISTURBANCES

In this section, two predefined-time convergent control laws are designed for a permanent magnet synchronous motor system in presence of a deterministic disturbance $\zeta(t)$ satisfying the Lipschitz condition with a constant $L$.

A. Control Design

Consider the equation (2), where $\zeta \neq 0$ and the control law is designed as

$$i_q = \frac{J}{K_t} (u_1(t) + u_2(t)), \quad (7)$$

where $u_1(t)$ is defined in Section III and $u_2(t)$ is to be assigned. Substituting $i_q(t)$ into (2) yields

$$\ddot{\omega}(t) = u_2(t) + \zeta(t), \quad (8)$$

where $u_2(t)$ is given by

$$u_2(t) = \begin{cases} 
-\lambda_1 |\omega(t)|^{1/2} \text{sign}(\omega(t)) - \lambda_2 |\omega(t)|^p \text{sign}(\omega(t)) \\
-\alpha \int_0^t \text{sign}(\omega(s)) ds \\
-\eta \frac{e^{\omega(t)}}{e^{\omega(t)}} \text{sign}(\omega(t)), \quad t_0 \leq t < t_f,
\end{cases}$$

$$-\lambda_1 |\omega(t)|^{1/2} \text{sign}(\omega(t)) - \lambda_2 |\omega(t)|^p \text{sign}(\omega(t)) \\
-\alpha \int_0^t \text{sign}(\omega(s)) ds, \quad t \geq t_f. \quad (8)$$

Here, $\lambda_1, \alpha > 0$, $\lambda_2 \geq 0$, $\eta > 1$, and $p > 1$. The gain $\lambda_2$ can be selected as $\lambda_2 = 0$ or $\lambda_2 > 0$.

Theorem 2: The control law (8) drives the state $\omega(t)$ of the system (4) and its derivative $\ddot{\omega}(t)$ to the origin for an a priori pre-assigned time $t_f$ and stays there afterwards for any $t \geq t_f$. In other words, the closed-loop system (4),(8) is predefined-time convergent to the origin in the presence of disturbance $\zeta(t)$ satisfying Lipschitz condition with a constant $L$, if the following conditions hold: $\eta > 1$, $\alpha > L$, $\lambda_1 \geq \sqrt{2\alpha}$, $\lambda_2 \geq 0$, and $p > 1$.

The resulting closed-loop control system can be represented in the conventional super-twisting form

$$\dot{\omega}(t) = \begin{cases} 
-\lambda_1 |\omega(t)|^{1/2} \text{sign}(\omega(t)) - \lambda_2 |\omega(t)|^p \text{sign}(\omega(t)) \\
-\eta \frac{e^{\omega(t)}}{e^{\omega(t)}} \text{sign}(\omega(t)) + y(t), \quad t_0 \leq t < t_f,
\end{cases}$$

$$\dot{\omega}(t) = \begin{cases} 
-\lambda_1 |\omega(t)|^{1/2} \text{sign}(\omega(t)) - \lambda_2 |\omega(t)|^p \text{sign}(\omega(t)) + y(t), \quad t \geq t_f,
\end{cases} \quad (9)$$

$$\dot{y}(t) = -\alpha \text{sign}(\omega(t)) + \dot{\zeta}(t), \quad \omega(t_0) = \omega_0, \quad y(t_0) = y_0.$$

Then, both states $\omega(t)$ and $y(t)$ converge to the origin for the predefined time $t_f$. 

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B. PMSM Simulations

To demonstrate efficiency of the proposed control law, numerical simulations have been performed for the PMSM system (2),(8). First, the case $\lambda_2 = 0$ is considered. The PMSM parameters used in the simulation are assigned as $J = 1.74 \times 10^{-4} kg \times m^2$, $B = 7.403 \times 10^{-5} N$, $n_p = 4$, $\phi_0 = 0.1167 wb$. The deterministic disturbance is selected as $\zeta(t) = \frac{1}{2}(0.1t + 0.01\cos(t))$; accordingly, the Lipschitz constant is set to $L = 633$. The control parameters are $\eta = 30$, $t_f = 20$, $\lambda_1 = 50$, $\alpha = 635$, which are selected to satisfy the conditions of Theorem 2. The treated initial conditions and the corresponding settling times obtained by simulation are given in Table II. Figures 5 presents the time histories of $\omega(t)$ and $y(t)$ for initial conditions $\omega_0 = 100$, $y_0 = -50$, which demonstrate predefined-time convergence of the PMSM states to the origin for the desired time $t_f$, as stated in Theorem 2. Figure 6 shows the control input (7) for the initial conditions $\omega_0 = 100$ and $y_0 = -50$.

Then, the case $\lambda_2 > 0$ is considered, where the values $\lambda_2 = 1$, $p = 1.5$ are selected to satisfy the conditions of Theorem 2 and the other control parameters are the same as in the preceding set of simulations. The treated initial conditions and the corresponding settling times obtained by simulation are given in Table III. Figures 7 presents the time histories of $\omega(t)$ and $y(t)$ for initial conditions $\omega_0 = 100$, $y_0 = -50$, which demonstrate predefined-time convergence of the PMSM states to the origin for the desired time $t_f$, as stated in Theorem 2. Figure 8 shows the control input (7) for the initial conditions $\omega_0 = 100$ and $y_0 = -50$. Note that in both cases the magnitude of the control input $i_q$ remains less than $3A$, which is acceptable in practice. Thus, both control laws provide reliable stabilization of the PMSM states to the origin in presence of a deterministic disturbance within the pre-assigned time of 20 sec. The second control law with $\lambda_2 \neq 0$ yields considerably lesser convergence times for large state initial values, although its magnitude is only slightly higher than that in the first case.

![Fig. 5. Time histories of $\omega(t)$ and $y(t)$ when $\lambda_2 = 0$ with initial conditions $\omega_0 = 100$ and $y_0 = -50$.](image1)

![Fig. 6. Control input with $\lambda_2 = 0$ and initial conditions $\omega_0 = 100$ and $y_0 = -50$.](image2)

### TABLE II

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### TABLE III

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<th>Convergence times vs. initial conditions.</th>
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### V. PREDEFINED-TIME STABILIZATION WITH DETERMINISTIC DISTURBANCES AND STOCHASTIC NOISES

#### A. Control design

In this section, a predefined-time convergent control law is designed for a permanent magnet synchronous motor system in presence of a deterministic disturbance $\zeta(t)$ satisfying the Lipschitz condition with a constant $L$ and a stochastic white noise.

Consider the permanent-magnet synchronous motor system (2) in presence of a deterministic disturbance $\zeta(t)$...
satisfying a Lipschitz condition and a stochastic white noise

\[ d\omega = \left( \frac{K_t}{J} i_q - \frac{B}{J} \omega + \zeta \right) dt + \sigma(t,\omega) dW(t), \]

\( \omega(0) = \omega_0, \)

where \( W(t) \) is a Wiener process defined in (3).

The control law is represented as

\[ i_q = \frac{J}{K_t} (u_1(t) + u_2(t)), \]

(11)

where \( u_1(t) \) is defined in Section III and \( u_2(t) \) is to be assigned. Substituting \( i_q(t) \) into (2) yields

\[ d\omega(t) = (u_2(t) + \zeta(t)) dt + \sigma(t,\omega) dW(t), \]

where \( u_2(t) \) is given by

\[ u_2(t) = \begin{cases} -\lambda_1|\omega(t)|^{1/2}\text{sign}(\omega(t)) - \lambda_2|\omega(t)|^{p}\text{sign}(\omega(t)) - \alpha \int_{0}^{t} \text{sign}(\omega(s)) ds, & 0 \leq t < t_f, \\ -\eta \frac{e^{T}}{e^{T} - 1} \text{sign}(\omega(t)), & t \geq t_f, \end{cases} \]

and \( \lambda_1 > 0, \lambda_2 > 0, \eta > 1, \text{ and } \rho > 1. \)

**Theorem 3:** The control law (11) drives the \( \rho-\text{th} \) initial moment \( E[|\omega(t)|^{\rho}] \) of the state \( \omega(t) \) of the system (10) to the origin for an \textit{a priori} pre-assigned time \( t_f \) and stays there afterwards for any \( t \geq t_f. \) In other words, the closed-loop system (10),(11) is predefined-time convergent to the origin in \( \rho \)-mean in the presence of disturbance \( \zeta(t) \) satisfying Lipschitz condition with a constant \( L \) and a stochastic white noise with diffusion \( \sigma(t,\omega) = |\omega|^p, \) if the following conditions hold: \( \rho > 1, \eta > 1, \alpha > L, \lambda_1 > \sqrt{2\lambda}, \lambda_2 \geq 0, p > 1, 2\lambda_1 > \rho - 1 > 0, 2\lambda_2 > \rho - 1 > 0, \) and \( \frac{3}{2} \leq 2r \leq (1 + p). \)

The resulting closed-loop control system can be represented in the conventional super-twisting form

\[ \dot{\omega}(t) = \begin{cases} -\lambda_1|\omega(t)|^{1/2}\text{sign}(\omega(t)) - \lambda_2|\omega(t)|^{p}\text{sign}(\omega(t)) - \eta \frac{e^{T}}{e^{T} - 1} \text{sign}(\omega(t)) + y(t) + \sigma(t,\omega) dW(t), & t \leq t < t_f, \\ -\lambda_1|\omega(t)|^{1/2}\text{sign}(\omega(t)) - \lambda_2|\omega(t)|^{p}\text{sign}(\omega(t)) + y(t) + \sigma(t,\omega) dW(t), & t \geq t_f, \end{cases} \]

(13)

\[ \dot{y}(t) = -\alpha \text{sign}(\omega(t)) + \zeta(t), \quad \omega(t_0) = \omega_0, \quad y(t_0) = y_0. \]

Then, both states \( \omega(t) \) and \( y(t) \) converge to the origin for the predefined-time \( t_f. \)

**B. PMSM Simulations**

To demonstrate efficiency of the proposed control law, numerical simulations have been performed for the PMSM system (10),(11). The PMSM parameters, deterministic disturbance, Lipschitz constant, and control parameters are the same as in Section IV. The stochastic noise parameter is given by \( r = 0.75. \) The stochastic convergence is regarded in mean-square sense, \( \rho = 2, \) to satisfy the conditions of Theorem 3. The treated initial conditions and the corresponding settling times obtained by simulation are given in Table IV. Figures 9 presents the time histories of \( \omega(t) \) and \( y(t) \) for initial conditions \( \omega_0 = 100, \ y_0 = -50, \) which demonstrate predefined-time convergence of the PMSM states to the origin in \( \rho \)-mean for the desired time \( t_f, \) as stated in Theorem 3. Figure 10 shows the control input (11) for the initial conditions \( \omega_0 = 100 \) and \( y_0 = -50. \) The magnitude of the control input \( i_q \) also remains less than 3A, which is acceptable in practice. Thus, both control laws provide reliable stabilization of the PMSM states to the origin in presence of a deterministic disturbance and a stochastic noise within the pre-assigned time of 20 sec.
The performance of the developed algorithm is verified with numerical simulations, which demonstrate reliable predefined-time convergence of the PMSM system states to the origin in all three cases. The employed values of the predefined-time convergent control inputs are applicable in practical cases of PMSM stabilization.

REFERENCES


VI. Conclusions

This paper has presented a predefined-time convergent continuous control algorithm for a permanent-magnet synchronous motor, which is free from the restrictions of exponential control growth and exact initial conditions knowledge. Three cases have been considered:

- disturbance-free,
- in presence of a deterministic disturbance satisfying a Lipschitz condition, and
- in presence of a deterministic disturbance satisfying a Lipschitz condition and a stochastic white noise.