Dynamical Modeling of a Large Range Compliant Stage Considering the Intermediate Stage Effect

Shuaishuai Lu$^1$, Pengbo Liu$^2$, Peng Yan$^{1*}$, Shuai Zhou$^1$

Abstract—This paper presents an improved dynamical modeling method for the double parallelogram mechanism (DPM) based motion stage with a large workspace, where the impacts of intermediate stages are considered. According to the mode shape analysis of the DPM by the Euler-Bernoulli beam theory, it is observed that the high order modes induced by the intermediate stages have significant impacts on the dynamical behavior of the DPM. With this in consideration, the matrix displacement method is applied to the modeling of the DPM based compliant stage, where the intermediate stages of DPMs are regarded as nodes with masses. Correspondingly, an improved dynamical model is proposed to better describe the complicated dynamical characteristics of the compliant stage, which is further verified by finite element analysis (FEA) and experiments on a prototype. Compared with existing methods, the proposed dynamical model offers a new look into the kinetostatic analysis and dynamical design of the compliant mechanism-type micro/nanostage.

I. INTRODUCTION

Compliant mechanism based micro/nano motion systems with large range have been extensively urgent in various high precision engineering areas such as spatial manipulation, micro-fluidics equipment design and micro/nano-positioning system [1], [2]. To achieve large workspace, the double parallelogram mechanism (DPM) type compliant stage is widely employed in the above fields benefited from excellent elastic deformation capability [2], where the DPM significantly suppresses the undesirable parasitic motion and has attracted significant research efforts [3], [4].

Note that the accurate dynamical model is the foundation of high-precision positioning and tracking performance [3]. A considerable number of theoretical approaches have been explored to describe the dynamic characteristic of the compliant mechanism, for instance, 5R dynamic PRBM [5], Hamilton principle based nonlinear dynamic model [6], where the intermediate stages are regarded as the equivalence masses based on the principle of dynamic equivalence [7], [8]. However, a unconstrained and non-controllable degree of freedom (DOF) is introduced by the intermediate stages. Therefore, the equivalent lumped mass ignoring the redundant DOF will result in the significant dynamical modeling errors of high-order modes [9], [10]. It is still an open problem to develop the accurate dynamical model for the compliant mechanism considering the effect of intermediate stages [11].

This paper investigates the effect of the intermediate stages and develop a dynamical modeling method of the DPM type compliant mechanism. Through this new method, a more accurate dynamical model with consideration of the high order modes induced by the intermediate stages can be achieved. Then the derived dynamical model is verified by the finite element analysis (FEA) as well as experiments on a prototype of the compliant stage. Comparisons between the theoretical and FEA and experimental results demonstrate the effectiveness of the developed modeling method.

The remainder of this paper is organized as follows. The mechanism description of the DPM based compliant stage is described in section II. The kinetostatic and dynamical modeling of the beam flexure-based DPM is derived in section III. In section IV, the dynamical model of the compliant stage is established. Section V verifies the established models by FEA method and the fabricated prototype. Finally, some concluding remarks are summarized in section VI.

II. A DPM-BASED COMPLIANT STAGE

As illustrated in Fig.1(a), a compliant stage with sufficient millimeter-level $X-Y$ motions and certain capability of rational motion is designed. The details of the mechanical design such as contribution and application can be referred to our previous work [3].

Recall the DPM shown in Fig.1(b), two outer beam flexures (numbered 3 and 4) connected to the secondary stage (i.e., intermediate stage) and the base form the outer parallelogram ($P$ unit). Similarly, the two inner beam flexures (numbered 1 and 2) connected to the primary stage and the secondary stage form the inner parallelogram ($P$ unit). For the illustrated DPM, the two $P$ units are connected in series by means of the intermediate stage [2] which will introduce the redundant DOF in the transverse direction and higher order mode [12]. When any general load is applied on the primary stage, two axial degrees of constraint (DOCs) motions with different directions, and two translational DOFs motions with same directions are produced by the two $P$ units located in reverse. Therefore, the DPM would provide an amplified output displacement in the DOF direction, meanwhile, the DOC motions can well be restricted.

1S. Lu, S. Zhou and P. Yan are with the Key Laboratory of High-efficiency and Clean Mechanical Manufacture (Shandong University), Ministry of Education, School of Mechanical Engineering, Shandong University, Jinan 250061, China
2P. Liu is with the School of Mechanical and Automotive Engineering, Qilu University of Technology, Jinan, Shandong, 250353, China
*Corresponding Author, yanpeng@sdu.edu.cn
III. ANALYSIS OF THE DYNAMICAL EFFECT OF THE INTERMEDIATE STAGES

In this section, we first analyze the DPM to obtain the theoretical dynamical effect of the intermediate stages. Based on the structure and constraint, we will use the modal analysis method to derive the mode shapes of beam flexures and then calculate the natural frequencies. Typically, the damping coefficient for such structures can be assumed to be neglected [5].

To specify the displacement components, the global and local coordinate systems \(oxy\) and \(o'x'y'\) (located on the secondary stage) are defined. The general loadings \(F_x, F_y\) and \(M_z\) applied at the primary stage, result in displacements \(U_1, U_2, W_1, W_2, \Theta_1\) and \(\Theta_2\) along \(x, y\) and around \(z\) axes respectively [2].

According to the Euler-Bernoulli beam theory, each beam flexure satisfies

\[
\frac{\partial^2}{\partial x^2} \left[ EI \frac{\partial^2 \omega(x,t)}{\partial x^2} \right] + \rho S \frac{\partial^2 \omega(x,t)}{\partial t^2} = 0, \tag{1}
\]

where \(\rho\) and \(S\) represent material density and area of cross section of beams, \(\omega(x,t)\) is the deflection.

By separating the variables, we can obtain

\[
\omega(x,t) = \phi(x)q(t). \tag{2}
\]

For the \(i\)-th beam, the general solution \(\phi_i(x)\) of the above equation can be assumed as

\[
\phi_i(x) = C_{i1}\cos\beta x + C_{i2}\sin\beta x + C_{i3}\cosh\beta x + C_{i4}\sinh\beta x, \tag{3}
\]

where \(\beta^4 = \rho S\omega^2/EI\), \(E\) and \(I\) represent the Young’s modulus of the material and the inertia moment of the beam flexure, \(\omega\) is the natural frequency.

Considering the symmetric structure, the two beam flexures in the upper half part will have the same mode shapes as their corresponding counterparts in the lower half. Therefore, we analyze the boundary conditions for beam 1 and beam 3 as depicted in Fig.1(b) for the case of free vibrations. Boundary conditions (subscript represents the beam number) can be obtained as

\[
\phi_1|_{x=0} = \phi_2|_{x=0}, \quad \frac{d\phi_1}{dx}|_{x=0} = \frac{d\phi_2}{dx}|_{x=0} = 0, \quad \frac{d^2\phi_1}{dx^2}|_{x=L} = \frac{d^2\phi_2}{dx^2}|_{x=L} = 0,
\]

\[
\frac{d^3\phi_1}{dx^3}|_{x=0} + \frac{d^3\phi_2}{dx^3}|_{x=L} = -\frac{M_p}{2m}b^4\phi_2|_{x=L},
\]

\[
\frac{d^4\phi_1}{dx^4}|_{x=0} = -\frac{M_1}{2m}b^4\phi_1|_{x=0}, \tag{4}
\]

where \(M_p\) and \(M_1\) are the masses of the primary and intermediate stages respectively.

As intuitively shown in Fig.2(a) and 2(b), the mode shapes show that the homodromous translation of the primary and intermediate stages occurs at the first mode. The second mode is inverted translational vibrations of the two rigid stages. Note that the lumped mass method can describe the dynamical behavior of the low-order modes on the DPM, but it is not...
well applicable for the high-order modes. In what follows the
dynamical model of the complicated the compliant stage will
be analyzed.

IV. MODELING AND ANALYSIS OF THE COMPLIANT STAGE

By regarding the intermediate stages as nodes with masses,
the matrix displacement method will be used to establish the
dynamical model of the proposed compliant stage in this
section. Considering the symmetrical structure, we select a
quarter of the stage to analyze and the discretized model is
illustrated in Fig. 3. The stage can be discretized into beam
flexures and lumped masses which are denoted serially from
(1) to (12) and are connected with nodes from 1 to 5, all the
clamped nodes are numbered as 0.

Then, the nodal force and nodal displacement of the i-th
beam flexure in the reference system can be first expressed as

\[
\begin{align*}
\{F_{i,j}(\omega)\} &= (R_i^T \cdot D^r(\omega) \cdot R_i) \cdot \{x_j(\omega)\}, \\
\{k_{i,1} & \quad k_{i,2} \quad k_{i,3} \quad k_{i,4}\} \cdot \{x_j(\omega)\} = \{x_k(\omega)\}. 
\end{align*}
\]

The dynamical stiffness matrix of the i-th beam flexure is

\[
D_i^r(\omega) = R_i^T \cdot D^r(\omega) \cdot R_i, 
\]

where the block submatrices \(k_{i,1}, k_{i,2}, k_{i,3}\) and \(k_{i,4}\) of the
dynamical stiffness matrix \(D_i^r(\omega)\) can be expressed as

\[
k_{i,1} = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & -d_4 & 0 \\ 0 & -d_3 & d_4 & 0 \\ d_5 & 0 & 0 & 0 \\ 0 & d_6 & d_7 & 0 \\ 0 & -d_7 & d_8 & 0 \end{bmatrix}, \quad k_{i,2} = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & d_3 & 0 \\ 0 & 0 & d_3 & d_4 \end{bmatrix}, \quad k_{i,3} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \text{the values of}
\]
d
\]

In Eq.(6), transformation matrix \(R_i\) is determined by the
orientation of the i-th beam flexure with respect to the refer-
ence system \(xy\theta\) (denoted as \(\theta_i\)) and can be written as

\[
R_i = \begin{bmatrix} \cos \theta_i & \sin \theta_i & 0 & 0 & 0 \\ -\sin \theta_i & \cos \theta_i & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos \theta_i & \sin \theta_i \\ 0 & 0 & 0 & -\sin \theta_i & \cos \theta_i \end{bmatrix}. 
\]

We select the representative element 1 and node 2 in Fig.3
to illustrate the equilibrium equations

\[
\begin{align*}
\{F_{1,0}(\omega)\} &= \{k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4}\} \cdot \{x_0(\omega)\}, \\
\{F_{1,1}(\omega)\} &= \{k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4}\} \cdot \{x_1(\omega)\}. 
\end{align*}
\]

\[
\begin{align*}
(k_{2,3} + k_{3,3}) \cdot x_1(\omega) \\
+k_{2,4} + k_{3,4} + k_{6,4} + k_{7,4} + k_{10,1} + k_{11,1} \cdot x_2(\omega) \\
+k_{6,3} + k_{7,3} \cdot x_3(\omega) + (k_{10,2} + k_{11,2}) \cdot x_4(\omega) \\
+m_2 x_2(\omega) = P_2. 
\end{align*}
\]

Node 2 is the lumped mass of intermediate stage \(m_2\) is
induced in Eq.(9). The dynamical stiffness matrix of the total
stage can be expressed as
where $X(\omega) = [x_1(\omega), x_2(\omega), x_3(\omega), x_4(\omega), x_5(\omega), \cdots]^T$ is the nodal displacement vector, $F(\omega) = [P_1, P_2, P_3, P_4, P_5, \cdots]^T$ denotes the vector of external forces directly applied on the nodes in the reference system.

The balance equations for other nodes can be obtained similarly. Considering all the force balance of the nodes similar to Eq.(9), the total dynamical model of the compliant stage by taking into account the nodal displacements as the variable can be developed as

$$D(\omega) \cdot X(\omega) = F(\omega). \quad (11)$$

The natural frequencies of the compliant stage are the roots of the determinant of the total dynamic stiffness matrix, $D(\omega)$, i.e.

$$\det(D(\omega)) = 0. \quad (12)$$

V. SIMULATION VERIFICATION AND EXPERIMENTAL TESTS

In this section, the dynamical analyses are carried out with the FEA software ANSYS Workbench 14.5. Meanwhile, verification experiments are also deployed on a prototype of the designed DPM based stage to verify the impact of the intermediate stage on the dynamical behavior and evaluate the effectiveness of the developed model for such mechanisms.

A. Simulation with FEA

<table>
<thead>
<tr>
<th>Mode</th>
<th>DPM</th>
<th>Compliant stage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modes</td>
<td>$1^{st}$</td>
<td>$2^{nd}$</td>
</tr>
<tr>
<td>FEA (Hz)</td>
<td>11.77</td>
<td>27.39</td>
</tr>
<tr>
<td>Theoretical (Hz)</td>
<td>10.9</td>
<td>26.71</td>
</tr>
</tbody>
</table>

The frequency modes of the DPM and compliant stage are illustrated in Fig.4. The corresponding frequencies are listed in Table I. It can be observed that the translations of the primary and intermediate stages of the DPM in DOF direction occur in the first and second modes with the model deviations of 7.0% and 2.0%. One can also see that the fundamental frequency of the stage in the DOF direction (take the $x$ axis for example) occurs in the first mode and can be accurately predicted by the developed model with the deviations of 5.9%. Note that the first mode is translational vibration of the end effector. The second mode is translational vibration of the intermediate stage and the prediction error is less than 1.2% for the higher-order natural frequency. It can be observed that the resonance of the intermediate stage is objectively in existence which can be well demonstrated.

Then the harmonic analysis of the DPM and stage are also conducted via FEA software and the proposed model, where the measured displacements are the primary stage of the DPM and the end effector of the stage along one axis, as illustrated in Fig.5. It can be observed that the theoretical natural frequencies values considering the intermediate stage are relatively consistent with the FEA results, which validates the developed dynamical model for the harmonic response analysis of the DPM and stage. The improvement is even more significant over the previous model [14], where 37.5% and 12.7% improvement is obtained.

Fig. 4. Mode shapes of the DPM and compliant stage

B. Experimental apparatus setup and results

Based on the design in section II, a prototype stage was monolithically machined using the wire electrical discharge machining (WEDM) technique thanks to its accuracy and precision. An experimental apparatus for verification experiments is then established as depicted in Fig.6. The stage is driven by the voice coil motors (BEI LA15-16-024). The laser sensors (from KEYENCE LK-H020) are installed for real time displacement measurement, where the measurement resolution is 20 nm. Moreover, the Matlab/Simulink package xPC Target and National Instruments (NI) PCI-6259 I/ O hardware is employed for real time reading data.

We first conduct an experiment of linear motion tests by applying linear currents in the range from 0A to 0.1A and 0.5A, which demonstrate the kinetostatic performance covering the intermediate and motion stages. It is demonstrated that the
achieved natural frequency is nearly close to the theoretical value 8.6 Hz. However the calculated value ignoring the intermediate stage is 7.4 Hz and the deviation is about 14%. Note that the decrease of the frequency values are mainly because of the additional load such as target mirrors. The developed method possesses higher computational accuracy of the natural frequency.

Finally, the dynamical displacement amplification of the stage is also experimentally evaluated with 8Hz triangular signal excitation, with the open loop control command as depicted in Fig.11. It can be clearly seen that the developed method demonstrates excellent accuracy of calculating the dynamical displacement amplification, while achieving around 40% improvement over the method of ignoring the intermediate stage.

VI. CONCLUSION

In this paper, we developed a modeling approach for the DPM type compliant mechanism and stage with consideration of the induced redundant DOF and dynamics by the intermediate stages. Specifically, the Euler-Bernoulli beam theory and the modal analysis method were utilized to derive the mode shapes of the DPM. Moreover, the matrix displacement method was applied to the dynamical modeling of a DPM type large range compliant stage. Comprehensive FEA simulations and experiments were conducted to effectively verify the theoretical model. Feedback control approaches will be explored for the DPM type compliant system on the basis of the current work such as high speed and long stroke precision scanning.
CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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