Path Following of Wheeled Mobile Robots Using Online Optimization Based Guidance Vector Field

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I. INTRODUCTION

Due to its wide applications in military and industrial scenarios, the control of wheeled mobile robots has been intensively studied and many well-developed methods can be found in the literatures. The control objectives for wheeled mobile robot can be classified into three basic categories [1]: posture stabilization [2], [3], trajectory tracking [4]–[7], and path following [8], [9]. In [2], the posture stabilization problem was solved by a Bézier smooth subline constraint based nonlinear model predictive control method which generates a smooth motion. An extended state observer based controller was given in [10] for trajectory tracking and obstacle avoidance of mobile robots. The authors in [7] developed a trajectory tracking controller for mobile robots by combining model predictive control and adaptive control to deal with kinematic constraints and model uncertainties. In [9], a prescribed performance bound technique was proposed to enhance the transient performance of path following and is also extended to formation control of mobile robots.

Some researchers focused on solving the above problems in a unified framework. For example, the authors in [11], [12] address both the posture stabilization and the tracking problems by using the same controller. In [13], the proposed neural network based control algorithm can be applied to all three problems.

In this present work, the path following problem of wheeled mobile robot is studied. Unlike the tracking problem for which the task is defined by a desired timed trajectory required to be tracked at every instant of time, the path following problem does not require the actual timing in the desired trajectory and emphasizes more on the coordination and synchronization between the different freedoms of the mobile robot [14]. Some applications of the wheeled mobile robots such as surveillance patrolling or pattern generation fall into this category. As pointed out in [14], the trajectory tracking error does not necessarily reflect how well a desired path is followed (See Fig. 1 for an illustration). Furthermore, a tracking controller for the path following problem leads to the so called radial reduction phenomenon, i.e., the actual path traced out has a smaller radius than the target path.

Recently, the guidance vector field (GVF) based methods have been shown effective [15], [16] for the path following problem of wheeled mobile robot. As a major advantage of this method, a GVF is appropriately designed to guide the system to approach the task path in a well behaved manner while guaranteeing provable stability and path following accuracy. The key is to encode such a GVF for the target path. For the case that the target path is a curve without self-intersections, the authors in [15] proposed a GVF encoding method where the target path in $\mathbb{R}^n$ is defined as the intersection of $n - 1$ surfaces with codimension [17] of one. This method utilizes the $n - 1$ surfaces to construct a potential function [18], the
In this paper, we consider a wheeled mobile robot with two degrees of freedom as shown in Fig. 2. The authors in [16] proposed a GVF encoding method and a nonlinear motion controller for the mobile robot such that the desired path is allowed to be an arbitrary smooth curve.

In this paper, the path following problem of wheeled mobile robot is also addressed by using a GVF. Unlike the above mentioned GVF encoding methods, we exploit an online optimization based GVF which is proposed in our previous work [19]. The online optimization procedure is adopted to estimate the path error such that the route traced out by the encoded GVF is close to the shortest route from a local position to the target path. Unlike [19] which assumes an ideal scenario where the optimization problem is solved immediately, this current work considers the running time of the online optimization. As a result, the optimization algorithm generates a sequence of closest points on the target path and they are pieced together in a smooth manner to ensure that the resulting GVF is at least $C^1$. Based on the proposed GVF, the path-following problem for wheeled mobile robots is studied in the presence of surface friction, unmodeled dynamics and unknown disturbances. To deal with these uncertainties, a sliding mode like controller is proposed to ensure a bounded path following result.

This paper is organized as follows. Section II presents the dynamic and kinematic model of the wheeled mobile robot with nonholonomic constraints and formulates the studied path following problem. Section III details the proposed online optimization based GVF encoding method. The path following controller for the mobile robot and a stability analysis procedure for the closed-loop system are given in Section IV. Section V provides the experiments for validation.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Nonholonomic Wheeled Mobile Robot Model

In this paper, we consider a wheeled mobile robot with two degrees of freedom as shown in Fig. 2. The authors in [16] proposed a GVF encoding method and a nonlinear motion controller for the mobile robot such that the desired path is allowed to be an arbitrary smooth curve.

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II. PRELIMINARIES AND PROBLEM FORMULATION

A. Nonholonomic Wheeled Mobile Robot Model

In this paper, we consider a wheeled mobile robot with two degrees of freedom as shown in Fig. 2. For the detailed definitions of the model parameters, we refer the reader to [13] and references therein.

The kinematic equation of the wheeled mobile robot is given as follows

$$\dot{q} = S(q)\nu(t)$$

where $S(q) \in \mathbb{R}^{3\times2}$ is a full rank Jacobian matrix given by

$$S(q) = \begin{bmatrix} \cos \theta & -d \sin \theta \\ \sin \theta & d \cos \theta \\ 0 & 1 \end{bmatrix}$$

and satisfies the following condition

$$S^T(q)A^T(q) = 0. \quad (5)$$

In order to ensure the controllability of the mobile robot, we require the following assumption (see e.g., [10]), which is satisfied if $\dot{q}$ is bounded.

Assumption 1: $F(\dot{q})$, $\tau_d(t, q, \dot{q})$ are bounded for all $q \in Q$ and all $t \geq 0$.

Similar to [13], we transform the system (1) into an alternative representation for control purpose. By differentiating (3) to obtain $\ddot{q}(t)$ and substituting it in (1), and then multiplying by $S^T$, the system (1) is rewritten as

$$\ddot{C}(q) + B_m(q, \dot{q})\nu + \tau_d = \ddot{r}$$

where $C(q) \triangleq S^TCS$, $B_m(q, \dot{q}) \triangleq S^T(C\dot{S} + B_mS)$, $F \triangleq S^TF$, $\tau_d \triangleq S^T(F + \tau_d)$, and $\ddot{r} \triangleq S^TE\tau$ is viewed as the new control input. Note that the constraint term $A^T(q)\lambda$ is eliminated by using (5). According to Assumption 1 and that all the elements in $S(q)$ is bounded for all $q \in Q$, the unknown disturbance $\ddot{r}_d$ is ensured to be bounded.
B. Path Following Problem

In this work, the task configuration of the path following problem is specified by a differentiable curve \( \gamma \), which is parameterized by a mapping \( \Pi : \mathcal{I} \rightarrow \mathbb{R}^n \) with \( \mathcal{I} \subset \mathbb{R} \). Therefore, the target path can be denoted as the following set

\[
\gamma \triangleq \{ x \in \mathbb{R}^n | x = \Pi(h), h \in \mathcal{I} \}. \tag{7}
\]

Here, it is assumed that \( \frac{d\Pi}{dh} \neq 0 \) and is bounded for all \( h \in \mathcal{I} \). Unlike [15], [20], [21], the target path \( \gamma \) is allowed to have self-intersections since it is defined in a parameterized manner. The control objective is to design a control input \( \bar{\tau} \) in (6) such that the mobile robot converges to the target path \( \gamma \) and moves along it.

III. OPTIMIZATION BASED GVF ENCODING

The GVF encoding method developed in our previous result [19] is modified in this work to facilitate the application to the wheeled mobile robot. Specifically, the GVF for the target path \( \gamma \) is given as follows:

\[
\dot{x} = f(x, t) = \frac{b}{\|x - \Pi(s(t))\| + \epsilon} \frac{d\Pi}{ds}(s(t)) - K(x - \Pi(s(t))) \tag{8}
\]

where \( \epsilon \) is a designable positive constant to prevent the denominator from approaching zero, \( b \) is a positive constant which decides the speed with which the target path will be followed, and \( K \in \mathbb{R}^{n \times n} \) is a positive definite matrix. In order to achieve a short-distance path following result, \( s(t) \in \mathbb{R} \) is updated as follows:

- if \( x(t) \notin \gamma \) for all \( t \geq t_0 \) with \( t_0 \) the initial time, then

\[
s(t) = \begin{cases} 
  s_0^*, & t \in [t_0, t_1); \\
  s_k^* - \frac{s_k^* - s_{k-1}^*}{\Delta t} \sin \left( \frac{(t - t_k^*) \pi}{\Delta t} - \frac{\pi}{2} \right) + 1, & t \in [t_k^*, t_{k+1}^* + \Delta t), \quad k \in \mathbb{N}_+; \\
  s_k^*, & t \in [t_k^* + \Delta t, t_{k+1}^*), \quad k \in \mathbb{N}_+.
\end{cases} \tag{9}
\]

where \( \Delta t \) is a time period which can be specified by the user, and

\[
s_k^* = \arg\min_{h \in \mathcal{I}} \| x(t_0) - \Pi(h) \|
\]

\[
s_{k-1}^* \leq h \leq s_{k-1}^* + \Delta s
\]

Here, \( \Delta s \) is designable positive constant, and \( t_k \) denotes the time when \( s_k^* \) is solved from (10). That is, the online optimization algorithm is conducted during \([t_{k-1} + \Delta t, t_k]\).

- if for some \( t' > t_{k-1} \) such that \( x(t') \in \gamma \), then

\[
s_k^* = \Pi^{-1}(x(t')), \quad t_k = \max\{t_{k-1} + \Delta t, t'\}. \tag{11}
\]

Recall that \( \Pi(h) \) is differentiable and \( \frac{d\Pi}{dh} \neq 0 \) for all \( h \in \mathcal{I} \), thus the left inverse mapping \( \Pi^{-1} : \mathbb{R}^n \rightarrow \mathcal{I} \) exists in a neighborhood of \( h \) such that \( \Pi(h) = x(t') \). In this case, (9) and (11) ensures that \( s(t_k + \Delta t) = s_k^* \). Then for \( t \geq t_k + \Delta \), \( s(t) \) is updated by

\[
\dot{s} = b/\epsilon. \tag{12}
\]

The above procedure ensures that \( s(t) \) is continuous. Note from (10) that we consider a local optimization problem here. Indeed, if without the constraint \( s_{k-1}^* \leq h \leq s_{k-1}^* + \Delta s \), the solution to (10) may not be unique especially when the target path has self-intersections or \( x(t_{k-1} + \Delta t) \) is far from the target path \( \gamma \). By choosing an appropriate \( \Delta s \) with respect to a specific target path, it is ensured that the solution to (10) always exists and is unique. This also ensures that \( s_k^* \) lies in a designed neighborhood of \( s_{k-1}^* \) and satisfies \( s_k^* \geq s_{k-1}^* \), and helps to generate a more smooth GVF according to (9).

The next result shows that \( x(t) \) whose dynamics is governed by (8) eventually converges to the target path \( \gamma \) and moves along it.

**Lemma 1:** For any initial condition \( x(t_0) \in \mathbb{R}^n \), the trajectory of (8) eventually converges to the target path \( \gamma \) and moves along it.

**Proof:** Note that the traditional Lyapunov method is not applicable in this case since an error signal between \( x(t) \) in (8) and the target path \( \gamma \) cannot be defined explicitly. Therefore, the matrix measure based contraction principle [22] is exploited for the subsequent analysis.

By (11) and (12), once \( x(t) \in \gamma \) for some \( t \), then \( s(t) \) in (8) will be updated as \( \dot{s} = b/\epsilon \). In this case, it is ensured that \( x(t) = \Pi(s(t)) \) is a solution of (8) since it reduces to

\[
\dot{x} = \frac{d\Pi}{ds}(s(t)) \dot{s}. \tag{13}
\]

For simplicity, denote this specific solution as \( x_\gamma(t) \).

From (9), the values of \( s(t) \) depend on \( x(t) \). Therefore, we rewrite \( s(t) \) as \( s(x) \) with a minor abuse of notation. Let \( x_\alpha(t) \) is a solution of (8), i.e.,

\[
\dot{x}_\alpha = \frac{b}{\|x_\alpha - \Pi(s(x_\alpha))\| + \epsilon} \frac{d\Pi}{ds}(s(x_\alpha)) - K(x_\alpha - \Pi(s(x_\alpha))). \tag{14}
\]

Then, we consider the following virtual system

\[
\dot{x} = f_a(x, s(x_a)) = \frac{b}{\|x_a - \Pi(s(x_a))\| + \epsilon} \frac{d\Pi}{ds}(s(x_a)) - K(x - \Pi(s(x_a))). \tag{15}
\]
Based on the above analysis, \( x_a(t) \) and \( x(t) \) are two solutions to (15). Note that \( f_a(x, s(x_a)) \) and \( \frac{\partial f_a}{\partial x} = -K \) are both continuous for all \( x \) and \( s(x_a) \). Additionally, \( s(x_a) \) is continuous. This satisfies the conditions of [22, Thm. 1]. Furthermore, the matrix measure of \( \frac{\partial f_a}{\partial x} \) with respect to \( L_2 \)-norm, denoted as \( \mu(\frac{\partial f_a}{\partial x}) \), satisfies

\[
\mu(\frac{\partial f_a}{\partial x}) = \mu(-K) = -\lambda_{\text{min}}(K)
\]

where \( \lambda_{\text{min}}(K) \) denotes the minimal eigenvalues of \( K \). Since \( K \) is positive definite, \( \lambda_{\text{min}}(K) > 0 \). By applying [22, Thm. 1], it arrives at

\[
\| x_a(t) - x(t) \| \leq e^{-\lambda_{\text{min}}(K)(t-t_0)}\| x_a(t_0) - x(t_0) \|
\]

which implies

\[
\lim_{t \to \infty} \| x_a(t) - x(t) \| = 0.
\]

That is, \( x_a(t) \) converges to \( x(t) \) eventually. Since \( x_a(t) \) is selected arbitrarily, it is ensured that every trajectory of (8) converges to and then moves along the target path \( \gamma \).

IV. CONTROL DESIGN AND STABILITY ANALYSIS

The desired velocity of the wheeled mobile robot, i.e., \( v_r(t) \) can be computed as follows

\[
v_r(t) = \eta(t) \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} f(x_w(t), t)
\]

where \( f(\cdot) \) is defined in (8), and

\[
x_w(t) = [x(t), y(t)]^T
\]

i.e., the current position of the wheeled mobile robot. The positive scalar \( \eta(t) \) is selected to ensure that \( v_r(t) \) is bounded for all \( t \in [t_0, \infty) \).

Remark 2: Similar to [23], [24], we can define

\[
\eta(t) = \frac{c_1}{\| f(x_w(t), t) \| + c_2}
\]

where \( c_1 \) and \( c_2 \) are designable positive constants. Here, (20) is known as the normalized term [23], [24]. By this design, the controlled object is expected to follow the path at a steady speed.

Based on the above analysis, an auxiliary velocity error is defined as

\[
e_v(t) = v(t) - v_r(t).
\]

From (6) and (21), the control input \( \bar{\tau} \) is designed as

\[
\bar{\tau} = \tilde{C}(q) \dot{v_r} + \tilde{B}_m(q, \dot{q}) v_r - k_1 e_v - k_2 \operatorname{Tanh}(k_3 e_v)
\]

where \( k_1, k_2, k_3 \) are positive constants, and \( \operatorname{Tanh}(k_3 e_v) = [\tanh(k_3 e_{v_1}), \tanh(k_3 e_{v_2})]^T \) with \( e_v \) denoting the \( i \)-th element of \( e_v(t) \).

Based on the Lemma 1, it is readily to claim that the introduction of \( \eta(t) \) in (19) still ensures that the trajectories of system (3) in closed-loop with \( \nu = v_r \) converge to and move along the target path \( \gamma \). This is because that the role of \( \eta(t) \) is related to a time parameterization.

Based on the above theoretical analysis, the following theorem proves that the path error can be made small enough by using the proposed control design in (22) with an appropriate control parameter setting.

**Theorem 1:** For the robot system defined in (3) and (6) with the control input (22), if Assumption 1 holds and \( k_2 \) is selected such that

\[
k_2 > \max(\| \bar{\tau}_{d1} \|_{\infty}, \| \bar{\tau}_{d2} \|_{\infty})
\]

then the target path \( \gamma \) is followed with an assignable error bound by selecting a large enough \( k_1 \).

**Proof:** From (6) and (22), the closed-loop dynamics of \( e_v \) is obtained as

\[
\dot{\bar{C}}(q) e_v = -\tilde{B}_m(q, \dot{q}) e_v - k_1 e_v - k_2 \operatorname{Tanh}(k_3 e_v) - \bar{\tau}_d.
\]

Then, consider the following nonnegative function

\[
V_1 = \frac{1}{2} e_v^T \tilde{C}(q) e_v.
\]

Note that \( \tilde{C}(q) \) is positive definite from its definition. By taking the derivative of \( V_1 \) along the trajectory of (24), we arrive at

\[
\dot{V}_1 = -k_1 \| e_v \|^2 - k_2 e_v^T \operatorname{Sgn}(e_v) - e_v^T \bar{\tau}_d
\]

According to [13], \( \tilde{C}(q) - 2\tilde{B}_m(q, \dot{q}) \) is skew symmetric. Thus, the first term of the right-hand side in (26) is zero. As a result, we have

\[
\dot{V}_1 = -k_1 \| e_v \|^2 - k_2 \| e_v \| \| \bar{\tau}_d \| + k_2 \| e_v \| \| \operatorname{Sgn}(e_v) - \operatorname{Tanh}(k_3 e_v) \| + k_2 \| e_v \| \| \bar{\tau}_d \|
\]

where \( \operatorname{Sgn}(e_v) = [\operatorname{sgn}(e_{v_1}), \operatorname{sgn}(e_{v_2})]^T \), and the condition (23) is utilized to obtain the second inequality. From (27), \( \dot{V}_1 \) is negative outside the set \( \{ e_v \in \mathbb{R}^2 \mid \| e_v \| \leq \sqrt{\frac{\lambda k_2}{k_1}} \} \). Additionally, \( \dot{V}_1 \leq -\sum_{i=1}^{2} \| k_2 - |\bar{\tau}_{di}| \| \| e_v \| + \sqrt{2} k_2 \| e_v \| \| \bar{\tau}_d \|
\]

Therefore, it is concluded that \( e_v(t) \) is uniformly ultimately bounded with the ultimate bound

\[
\| e_v \| \leq \sqrt{\frac{2 \lambda k_2}{k_1}}
\]

where \( \lambda, \lambda \) satisfy \( \lambda \| e_v \| \leq V_1 \leq \lambda \| e_v \| \). It is worth pointing out that the bound in (28) can be made arbitrarily small by selecting a large enough \( k_1 \). According to a Lyapunov-like theorem [25, Thm. 4.18], for a small \( \varepsilon > 0 \), there exists \( T_\varepsilon > 0 \) such that

\[
\| e_v(t) \| \leq r_m \leq \sqrt{\frac{2 \lambda k_2}{k_1}} + \varepsilon, \quad \forall t \geq t_0 + T_\varepsilon.
\]

From (8) and (19), \( v_r \in L_\infty \). Therefore, it is concluded that \( v(t) \in L_\infty \), this implies that \( q(t) \) is uniformly continuous in \( t \).
according to (3) and noting that \( S(q) \) is also bounded. Thus, \( q(t) \) with a bounded initial condition \( q(t_0) \) is ensured to be bounded for a finite time.

To facilitate the subsequent analysis, we introduce a virtual error signal \( e_\gamma(q(t)) \) ∈ \( \mathbb{R}^3 \) such that if the mobile robot configuration \( q(t) \) is on the target path \( \gamma \) and moves along it at a desired speed, then \( e_\gamma(q) = 0 \). From Lemma 1, there exists a class \( K \) function \( \beta \) such that the trajectories of the closed-loop system

\[
\dot{q} = S(q)\nu_r(t)
\]

satisfy

\[
\|e_\gamma(q(t))\| \leq \beta(\|e_\gamma(q(t_0))\|, t - t_0), \ \forall t \geq t_0 \geq 0.
\]

Then, according to the converse Lyapunov function theorem [25, Thm. 4.16], Eq. (31) ensures that there is a continuously differentiable function \( V_2 : [t_0, \infty) \times \mathbb{R}^3 \rightarrow \mathbb{R}_{\geq 0} \) such that the following conditions hold

\[
\alpha_1(\|e_\gamma(q)\|) \leq V_2(t, q) \leq \alpha_2(\|e_\gamma(q)\|)
\]

\[
\frac{\partial V_2}{\partial t} + \frac{\partial V_2}{\partial q}S(q)\nu_r(t) \leq -\alpha_3(\|e_\gamma(q)\|)
\]

\[
\left\| \frac{\partial V_2}{\partial q} \right\| \leq \alpha_4(\|e_\gamma(q)\|)
\]

where \( \alpha_i, i = 1, \cdots, 4 \), are class \( K \) functions. Then, by taking the derivative of \( V_2(t, q) \) along the trajectories of (3) with \( \nu(t) \) generated by the closed-loop system (24), we have

\[
\dot{V}_2 = \frac{\partial V_2}{\partial t} + \frac{\partial V_2}{\partial q}S(q)\nu_r(t)
\]

\[
= \frac{\partial V_2}{\partial t} + \frac{\partial V_2}{\partial q}S(q)(\nu_r + \nu_e)
\]

\[
\leq -\alpha_3(\|e_\gamma(q)\|) + \alpha_4(\|e_\gamma(q)\|)\|S(q)\|\|\nu_e\|.
\]

where (32) is used for the inequality. Based on the previous discussions, it is guaranteed that \( e_\gamma(q(t)) \) is bounded for \( t \in [t_0, t_0 + T_e] \) since \( q(t) \) is bounded during this period. For \( t \geq t_0 + T_e \), (29) and (33) implies that

\[
\dot{V}_2 \leq -\alpha_3(\|e_\gamma(q)\|) + r_m\alpha_4(\|e_\gamma(q)\|)\|S(q)\|.
\]

Since \( \varepsilon \) is a virtual variable introduced for analysis, \( r_m \) can be made arbitrary small by selecting a large enough \( k_1 \) and picking a small \( \varepsilon \). Furthermore, according to the property of the class \( K \) function, there exists a small enough \( \tilde{r}_m > 0 \) such that

\[
\dot{V}_2 \leq -\alpha_3(\|e_\gamma(q)\|) + \tilde{r}_m\alpha_4(\|e_\gamma(q)\|)\|S(q)\| < 0,
\]

\( \forall e_\gamma(q) \geq \mu > 0 \)

where \( \mu \) is dependent on \( \tilde{r}_m > 0 \). Therefore, by making \( r_m \leq \tilde{r}_m \), it is ensured that \( e_\gamma(q) \) is uniformly ultimately bounded with the ultimate bound \( \|e_\gamma(q)\| \leq \alpha_1^{-1}(\alpha_2(\mu)) \).

V. EXPERIMENTAL VALIDATIONS

To illustrate the effectiveness of the proposed path following controller, the experiments are conducted on a Summit-XL four-wheeled mobile robot as shown in Fig. 3. The location information of the robot is obtained by using an Unicorecomm-UB482 satellite locator with accuracy less than 0.05 m. The model parameters in Fig. 2 are as \( R = 0.228 \) m, \( r = 0.085 \) m, \( d = 0.1 \) m. The mass of the mobile robot including the locator is 71 kg. If the value of \( s_{\gamma}^* \) in (10) is not unique for certain initial locations \( x(t_0) \), an embedded program just selects one of the values considering the path direction and the initial heading direction of the mobile robot.

Note that the proposed controller in (22) defines the torques applied to the robot. However, only the linear and angular velocities of the experimental robot can be directly controlled. Therefore, based on (6) and (22), the following approximation of the desired controller is utilized

\[
\nu(t) = \nu_r(t) - \int_{t_0}^{t} C^{-1}(q(\tau))\left[ B_m(q, \dot{q})e_r(\tau) + k_1e_r(\tau) + k_2\tanh(k_3e_r(\tau)) \right] d\tau.
\]

The control gains are selected as \( k_1 = 1, k_2 = 4, \) and \( k_3 = 2 \) based on trials and errors.

The experimental results are presented in Fig. 4-5. It is shown that the controlled mobile robot can follow the target path with an admissible error. Furthermore, Fig. 4 depicts that a short-distance path following result can be achieved especially when the initial location of the robot is relatively far away from the target path.

VI. CONCLUSION

The path following problem for a wheeled mobile robot with nonholonomic constraints is investigated in this paper. A guidance vector field (GVF) based controller is proposed such that the mobile robot can follow the desired path with an admissible error in the presence of uncertainties including surface friction, unmodeled dynamics, and disturbances. In order to achieve a short-distance path following result, an online optimization procedure is adopted to estimate the path.

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Fig. 3: Experimental platform: a Summit-XL four wheeled mobile robot and an Unicorecomm-UB482 satellite locator.
Fig. 4: Path following results for an 8-shaped path (red dashed line) with different initial conditions (red star).

Fig. 5: Sequence of snapshots from the experiment for the desired 8-shaped path.

to the target path eventually. Then, a sliding mode like control curves of the encoded GVF with online optimization converge based method. For this reason, the matrix measure based convergence properties compared to the conventional GVF error, and this leads to a new theoretical problem regarding the desired 8-shaped path.

Then, a sliding mode like control method is designed to track the encoded GVF.

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