Multiple Working Mode Control of Mobile Serial Robot with Online Mode Switching for Operations on Mechanisms with Unmodelled Constraints

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Abstract—In this paper, a multiple working mode approach is proposed to control a mobile serial robot manipulator to perform operations on mechanisms with unmodelled constraints. A constrained robot manipulator is an over-actuated system in which kinematic modelling errors lead to high internal and external reaction forces. With the proposed approach, problems caused by modelling inaccuracies are avoided by reducing the number of position-controlled joints during task execution. As a result, the manipulator becomes a minimally actuated system, where the uncontrolled generalized variables are task-actuated (passive), and the remaining (active) variables are assigned to a priority-based task hierarchy. Three practical criteria are derived for the assignment of active and passive control modes; two based on the coordinate partitioning method for multidynami, and another based on minimizing task friction forces. The proposed control method is implemented on a mobile reconfigurable robot, performing constrained motion along an elliptical trajectory, similar to door-opening task. The experimental results have demonstrated the effectiveness of the proposed control approach.

Keywords—Constrained robot control, multiple working mode, holonomic constraints, projective criterion, Lagrange multiplier

I. INTRODUCTION

The problem of constrained interaction with environment is one of resolving actuation redundancy that arises due to the reduction of independent generalized coordinates in constrained motion [1]. Known approaches to this problem can be classified into two non-exclusive categories; the first is control along the path of least resistance, while the second is the modification of system dynamics by incorporation of constraint equations. The former is well suited to a variety of tasks in unstructured environments, albeit the constraints that can be handled robustly are simple; point contact, Cartesian, with limited coupling between the task degrees of freedom [2]. Provided that a model of the constraint is available, the latter approach can handle holonomic and non-holonomic constraints (parallel and mobile manipulators) and can incorporate the constrained system dynamics for better tracking performance [3][4]. Constrained mechanism manipulation in unstructured environments is governed by unknown holonomic constraints, and as such, control along the path of least resistance appears to be the more common approach in the literature.

Without an algebraic model of a constraint, one can specify a desired speed for the end effector and modify the direction vector to lie within the allowable motion manifold [5]. However, for mobile robots, estimation of constraint kinematics is further obfuscated by the uncertainty in self-localization of the mobile base. To reduce unwanted interaction forces, approaches that are based on more dynamically complete formulations such as impedance, admittance, hybrid control schemes, robust and adaptive impedance control, and hybrid impedance control were proposed by numerous researchers [6]-[10]. Furthermore, environment friction compensation was proposed in a hybrid control scheme to reduce tracking error [11].

In this work, we extend the multiple working mode approach, reported originally in [12], to control of modular, and kinematically redundant reconfigurable robots subjected to holonomic and non-holonomic constraints. During task execution, joints that are found to contribute the least to the current task are commanded by supervisory control to switch the joint controller to passive working mode, in which the joint with friction compensation appears to be unactuated when reacting to constraint-generated forces.

The contributions of this work include the development of criteria for online switching of joint working modes. For the assignment of active and passive control modes, two practical criteria based on the coordinate partitioning method for multibody dynamics, and one other criterion based on minimizing task friction forces are derived. Significantly, with the proposed method, accurate knowledge of the constraint geometry is not required to perform motion control along the tangent manifold of the constraint. Problems caused by modelling inaccuracies are avoided by reducing the number of position-controlled joints during task execution. The constraint modelling errors are accommodated by joints working in passive mode, thereby bounding the constraint reaction forces.

The development of the proposed working modes, as well as the specific steps in modelling and criteria evaluation are presented as follows: Section II formulates the problem; we describe the manipulator and joint motor dynamics models. Subsequently, Section III presents the controllers that achieve active and passive working modes, as well as their unification in a single formulation. In this section, the conditions on controller performance are presented and joint working mode selection criteria are developed. Section IV introduces the hardware used in experimental validation of the Multiple Working Mode approach and presents the results of the mobile manipulator opening a door. Lastly, Section V draws conclusions regarding the feasibility and applicability of the proposed approach to various interaction tasks based on the experimental results and general insight into the problem.

II. PROBLEM FORMULATION AND MODELLING

The dynamics of a robot manipulator are usually expressed as a system of second order ODEs, which follow from the Lagrange formulation [1]. This system can be represented by

\[ \dot{M}(q) \ddot{q} + h(q, \dot{q}) + \tau_r = \tau_c + \tau_f, \]

where \( M \in \mathbb{R}^{n \times n} \) is the manipulator mass matrix, \( h \in \mathbb{R}^n \) is the vector of the non-linear inertial and gravity forces, \( \tau_f \in \mathbb{R}^n \) is the vector of joint friction forces, \( \tau_c \in \mathbb{R}^n \), is the vector of mechanism operating
forces and $\mathbf{r}_c \in \mathbb{R}^n$ is the vector of generalized actuator forces.

With the model considered here, as illustrated in Fig. 1, by replacing the generalized actuator force with the torque at the joint torque sensor location, $\mathbf{r}_s$, the manipulator dynamics (1) can be partitioned into link and motor components as

$$
M_i(q)\ddot{q} + h_i(q, \dot{q}) - \tau_c = \mathbf{r}_s,
$$

$$
M_m(q)\ddot{q} + h_m(q, \dot{q}) + \tau_f + \mathbf{r}_s = \Gamma_m
$$

where subscriptsing with $\mathbf{m}$ and $\mathbf{j}$ denotes motor and link-side dynamics terms, respectively, $\Gamma = \text{diag}[\gamma_1 \ldots \gamma_n]$ is the diagonal matrix of motor gear ratios, and $\tau_m \in \mathbb{R}^n$ is the motor torque. As per [13], the dynamics (3) for motor $i \geq 3$ are

$$
I_{mi}\dot{\omega} + \sum_{j=1}^{i-1} J_m^m \dot{\omega} j + \sum_{j=1}^{i-1} \sum_{k=1}^{i-1} J_m^m (\tau_k \times \omega_j) + y_j \tau_{fi} + \tau_{ri} = y_i \tau_{mi}
$$

For joint $i$, $I_{mi}$ is the joint motor inertia, $z_{mi}$ and $z_i$ are the rotor axis and joint axis direction vectors, respectively. For $i = 2$, the Coriolis forces summation term is excluded, and for $i = 1$, both summation terms for non-linear inertial forces are excluded.

$$
\text{Fig. 1. Schematic diagram for the motor dynamics formulation}
$$

The friction term, $\tau_f$, is assumed to be a function of the joint velocity and is modeled by [14][15]

$$
\tau_f = \left[ f_{ki} + f_{ai} \exp(-f_{ai}q_i^2) \right] \text{sgn}(q_i) + b_i q_i
$$

where $f_{ki}, f_{ai}, b_i$ represent the Coulomb, Static, Strieber and viscous components, respectively.

Defining a projection operator, $P \in \mathbb{R}^{n \times n} \mid \text{ker}(C) = \text{image}(P) = P^T, P = P^2 \Rightarrow (P^T \times -C^T C)^\perp [16]$, where $P^T \times$ is the identity matrix and $(\cdot)^\perp$ denotes the Moore–Penrose pseudoinverse, allows one to partition the dynamics of $(1)$ onto two orthogonal subspaces, spanned by $\mathbf{r}_s$ and $\mathbf{r}_c$, respectively,

$$
\tau_c = P \tau_c + (I - P) \tau_c = \tau_1 + \tau_2.
$$

The orthogonal component, $\tau_1$, lies in the image of the constraint Jacobian, $T_1 = \text{image}(C^\perp)$. However, the tangent component, $\tau_2$, is the component of actuator torque which lies in $T_1$, tangent to the constraint; not the cotangent space, $T_1^*$.

Furthermore, we define a mapping, $\mathbf{D} : \mathbb{R}^{n \times m} \rightarrow \mathbb{R}^n$ such that $q = \mathbf{D} q_i$, where $q_i \in \mathbb{R}^{n \times m}$ is an independent set of generalized coordinates. This mapping allows the manipulator model in (1) to be expressed via an independent set of equations as

$$
D^T M D q_i + D^T (h - \tau_c + \tau_f - \xi) = D^T \tau_c
$$

where the non-linear term, $\xi = M D q_i$, is due to differentiation of the mapping operator. Significantly, $D^T M D$ is the metric tensor of the reduced tangent space, $T_1^*$, which will later utilize in establishing a criterion for active joint selection.

III. MULTIPLE WORKING MODE APPROACH

A. Controller Design

The system to be controlled is represented by the combination of (2) and (3). Using estimates of the motor-side dynamics model parameters, denoted by $(\cdot)^\hat{}$, and the joint torque at the sensor location $\hat{\tau}_s$, we can define a control law for (14) as

$$
\Gamma_m = \tau_m^a = \tau_f + \hat{\mathbf{r}}_m + \mathbf{u}_c
$$

$$
u_c = M_m \ddot{q}_d + \mathbf{K}_d e_q + \mathbf{K}_p e_q + \mathbf{u}_y
$$

where $\mathbf{r}_f$ and $\hat{\mathbf{r}}_m$ are estimated vectors of joint friction forces and nonlinear motor-side inertial and gravity forces, respectively. $\ddot{q}_d$ is the desired acceleration of joint position, $\mathbf{K}_d$ and $\mathbf{K}_p$ are positive definite control gain matrices, and $e_q = q - \dot{q}_d$ is the joint position error. The control input, $\mathbf{u}_y$, such as the parametric robust compensator developed by [13], compensates for uncertainties in friction and manipulator configuration. The controller $\mathbf{u}_y$ is designed using the saturation-based robust control method with a tunable positive control parameter that is used to compensate for uncertainties that cannot be estimated in real time. However, it is omitted here for constrained motion tasks as the constraint modeling errors are of greater consequence in position-controlled manipulators. The dynamics resulting from (2,3) and (8,9) can be shown to reduce to the error dynamics

$$
\ddot{e} + M_m^{-1} \mathbf{K}_d \ddot{e} + M_m^{-1} \mathbf{K}_p e = M_m^{-1} (\ddot{\tau}_f + \ddot{\mathbf{r}}_m - \mathbf{u}_y)
$$

The controller defined by (8) is active. Notice that the uncompensated components contained in (20) are not bounded since they contain the constraint reaction, $\mathbf{C}^\perp \lambda [1]$. The system can be made stable with a compliant controller, or if the reaction force in (20) can be modelled by an environmental impedance. With the goal of ensuring that the reaction forces are bounded, we define a joint-level compliant controller as

$$
\Gamma_m = \tau_m^a = \tau_f + \hat{\mathbf{r}}_m
$$

Temporarily, we can model the measured motor-link coupling torque, $\tau_m$, as the torque transmitted across a joint with constant stiffness $K_s \gg K_p$. Thus, one can relate the joint and link generalized coordinates $\mathbf{q}_m$ and $\mathbf{q}_i$, as well as the position error, $\mathbf{q}_i - \mathbf{q}_m = e_s$ as follows.

$$
\tau_s = K_s (\mathbf{q}_i - \mathbf{q}_m) = K_s e_s
$$

Using (13) in (12), with $\dot{\tau}_f (\hat{e}_c) = \tau_f - \tau_f$, joint dynamics are

$$
\dot{e}_c + M_m^{-1} \mathbf{K}_d \dot{e}_c + M_m^{-1} \mathbf{K}_p e_c = 0
$$

Clearly $e_c \rightarrow 0$ as $t \rightarrow \infty$ (hence $\tau_y \rightarrow 0$), with a time constant that is much smaller than the time constant in (10), owing to $K_s \gg K_p$ (this condition is nearly ubiquitous due to avoidance of excitation of high-order flexible body dynamics). Also, by defining a Lyapunov function candidate such as in [17] for the active control law defined in eq. 18, it can be proved that the tracking error of each joint is ultimately uniformly bounded. We can now unify the active and passive control modes by employing a selection matrix, $S \in \mathbb{R}^{n \times n}$:

$$
S = \text{diag}(s_i)
$$

where such columns $k$ in $S$ is a unit vector in $\mathbb{R}^n$ along the $k$-th joint axis for a joint in active working mode. The selection $s_i$ is defined by the $i$-th unique permutation of the selection vector as

$$
s_i = \mathbf{P}_i \left[ (1^{n\times m}, 0^{n\times n})^T \right]
$$

The overall control input is therefore

$$
\Gamma_m = S \tau_m^a + (I - S) \tau_m^a
$$

One can rewrite (8) and (11) as a unified control law as

$$
\Gamma_m = S \dot{\tau}_m + (I - S) \dot{\tau}_m + \hat{\mathbf{r}}_m + S M_s \ddot{\mathbf{q}}_m + K_d \dot{e}_q + K_p e_q + S u_y
$$

Combining (18) and (3), and further substituting the result into (2),
the dynamics in (1) reduce to
\[
S(\dot{e} + M_m^{-1}K_p\dot{e} + M_m^{-1}K_e e) + 
\]
\[
(I - S)(\dot{e} + M_m^{-1}K_t(\dot{e}) + M_m^{-1}K_e e) = 0
\]
(19)
This result is due to the assumption of negligible error in parameter estimates such that \(M_m^{1/2}(\dot{e}_p + \dot{\hat{e}}_m - u_e) = 0\). Nevertheless, the combination of active and passive controllers remains asymptotically stable under the control error specification for (10) and (14). Controllability of the end effector in task space is not assured, unless the artificial constraints imposed by the joint-level position controller are complemented by natural constraints of the environment [18].

B. Active Joint Selection Criteria

By choosing to set to zero the controller error terms per the passive joints’ selection vector, \(s_p = (I - s)\), we have implicitly reduced the number of independently controlled degrees of freedom from \(n\) to \(n - m\), potentially resolving the quandary of overactuation. One can evaluate the suitability of a specific selection based on pertinent control objectives, such as
\[
(s_i)_{opt} = \arg\min_s (\lambda, t_i)\Sigma, \quad \lambda_i = \frac{n}{\sum\lambda_i}
\]
(20)
which minimizes the metric-weighted norm of the control input from a set of all permutations of \(s_i\). Considering the result from (14), \((e_o \to 0)\), and noticing that joint position error, \(e\), is actively controlled, while \(e_o\) is a consequence of the passive dynamics, we can observe that our unified controller enforces
\[
\tau_c = St_c
\]
(21)
Recalling the orthogonal partitioning of dynamics in (6), it can be shown that \(\min(\hat{\tau}_c)\) occurs when \(PS = S\), i.e., the generalized force vector lies tangent to the constraint. In general, \(\hat{\tau}_c \geq \tau_c^P\).

For emphasis, the dependence of the control torque, \(\tau_c\), on the particular selection of active joints is indicated explicitly. A non-trivial solution exists when the projection of the constraint null space onto the passive coordinates is contained within the projection of the null space orthogonal onto the passive coordinates:
\[
\text{img}(I - S)P \subseteq \text{img}(I - S)(I - P)
\]
(23)
Intuitively, this condition requires that generalized forces on passive joints that arise from force/motion along the unconstrained task directions can be negated by constraint forces, thereby ensuring static equilibrium of the passive joint variables. This condition is satisfied if \(\text{rank}(PS) = \text{rank}(P) = n - m\). Furthermore, if \(\text{rank}(J) = p\), there is always a vector \(s_i\) that will result in a selection matrix \(S\) such that this condition is satisfied; \(\text{img}(P)\) is a subspace of \(\text{img}(J^T)\), and columns formed by \(JS_i\), \(\forall i\), span \(\text{img}(J^T)\).

In order to obtain the optimal selection according to (20), consider the partitioning of (10) along with (21) and (22),
\[
\tau_c = \tau_t + \tau_l = (I - P)\Sigma\tau_c = (I - H)PS\tau_c
\]
(24)
where \((I - H)\) is the term in square brackets of (22). To minimize the orthogonal component, \(\tau_l\), which does not contribute to motion along the constraint, it is necessary to diminish the scaling of the control torque due to the mapping operator, \(PS\); this corresponds to the eigenvalue problem,
\[
(PS)x_i = \lambda_i x_i,
\]
(25)
in which we desire to maximize the eigenvalues, \(\lambda_i\) corresponding to the eigenvectors \(x_i\). The eigenvectors of \(P, S, \text{and } PS\) corresponding to non-zero eigenvalues are orthonormal, and the eigenvalues \(\text{eig}(PS) \leq 1\). Since the sum of eigenvalues of a matrix is equal to the trace of that matrix,
\[
\text{tr}(PS) = \sum_i \lambda_i
\]
(26)
the joint indices of the active joints are then obtained by selecting the indices of top \(n - m\) elements of the \(P\) diagonal as
\[
a = (a_i|(P)_{ii} > 0) \cup (P > 0), \quad i \in \{1 \cdots n - m\}, j \in \{2 \cdots n\}
\]
(27)
This selection is akin to a greedy search algorithm to maximize the sum of the eigenvalues. For any other selection, represented here as an arbitrary permutation, \(P\), of the selection set, the sum of eigenvalues of the resulting projection \(PS\) is smaller:
\[
S' = \text{diag}(\text{P}_{S})
\]
(28)
\[
\sum_i \text{eig}(PS') < \sum_i \text{eig}(PS)
\]
(29)
However, such a selection is not optimal as it may result in an uneven distribution of eigenvalues such that a task along some directions of the constraint null space will require unrealizable scaling of the control input. We propose a task-priority based selection of the active joints that attempts to allocate the active joints to the highest priority tasks. For a total of \(l\) independent tasks, we can define the pair
\[
(J_k \in \mathbb{R}^{lxn}, x_k \in \mathbb{R}^p) \mid \sum_k p_k = t
\]
(30)
The corresponding minimum norm solution that satisfies the task and constraint is therefore
\[
q_k = w_k P J_k^T x_k
\]
(31)
where \(w_k \in \mathbb{R}^l\) is a task scaling parameter between 0 and 1 representing task priority. The combination of \(l\) tasks thus spans a space given by
\[
D = (q_1 \cdots q_l) \in \mathbb{R}^{lxn}, \quad \text{rank}(T) \leq \text{rank}(P), \quad T = D(D^T D)^{-1}WD^T
\]
(32)
where \(W \in \mathbb{R}^{lxl}\), containing the \(w_k\) scaling parameters on the diagonal, inserts eigenvalues into the resulting projector \(T\). We can re-evaluate the selection of active joints according to (27), replacing \(P\) with \(T\). Since low priority tasks contribute a smaller eigenvalue to the total in (26), active joints thus select smaller terms from the diagonal of \(T\). However, since the \(l\) task directions in (32) are not orthogonal, the priority assignment does not guarantee a hierarchy, and only serves to improve the selection.

Another approach stems from the realization that maximization of the product of eigenvalues should result in a more even distribution compared to (27). Since the product of eigenvalues of a matrix is equal to the determinant of that matrix, we are seeking the submatrix of \(P, P(s)\), whose columns and rows are according to the selection indices in \(s\), such that \(\det(P(s)) \geq P(P(s))\). Efficient selection can be performed recursively by selecting an active joint according to the maximum diagonal element, removing its projection from the constraint null space projector \(P\), resizing the operator by excising the zero row and columns, and repeating the process:
\[
a_k = \arg\max_i |P_{kk}|, \quad \forall i \in \{1 \cdots n\}, k \in \{1 \cdots n - m\}
\]
(34)
Another pertinent interpretation of this criteria is through the definition for manipulability measure, and the desire to maximize manipulability of the set of active joints with respect to the constraint tangent space. Here, this manipulability measure, \( w \), can be expressed as the product of eigenvalues of \( PS \), which (34) attempts to maximize:

\[
 w = \det(D^T S) = \left| \prod_{i=1}^{\n} \lambda_i \right| \tag{35}
\]

**C. Consideration of Unstructured Environment and Friction**

In general, the projection operator, \( P \), is not readily obtained when operating in unstructured environments; however, when the end effector of the manipulator is in motion, the vector of generalized velocities necessarily lies on the constraint motion manifold, \( \dot{q} \in \ker(C) \). We can express the generalized velocity vector as a linear combination of eigenvectors of the nullspace projection operator:

\[
 \dot{q} = v_1\dot{v}_1 + v_2\dot{v}_2 + v_{n-m}\dot{v}_{n-m} \tag{36}
\]

where \( v_i \) are scalars, and \( \dot{v}_i \) are the eigenvectors of \( P \). Furthermore, if a continuous trajectory can always be parametrized by a scalar, we can momentarily consider the motion to be constrained to a 1-DOF submanifold \( \dot{q}_i \in \mathbb{R} \). Thus, a local mapping \( D: \mathbb{R}^n \rightarrow \mathbb{R} = \dot{q}/\dot{q}_i \), which is readily available from measurement of joint velocities, can be used to rewrite (36) as:

\[
 \dot{q} = \dot{q}_i D \tag{37}
\]

Furthermore, the projection operator can now be obtained as:

\[
 P = vv^T = D^T D = D(D^T D)^{-1} D^T \tag{38}
\]

and we can rewrite (25) as:

\[
 v_i^T S x = D^T D \dot{x} \tag{39}
\]

The eigenvalues of \( PS, \lambda \), are maximized when the projection of active joint directions in actuator space onto the tangent space, \( T_i \), is maximized. It is therefore possible to replace \( a_k \) of (34) with:

\[
 a_i = \text{argmax}(\|D_i\|) = \text{argmax}(\|q_i\|), \tag{40}
\]

Since we require \( n - m \) active joints to circumvent the issue of overactuation, the mapping, \( D \), is parametrized on some knowledge of the task for \( n - m \) task directions that are not expected to violate the constraint definition such as:

\[
 \dot{D} = (q_1, \ldots q_{n-m}) = \dot{P} (x_1, \ldots x_{n-m}) \tag{41}
\]

As with the task specification in (31), these knowledge of the constraint is required; here \( P \) is an estimate of the real constraint null space projection, requiring only that \( \text{rank}(P\dot{P}) = \text{rank}(P) \). Assembling a single mapping operator \( \dot{D}: \mathbb{R}^{n-m} \rightarrow \mathbb{R}^n \mid \dot{q} = \dot{D} q_i, q_i \in \mathbb{R}^{n-m}, \dot{D} = k_i^0, \ldots, k_{n-m}^0 \) allows one to recursively obtain the active joint index for \( n - m \) joints as:

\[
 a_k = \text{argmax} \left( \left( k_i^0 \right)^T k_i^0 \right), \forall i \in \{1 \ldots n\}, k \in \{1 \ldots n-m\} \tag{42}
\]

The error in the mapping \( \dot{D} \) does not preclude one from using the criterion in (42), provided that the estimates of the task directions project onto \( T_i \), i.e., \( \text{rank}(P\dot{D}) = \text{rank}(P) \). Notice that the mapping, \( D \), complements the constraint Jacobian, \( C \), such that \( CD = 0 \). In fact, \( D: T_i \rightarrow T_i \). Also, from the definition of \( C \), columns of \( D \) must be the contravariant vectors in tangent space \( T_i \). However, from (7), the columns of \( D^T, k_i^0 \in \mathbb{R}^{n-m}, \) are the covariant vectors in the reduced representational space, \( D^T \). Therefore, (42) is invariant only when its argument is as:

\[
 (k_1^0, k_{n-m}^0)^T = (k_i^0)^T M_i^0 k_i^0, \tag{43}
\]

where \( M_i^0 = (D^T M D)^{-1} \) is the tensor metric of the reduced representation space. Similarly, the columns of the selection matrix, \( S \), are the covariant vectors, \( k_i^0 \in \mathbb{R}^n \), with the natural space metric, \( M^0 = M^{-1} \), and their magnitude is as:

\[
 (k_i^0)^T M_i^0 k_i^0 = (k_i^0)^T M_i^0 k_i^0 \tag{44}
\]

The inner product, \( (D, S) \), thus forms the geometric basis for another criterion. Observe that \( k_i^0 \) is the projection of \( k_i^0 \) onto \( D_i \):

\[
 (k_i^0, k_i^0) S_i = \left( k_i^0, k_i^0 \right) M_i^0(k_i^0, k_i^0)^T \cos^2 a_i \tag{45}
\]

where \( \alpha \) has the interpretation of being the angle in Riemann space between vectors \( k_i^0 \) and \( k_i^0 \) in the tangent space \( T_i \). Using (45) and (42), we obtain the metric consistent criterion as:

\[
 a_k = \text{argmax}(\cos^2(a_i)) \tag{46}
\]

This is equivalent to the coordinate reduction criteria in [19]. As previously, \( a_k \) is used to establish the active joint index, and the recursive approach of (42) is used to create the selection matrix \( S \).

**D. Extension to Kinematically Redundant Manipulators**

An interesting case arises when \( n > p \), i.e., a redundant manipulator such as a mobile platform with an arm, which allows one to control the robot configuration without affecting the task. Is according to the Pfaffian form of the mechanism constraint Jacobian, our constraint model for an end effector interaction task can have at most \( p \) degrees of freedom, which implies that the task specification requires \( k = p \) independent components. Since the manipulator in a non-singular configuration with \( n \) joints has \( n - m \) degrees of freedom for motion specification, there exists a \( N = n - p \) nullspace motion manifold with respect to the task. We refer to this manifold as the reconfiguration space of the manipulator, \( T_N \) \( \in \mathbb{R}^{N} \):

\[
 T_N = \{ q \in \mathbb{R} | \{J(q)q = 0, \forall q \in T_q \} \} = \ker(J) \tag{48}
\]

Curiously, this reconfiguration space is in fact a subspace of the constraint tangent space, \( T_i \). A partitioning of the generalized variables into \( \dot{q} = \dot{q}_i + \dot{q}_s \) is possible, where the reconfiguration component \( \dot{q}_s \in T_N \) such that a secondary control task uses a subset \( T_s \in \mathbb{R}^{N} \subset T \). To minimize an objective function \( H(q) \). As per (21), the control torque must satisfy:

\[
 \tau_c = \tau_s \tau_c + \tau_N \tau_c \tag{49}
\]

where \( S_l \) and \( S_N \) are selection matrices that result from partitioning of the set \( a \) from (34) into two subsets with \( t \) and \( N \) elements, respectively. Recalling the partitioning from (24), we can also rewrite the control torque to include the reconfiguration input as:

\[
 \tau_c = \tau_s \tau_c + \tau_N \tau_c \tag{49}
\]
\[τ_c = τ_1 + τ_2 + τ_3 \quad (50)\]
where \(τ_0 = P(J^+J)τ_c\) and \(τ_2 = P(I - J^+J)τ_c\). \(τ_1 = (I - P)τ_c\) [13].

The three selection criteria can be repurposed: using \(P_1 = P(J^+J)\) projection operator, \(t\) joints are assigned to active working mode to perform motion of the end effector, and using \(P_n = P(I - J^+J) = (I - J^+J)\) projection operator, \(N\) joints are assigned to perform self-motion to reconfigure the manipulator and improve some pertinent objective function \(H(q)\). The active joint selection criteria defined by (42) and (46), are unaffected by this partitioning. However, we consider another criterion which directly selects passive joints without the added consideration of the null space motion of the passive manipulator; passive joints can be selected based on their projection into the constraint orthogonal submanifold, \(T_2^c\).

An estimate of the constraint directions, \(C^c\), is as
\[
\hat{C}^c = \arg\max (k_i^c)^T k_i^c \quad (51)
\] and the metric consistent active joint selection criterion is as
\[
\bar{C}_{k+1} = \left( I - k_i^c \left( (k_i^c)^T k_i^c \right)^{-1} (k_i^c)^T \right) \bar{C}_k \quad \forall i \in \{1 \ldots n\}, k \in \{1 \ldots m\},
\]
where \(k_i^c\) is the i-th column of \(\bar{C}\), and \(\bar{C}_k = \bar{C}\). This criterion is computationally more efficient when \(m < n - n\).

Lastly, we suggest that the objective function \(H(q)\) for redundancy resolution should be based on the idea that passive joints accommodate displacement along the range of \(C^c\). Modelling and measurement errors result in small-angle rotation of the constraint direction estimates, \(C^c\), that result in displacements
\[
\Delta q \leq \imath(q(C^c)) \quad (53)
\]
Although it is guaranteed that these displacements are contained in the space spanned by the passive joints, we desire a configuration \(q^*\) that also minimizes the effect of passive joint motion on the task which is spanned by \(D\). As is evident from (24), we desire that
\[
P(I - S) = 0 \Leftrightarrow (I - P)(I - S) = (I - S) \quad (54)
\]
Expanding the definition for \(P\) using its constituent parts,
\[
(\Phi\Phi^T)^+ (\Phi\Phi^T) (I - S) = (I - S) \quad (55)
\]
This expression is true only if for the \(m\)-non-zero eigenvalues of \((\Phi\Phi^T)^+ \Phi\Phi^T\), the corresponding eigenvectors, denoted \(v^i\), span the range of \(J(I - S)\),
\[
\imath(v^i) \leq \imath(k_i^c) \quad (56)
\]
where columns of \(J(q)\)(I - S) are denoted by \(k_i^c\). For a fixed set of passive joints, and a particular constraint or mechanism configuration, \(\Phi(X)\), there must exist a \(q^*\) that realizing the required \(J(q^*)\) to achieve this condition. Since eigenvectors of \((\Phi\Phi^T)^+ \Phi\Phi^T\) represent the normal directions of constraint surfaces (or contact pairs), a trivial solution that ensures (54) is the one that positions passive joint axes in such a way that sets
\[
v^i = k_i^c \quad \forall i \in \{1 \ldots m\} \quad (57)
\]
Solving for the resulting \(q^*\) is not trivial, but a gradient descent approach may be utilized where the objective function \(H(q)\) drives the joint positions towards \(\max(v_i^c)\) \(k_i^c\).

Another interpretation of the statement in (24) is that passive joints amplify the constraint orthogonal component of input torque, \(τ_3\), when (54) is not true; the projection of passive joint axis, \(k_i^c\), onto the constraint tangent space, \(T_3\) is non-zero. Thus, ensuring (57) also eliminates the static reaction forces against the constraint surfaces.

\[\text{IV. EXPERIMENTS AND RESULTS}\]

The proposed multiple working mode approach has been experimentally studied for constrained interaction tasks with autonomous partitioning of the control input into active and passive working modes. A five DOF arm consisting of 4 joints – 3 revolute and 1 universal – is mounted on a differential drive PowerBot platform which can be seen in Fig. 3. Each joint contains a DC brushless motor, a motor controller integrated into a DSP board, a motor-side brake and encoder, a harmonic reduction gear with integrated torque sensor, and an ADC amplifier and conversion board [20]. Joints 2 and 3 are equipped with SHD17-100 HD [21].

![A photo of the experimental setup](image-url)

A 6-axis force-torque sensor is mounted before the gripper to evaluate the interaction forces. Multiple working modes are implemented on each joint module; each module can, therefore, independently work in active mode with position or force control, or passive mode with friction compensation. By sending commands to the DSP card embedded, the corresponding joint module can be switched online to work in either the active or the passive mode. The MRR wrist is developed based on the 2-DOF double active universal joint (DAUJ) mechanism [22]. The gripper grasps onto a one-link mechanism that represents a typical door; the contact is assumed to restrict 2-DOF, with \(m = 2\), such that rotations about the doorknob are not constrained (although friction is not negligible). Parameters of the mechanism such as height, width, and friction of the door are adjustable.

The task is set up such that grasping has already occurred, but with a configuration that is suboptimal according to the statement of (54) regarding the manipulability criteria of the passive joints. The initial desired task velocity, \(\dot{x}_d\), is set to move the end effector directly towards the mobile base center of gravity. Motion continues until one full revolution of the door to demonstrate convergence of the manipulator configuration. The task for the manipulator is specified as
\[
\dot{S}_q \hat{d} = (J_{S}q)^\# k_d \quad \frac{\dot{x}_d}{\dot{z}_d} + S_n(I - (J_{2}q)\nu_n \dot{q}_n + \nu_n) \quad (58)
\]
The first term specifies a constant task space velocity in the direction in which the constraint is guiding the end effector, 
\[
(J_{k} \in \mathbb{R}^{3 \times n}, \dot{x}_d \in \mathbb{R}^{3}) \sum_{k=1}^{n} P_k = t \quad (59)
\]
where the diagonal matrix \(x_d \in \mathbb{R}^{3 \times 3}\)(\(\nu\)) specifies the desired task velocity. The second term guides the manipulator along the self-motion manifold towards a more advantageous configuration described by (54). This is simply a restoring velocity which drives the configuration towards the unique solution given by \(q^*\); and
\[
\nu(q - q^*) = W(q - q^*) \quad (60)
\]
where \(W \in \mathbb{R}^{n \times n}\) is a weighing matrix. This solution intersects the passive joint axes with the horizontal plane and the current end-effector velocity vector, \(\dot{x}\). The motion of the mobile base is thus
dependent on the motion of the end effector, such that the base is restricted to follow the arc traced out by the grasping point. In practice, the mobile base actuators are selected for active working mode, and the active manipulator joints are assigned to control of the reconfiguration space. Lastly, the $q_{nbh}$ term represents the input from trajectory planning for non-holonomic manipulators. We use a parametric planner, whose details are left out for brevity, for generating short, non-discontinuous trajectory segments that guide the mobile platform along the gradient of the objective function in (70). A supervisory loop encloses the joint-level controller that re-evaluates the selection criteria and the control input of (68) at a frequency of 1Hz. Low-level control on each active joint works as per the control law in (8) at a frequency of 500Hz. Lastly, the 6-axis force-torque sensor is used to detect overload and trigger motion stop when necessary. Position, of the joints are shown in Fig. 4. All positions are in the workspace of the manipulator joints. Control input torque of the manipulator joints are shown in Fig. 5, which shows that the control torques required to maintain the task motion are bounded for all joints. During the experiment, joint 2 and joint 3 are switched to passive mode, and joint 1 remains in active mode. As a result of switching, the interaction force is reduced in passive joints.

![Manipulator joint positions](image)

**Fig. 4.** Manipulator joint positions

![Control input joint torques](image)

**Fig. 5.** Control input joint torques

The experiment results have demonstrated the ability of the mobile manipulator to interact with the constrained environment with the proposed control approach, reconfiguring the mobile base and manipulator to maintain bounded interaction forces and joint control torques that are required to maintain the task motion.

V. CONCLUSION

A method for controlling a serial robot in constrained motion has been presented in this paper. Multiple working mode control has been extended to handle unmodelled constraints by autonomously selecting joints for operation in passive and active modes. With this method, constrained motion tasks can be accomplished by joint-level multiple working mode control with supervisory mode switching. We have shown three criteria for active joint selection and evaluated the performance through simulations and experiments; the results show that reaction forces remain bounded, while accurate knowledge of the constraint geometry is not required. Furthermore, this work expands on the previous work of our research group with a redundancy resolution scheme that guides the mobile base while simultaneously reducing the reaction forces against the constraint.

While three different joint selection criteria were presented in this research, more criteria may be needed to deal with all kinds of problems with unstructured environments and constraints, which will form part of our future work.

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