A Novel Wolfrom-based Gearbox for Robotic Actuators

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Abstract—In many robotic applications, high torque density and highly efficient actuators are required to handle the high torque motion without hampering mobility through excessive weight and footprint. Traditionally, engineers designing actuators for these types of applications use high speed electrical motors in combination with high ratio speed reducers such as harmonic- and cycloid drives or lever arms in order to achieve the required torques. However, these systems are not ideal since they suffer from one or more of the following limitations: high cost, load hysteresis, non-linear behaviour and torque ripple. An alternative to achieve the required torque increase is by the use of an interesting type of Compound Planetary Gear Trains (C-PGTs). In this paper a novel type of such a C-PGT, based on the Wolfrom topology is presented. An extra degree of freedom is created which can be used to increase the performance of the overall gearbox, i.e. increase its efficiency, reduce its inertia, increase its torque capability, etc. This modified Wolfrom is discussed in detail by deriving a general framework for the design of such a transmission system. This framework consists of (1) listing the specific fitting conditions for such a Wolfrom PGT, (2) the derivation of an efficiency model based on the Rolling Power concept and (3) a load analysis to determine the weakest gearing element. Finally, a prototype was built to demonstrate the design framework and the feasibility of the Wolfrom variant. Additionally, the trend of the efficiency model was confirmed.

Index Terms—Efficient Actuators, Robotic Actuators, High-Ratio Gearboxes, Planetary Gear Trains, Compound Planetary Gear Trains, Wolffrom Gearbox

I. INTRODUCTION

Gearboxes are essential elements of a drive-train. They enable better efficiency, and higher power and torque densities.

Conventional industrial robots require extreme positioning accuracy. This contributed to the development of new high-ratio gearboxes as Cycloid Drives or Harmonic Drives (HDs). Today, in modern robotic devices like cobots, mobile robots, exoskeletons, or prostheses, the main focus has shifted towards torque density. Comparing performances in Fig. 1, we see how the lower weight of HDs provides a torque density advantage that explains its dominance in these new robotic areas.

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![Fig. 1: Performance comparison of the main high-ratio gearbox technologies, from [1] except novel Wolfrom (estimated). While other technologies have clearer strong and weak points, the novel Wolfrom provides a more balanced performance.](image-url)

Higher reduction ratios reduce motor weight and could also improve torque density [2], but low efficiency currently prevents this possibility. High-ratio gearboxes can reach peak efficiencies close to 80%. But, when subject to robotic cycles, dominated by below nominal torques, their mean efficiencies drop rapidly below 50% [3].

The need for ultralight, efficient actuation has motivated extensive research in robotics, including (i) incorporating elastic elements, clutches, etc. to improve energy efficiency [4], [5], [6], (ii) optimising the torque density of motors [7], or (iii) new gearbox concepts [1]. Beyond robotics, other modern applications like wind turbines, electric, and hybrid cars need also ultralight, efficient motorization [8], [9].

In this paper we introduce a novel compound planetary gearbox - patent-pending - based on a configuration initially proposed in 1912 by U. Wolfrom. The original design provides very high transmission ratios in a compact shape, but suffers from efficiency limitations, as Cycloids and HDs [10]. Our contribution is situated in exploiting the additionally introduced design freedom to balance Hertzian and bending tooth loads, reducing weight and optimising efficiency. The result is an ultralight gearbox enabling high torque density through very high ratios and good efficiencies.

This paper is organized as follows: Section II proposes a framework for the design process, including assessing the efficiency and tooth load distribution. Section III presents the design’s implementation in a proof-of-concept prototype with a reduction ratio about 50% higher than those typically found in ankle prostheses [5]. The tests to validate the technology’s feasibility, reduction ratio, and efficiency trends are described in section IV, while section V finally presents our conclusions.
II. DESIGN FRAMEWORK

Fig. 2 shows the standard Wolfrom layout which consists of two stages (hence sometimes also referred to as the 2-stage Wolfrom hereafter), (a) a full planetary gear stage with one sun-, ring- and planet gear and (b) a second stage with only a planet- and ring- gear. Both planets are connected and supported by a common carrier. Generally the sun gear acts as the input and ring b as the output of the Planetary Gear Train (PGT).

Since only one meshing takes place in the (b) stage, the micro-and macro geometry of gears $R_b$ and $P_b$ can be freely dimensioned to cope with the forces while its meshing efficiency can be maximised. However, this is not the case in the first stage (a) where a trade-off between the S-Pa- and Pa-Ra meshing is unavoidable. This results in an overdimensioning of the sun or ring gear and/or a lower overall efficiency.

The novel Wolfrom discussed in this paper is shown in Fig. 3. As can be seen, it incorporates one additional stage and hence all three meshings happen in a different stage, (s) the first stage with a sun- and first planet gear, (a) and (b) with both a planet- and a ring gearwheel. As such each individual meshing can be designed individually. This extra degree of freedom can hence be used to increase the performance of the overall gearbox, i.e. increase its efficiency, reduce its inertia -by selecting a lighter material or smaller planets-, increase the torque capability, etc.

A. Kinematic lay-out

To start the detailed analysis of the 3-stage Wolfrom, the transmission ratio, i, will be calculated. This is done by identifying the three Willis equations, one for each meshing, and solving the resulting system:

\[
\frac{\omega_S - \omega_C}{\omega_P - \omega_C} = \frac{Z_{P_b}}{Z_s} ; \quad \frac{\omega_P - \omega_C}{\omega_{R_a} - \omega_C} = \frac{Z_{R_a}}{Z_{P_a}} ; \quad \frac{\omega_P - \omega_C}{\omega_{R_b} - \omega_C} = \frac{Z_{R_b}}{Z_{P_b}}
\]

(1)

with $\omega_x$ the absolute rotational speed and $Z_x$ the number of teeth of gear $x$.

As the first ring gear is grounded ($\omega_{R_a} = 0$), the total gear ratio $i$, is given by:

\[
i = \frac{\omega_S}{\omega_{R_b}} = 1 + \frac{Z_{P_a} Z_{P_b}}{Z_{P_a} Z_{P_b}} ; \quad i_0 = \frac{\omega_S}{\omega_{R_a}} = 1 + \frac{Z_{R_a}}{Z_{P_a} Z_{R_b}} \frac{Z_{P_b}}{Z_{P_a}} \frac{Z_{P_b}}{Z_{P_b}}
\]

(2)

With $i_0$ and $i_w$ were introduced to facilitate the expressions later on. Hence, the total gear ratio is simply the product of these two components: $i = i_0 i_w$. Note that in the case of the 2-stage Wolfrom, $Z_{P_s} = Z_{P_a}$.

Moreover, it can be seen that the second gear ratio, $i_w$, can become very high, especially when $\frac{Z_{P_a} Z_{R_a}}{Z_{P_b} Z_{R_b}}$ approaches one, and hence explains why this type of C-PGTs can obtain very high gear ratios while remaining compact. However, since the efficiency is very sensitive to a change of this parameter, as will be shown later in section II-C, one should be careful when selecting its value.

B. Fitting conditions

Since the Wolfrom is a compound planetary gearbox, not all combinations of number of teeth and moduli fit and hence are possible. The fitting conditions are summarised below.

1) Diameter Fitting: The first condition which needs to be respected is that three gearwheels of each compound planet need to be coaxial.

\[
r_{w,S} + r_{w,P_s} = r_{w,R_a} - r_{w,P_a} = r_{w,R_b} - r_{w,P_b}
\]

(3)

Where $r_{w,x}$ is the working pitch radius of gear $x$.

2) Neighbouring condition: Secondly, it should also be guaranteed that the teeth of the different planet gears in each stage will not interfere with each other. The tip-to-tip interference can be derived by looking at Fig. 4. From this figure it can be seen that following conditions need to be met:

- **Sun-Planet meshing:**

\[
(r_{S} + r_{P_a}) \sin \left( \frac{\pi}{n_w} \right) \geq r_{P_a} + h_a + c
\]

(4)

\[
\Rightarrow \frac{r_{P_a}}{r_S} \geq \frac{Z_{P_a}}{Z_s} \leq \frac{\sin \left( \frac{\pi}{n_w} \right)}{1 - \sin \left( \frac{\pi}{n_w} \right)} - \frac{2(1+c^*)}{Z_s(1-\sin \left( \frac{\pi}{n_w} \right))}
\]

With $r_S$, the pitch circle of gear $x$, $n_w$ the number of planets in one stage, $h_a$ the addendum height of the gear, which for standard gears is equal to the module, $m$, and $c = mc^*$ is a clearance factor introduced to add some clearance between the two planet gears. Equation (4) shows that ratio $\frac{Z_{P_a}}{Z_s}$ increases with the number of teeth of the sun but will never be larger than 0.46. Except in the case where the number of planets in one stage is one or two.

- **Ring-Planet meshings** For each of the two last stages, i.e. $x \in \{a, b\}$, the following condition needs to be valid:

\[
(r_{R_x} - r_{P_x}) \sin \left( \frac{\pi}{n_w} \right) \geq r_{P_x} + h_a + c
\]

(5)

\[
\Rightarrow \frac{r_{P_x}}{r_{R_x}} \leq \frac{\sin \left( \frac{\pi}{n_w} \right)}{1 + \sin \left( \frac{\pi}{n_w} \right)} - \frac{2(1+c^*)}{Z_r \left(1 + \sin \left( \frac{\pi}{n_w} \right) \right)}
\]

3) Angular fitting: Since it is possible that the number of teeth of the ring- and planetary gears of stage a differ from the ones in stage b, the planets often need to get a phase shift in order to fit [11]. This offset is visualised in Fig. 5.
Fig. 3: Example of a 3-stage Wolfrom with hence only one meshing in each stage, adding an extra degree of freedom to the standard, 2-stage, Wolfrom. In Fig. 3c, $S$ represents the sun-, $Ra$ the ring- and $Rb$ the ring gear of respectively stages (s), (a), (b). $C$ denotes the common carrier supporting the planetary gears $Ps, Pa, Pb$.

Fig. 4: Nomenclature used for the tip-to-tip interference condition.

By imposing rolling without slipping of the planets, the phase shift between the different planet pairs can be expressed by:

$$
\begin{align*}
\gamma_{s,a} &= \frac{2\pi k}{n_w} \left( \frac{Z_{Pa}}{Z_{Pb}} - \frac{Z_{Ps}}{Z_{Pb}} \right) + 2\pi \left( \frac{\omega_s}{Z_{Ps}} + \frac{\omega_a}{Z_{Pb}} \right) \\
\gamma_{s,b} &= \frac{2\pi k}{n_w} \left( \frac{Z_{Pa}}{Z_{Ps}} - \frac{Z_{Pb}}{Z_{Ps}} \right) + 2\pi \left( \frac{\omega_s}{Z_{Pb}} + \frac{\omega_b}{Z_{Ps}} \right)
\end{align*}
$$

(6)

Where $\gamma_{s,a}$ and $\gamma_{s,b}$ are the angles between the planet gears in stage (a), (b) and stage (s) –defined according to Fig. 5– and $k$ the respective planet.

As a consequence of (6) all planets can have a different phase shift and hence increase the production cost. If one chooses to make identical planets ($\gamma_{s,a} = \gamma_{s,b} = 0$) an extra set of fitting conditions is required [12], given by:

$$
\begin{align*}
Z_{Ps}Z_{Ra} + Z_{Ps}Z_{S} &= l_1 |l_1| \in \mathbb{Z} \\
Z_{Ps}Z_{Rb} - Z_{Pb}Z_{Ra} &= l_2 |l_2| \in \mathbb{Z}
\end{align*}
$$

(7)

C. Efficiency Analysis

According to Müller [13], each planetary motion can be decomposed into a rolling motion and a coupled motion. The former describes the rotation of all gears in the reference frame of the common carrier while the latter describes the motion of all gears together with the carrier as if the gear train rotates as a rigid coupling. Mathematically, this is expressed as follows:

$$\omega_x = \omega'_x + \omega_C$$

(8)

Where $\omega_x$ and $\omega'_x$ denotes the absolute- and rolling speed of gear $x$, respectively and $\omega_C$ the absolute speed of the carrier.

Since there are no relative speeds in the coupled motion between two gears, the latter does not introduce any meshing losses which, as a consequence, are completely determined by the rolling motion and the corresponding rolling power. Hence, the total meshing losses, $L$, are given by the sum of the rolling powers, $P'_x$, in each meshing multiplied with their respective generalised loss factor, $Q_{L,x}$ [13]:

$$L = - \sum_x P'_x \cdot Q_{L,x}$$

(9)

Note that, by convention, a negative sign for the losses (and other output powers) is used. In (9), the generalised loss factor depends on the sign of the rolling power:

$$Q_{L,x} = \begin{cases} f_{L,x} & \text{if } P'_x > 0 \\ -\frac{f_{L,x}}{f_{L,x}} & \text{if } P'_x < 0 \end{cases}$$

(10)

This loss factor, $f_{L,x}$, is often split into two factors [14]. The first is related to the teeth geometry and the second one depends on the friction between the two gears.

For the Wolfrom gearbox, the rolling powers of the three meshings are equal to:

$$
\begin{align*}
P'_S &= T_S\omega'_S \quad (S-Pa \text{ meshing}) \\
P'_Ra &= T_{Ra}\omega'_Ra \quad (Pa-Ra \text{ meshing}) \\
P'_Rb &= T_{Rb}\omega'_Rb \quad (Pb-Rb \text{ meshing})
\end{align*}
$$

(11)

Where $T_{Ra}$, $T_{Rb}$ and $T_S$, stand for the torque of ring $Ra$, ring $Rb$ and the sun $S$ respectively. The rolling speeds can be
written as a function of the gear parameters by using (2):
\[
\begin{align*}
\omega'_S & \triangleq \omega_S - \omega_C = \left(1 - \frac{1}{i_0}\right) \omega_S \\
\omega'_{Ra} & \triangleq \omega_{Ra} - \omega_C = -\frac{\omega'_S}{i_0} \\
\omega'_{Rb} & \triangleq \omega_{Rb} - \omega_C = \left(1 - \frac{1}{i_0}\right) \omega_S
\end{align*}
\] (12)

The torques of the system on the other hand, can be determined by expressing the torque equilibrium and the definition of the total efficiency:
\[
\begin{align*}
T_{Ra} + T_{Rb} + T_S &= 0 \\
\eta_{tot} &\triangleq -\frac{P_{out}}{P_{in}} = -\frac{P_{Ra}}{P_S} = -\frac{1}{i_0} T_{Rb}
\end{align*}
\] (13)

By solving (13) the following expressions for the torques can be derived:
\[
\begin{align*}
T_{Ra} &= (i\eta_{tot} - 1) T_S \\
T_{Rb} &= -i\eta_{tot} T_S
\end{align*}
\] (14)

Hence, when inserting (12) and (14) into expression (11), the rolling powers can be expressed as a function of the gear ratio components \(i_0\) and \(i_w\):
\[
\begin{align*}
P_S &= \left(1 - \frac{1}{i_0}\right) P_S \\
P_{Ra} &= \left(1 - \frac{1}{i_0} i_w\eta_{tot}\right) P_S \\
P_{Rb} &= \eta_{tot} (i_w - 1) P_S
\end{align*}
\] (15)

Note that the values of \(P'_{Ra}\) and \(P'_{Rb}\) are approximately equal to \(i_w P_s\) and hence a multiple of the input power. Therefore, the losses generated by the two ring meshings can grow rapidly for high values of \(i_w\). Consequently, it is especially interesting to optimise the gear profiles to maximise the meshing efficiencies. In and [15] this has been done for the standard Wolfrom and can easily be extended to the modified version presented in this paper.

In general, each of these rolling powers should be checked on its sign to retrieve the correct value of the generalised losses factor according to (10). However, since the meshing efficiency of a correctly optimised spur- or helical gear contact is typically around 99\% and higher [16], both formulations of (10) are approximately the same, except from the sign. Hence, to simplify the equations, the meshing losses are assumed to be: \(L_x = -|P_x^f| \cdot f_{k,x}\).

As a consequence, the total loss ratio is given by:
\[
\frac{L_x}{P_S} = \left|1 - \frac{1}{i_0} |f_{k,S}| - \frac{1}{i_0} i_w\eta_{tot} |f_{L,Ra} - |\eta_{tot}(i_w - 1)|f_{L,Rb}\right| (16)
\]

Under some conditions a simple expression can be found for the total efficiency.

- It can be noted that when \(\frac{Z_{Ra}}{Z_{Pb}} < \frac{Z_{Rb}}{Z_{Pa}}\) this results in \(i_w < 0\), which according to (16) leads to higher losses compared to \(i_w > 0\). Therefore, we focus our analysis on the latter.
- As high gear ratios are targeted, it is assumed that \(i > 1/\eta_{tot}\) and \(i_w > 1\).
- By definition, \(i_0\) will always be larger than 1.

Under the aforementioned assumptions it can be calculated that the signs of the rolling powers are given by:
\[
\begin{align*}
sign(P'_S) &= +sign(P_S) \\
sign(P'_{Ra}) &= -sign(P_S) \\
sign(P'_{Rb}) &= +sign(P_S)
\end{align*}
\] (17)

And consequently an expression for the efficiency can be found, using (15), (16) and (17):
\[
\eta_{tot} = 1 + \frac{L}{P_S} = \frac{1 + f_{L,Ra} i_w - f_{L,S} (1 - \frac{1}{i_0})}{1 + f_{L,Ra} i_w + f_{L,Rb} (i_w - 1)}
\] (18)

Equation (18) is visualised in Fig. 6 where it is assumed that the internal meshing losses are half of the external meshing losses [13]. It can clearly be seen that a higher \(i_0\) has a positive effect on the overall efficiency.

![Fig. 6: The total efficiency of the Wolfrom PGT as a function of \(i_0\) for an internal meshing efficiency of 99\%. As can be seen, the efficiency decreases rapidly with an increasing gear ratio but \(i_0\) has a positive effect on the efficiency.](image)

**D. Load Analysis**

One advantage of splitting the stages of a Wolfrom is that each meshing can be dimensioned for the load it has to transmit. Indeed, with only two stages, the planet gear in stage (a) interacts both with a sun- and ring gear. Therefore, a compromise has to be made and the two latter meshings can not be optimised for load and efficiency independently of each other. With the introduction of the third stage this is no longer an issue.

In Fig. 2 it can be seen that the Wolfrom gearbox has in total three different meshing contacts, two in the input stage (a) and one in the output stage (b). Hence, in the output stage the module can be easily optimised based on the load. On the other hand, since there are two different meshings in the first stage the module and the gear geometry of the planet gear is common for both the sun- and ring gear. In order to analyse which meshing will have to be considered for dimensioning the latter, the stresses in each of the meshings will be computed below.
1) Forces acting on the gears: The tangential forces acting on the sun- and ring gears can be derived from their torques. Using (14) the following relationships can be found:

\[
\begin{align*}
F_{Ra,S} &= Z_{Ra,S} (i \eta_{tot} - 1) \\
F_{S,Ra} &= -Z_{Ra,S} i \eta_{tot}
\end{align*}
\]  

Again, as the meshing efficiency is very high for gear contacts, the forces acting on the sun and rings are opposite and approximately equal to the forces acting on their corresponding planet gears. Hence,

\[
\begin{align*}
F_{Ra,Ra} &\approx -F_{Ra,S} \\
F_{S,Ra} &\approx -F_{S,Ra}
\end{align*}
\]  

Where the notation \( F_{x,y} \) represents the force on planet \( x \) when it is meshing with central gear \( y \).

Now two types of stress failure can be considered, namely the contact stress and the bending stress.

2) Contact stress \((\sigma_H)\): The Hertzian contact stress for a gear pair, \(\sigma_H\), is the same for both gears and given by [17]:

\[
\sigma_H^2 = Z^2 K |F_n| \left( Z_2 + \frac{Z_1}{mbZ_1Z_2} \right) 
\]  

The positive sign should be used in the case of an external gear pair, while the negative sign should be used for an internal gear pair (in the latter case \( Z_2 \) is the number of teeth of the ring gear). In (21) \( F_n \) represents the tangential force acting on the gears, \( b \) the width and \( m \) stands for the module of the gear pair. \( K \) and \( Z \) are material, geometry and load distribution factors which consist of:

\[
\begin{cases}
Z^2 = Z_H^2 Z_M^2 Z_c^2 \\
Z_H^2 = \frac{Z_2}{mZ_1} \\
Z_M^2 = \frac{2}{(1-\nu^2)} \\
Z_c^2 = \frac{4 - \nu - 6}{9}
\end{cases}
\]  

In (22), \( \alpha \) represents the pressure angle, \( E \) and \( \nu \) the Young’s module and Poisson coefficient respectively and \( \epsilon_\alpha \) the contact ratio.

Generally \( K_{H\alpha} \) (which takes the load distribution between the teeth into account) can be set to 1 for spur gears, while \( K_{H\beta} \) (takes the unevenness of the load distribution along the length of the contact lines into account) is more complicated to determine and can only be chosen to be 1 when the gears are perfectly manufactured. For simplicity reasons, this is set to 1.3, which is a correct value for most operating conditions [17].

When this is applied to both the sun \((\sigma_{H,S})\) and ring \((\sigma_{H,Ra})\) meshings, the ratio of the contact stresses is given by:

\[
\sigma_{H,Ra,S}^2 = \frac{Z_{Ra,S}^2}{Z_S^2} \left( \frac{F_{Ra,S}}{F_S} \right) \frac{(Z_{Ra} - Z_{Pa})Z_S}{(Z_S + Z_{Pa})Z_{Ra}}
\]

Which is, by using \( Z_{Ra} = Z_S + 2Z_{Pa} \) and (19), given by:

\[
\sigma_{H,Ra} = \frac{Z_{Ra} \sqrt{i \eta_{tot} - 1}}{Z_S} \approx \frac{\sqrt{i \eta_{tot} - 1}}{1 + 2\frac{Z_{Pa}}{Z_S}}
\]

Since in practise the contact ratios of both meshings are almost the same and all gears in the same stage are made of the same material, the ratio of \( \frac{Z_{Ra,S}}{Z_{S,Ra}} \) is approximately 1. Note that \( i \eta_{tot} \) is also referred to as the torque ratio, i.e. \( i_T \triangleq i \eta_{tot} [13] \).

A visualisation of (24), for different torque ratios, is given in Fig. 7. It can be seen that when the ratio of the teeth increases, the ratio of the stresses decreases. Moreover, only when the absolute value of the torque ratio is higher than \(|i_T\text{min}| = 195\), the contact stress on the ring is higher than the contact stress on the sun when at least three planets are used, i.e. \( \frac{Z_{Pa}}{Z_S} < 6.46 \).

3) Bending stress \((\sigma_B)\): The bending stress is given –with the same definitions of \( K, F_n, b, m \) and \( \epsilon_\alpha \) as in (21)– by [17]:

\[
\sigma_B = YK \frac{|F_n|}{bm}
\]

Where \( Y \) is given by the following expression:

\[
Y = \begin{cases} 
\frac{2 + 3.1e^{-14}}{2.06} & \text{for external gears} \\
2.06 & \text{for internal gears}
\end{cases}
\]

To ease the equations, the scaling factor \( Q_s = \frac{Z_{Pa}}{Z_S} \) is introduced. As such, the bending stresses on the different gears, normalised with the scaling and load distribution factors, are given by:

\[
\begin{align*}
\sigma_{B,Pa,s} &= \frac{2 + 3.1e^{-\frac{Z_{Pa}}{Z_S}}}{Z_{Ra}} & : Pa \text{ at Sun meshing} \\
\sigma_{B,Pa,Ra} &= \frac{2 + 3.1e^{-\frac{Z_{Pa}}{Z_S}}}{Z_{Ra}F_{Ra}} & : Pa \text{ at Ring meshing} \\
\sigma_{B,Ra,s} &= \frac{2 + 3.1e^{-\frac{Z_{Pa}}{Z_S}}}{Z_{Ra}} & : Sun \\
\sigma_{B,Ra,Ra} &= \frac{2 + 3.1e^{-\frac{Z_{Pa}}{Z_S}}}{Z_{Ra}F_{Ra}} & : Ring
\end{align*}
\]

Some conclusions can be drawn from these equations:

- The bending stress on ring \( Ra \) will always be smaller than the stress on the corresponding planet \( Pa \), due to the calculation of \( Y_T \) given in (26).
When $Z_{Pa}/Z_S > 1$, which is desirable from the perspective of the efficiency $\frac{\sigma_{B,Pa}}{\sigma_{B,S}} > 1$. Indeed if this is not the case $i_w$ will be lower and consequently $i_w$ higher which, according to section II-C, results in a lower overall efficiency.

- Recall from section II-B2 that in the case where the first stage consists of at least three planets, $Z_{Pa}/Z_S < 6.46$. When additionally the torque ratio, $i_T > 26$ and assuming standard gears (without profile shift and a pressure angle of 20°), it can be calculated --using (27)-- that $\frac{\sigma_{B,Pa,Ra}}{\sigma_{B,S}} > 1$.

Under these conditions, the highest bending stress will occur at the planet teeth where it meshes with the ring gear, as can be seen in Fig. 8.

![Fig. 8: Ratio of the bending stresses of the planet (at the ring interface) and the sun for different values of $Z_S$ and $Z_{Pa}$ for $i_{tot} = 26$. Note that $\sigma_{B,Pa,Ra}$ is always larger than $\sigma_{B,S}$.

Evaluating the width and module for stage (b) can be done on the planet gear only, since --again-- the bending stress on ring $Rb$ will always be smaller than the bending stress on planet $Pb$, while the contact stress is the same.

As a result of splitting the first stage into two, the gear parameters of each meshing can be chosen to balance the bending and contact stress. This results in a more compact overall solution in radial direction since the module is no longer overdimensioned to cope with the highest of the two stresses. An example of this is given in table V, where a standard Wolfrom is compared with a prototype of the novel Wolfrom. For the same size and gear ratio the rated torque could be increased with 31%.

### III. DESIGN IMPLEMENTATION

Ankle prostheses have high torque density, which motivated us to select them as a reference application for our proof-of-concept. We further increased the reduction ratio, to demonstrate its impact, and adapted the testing speeds and torques to accommodate the somewhat lower capacities of the materials and manufacturing means available. The main parameters of the proof-of-concept are presented in table I, while its internal configuration and external aspect are shown in Fig. 9. This implementation is based on an FMEA study [18] which identified various key design aspects:

- 3D printed polyamide for the two ring housings and the disc support of the carrier

#### A. Floating Elements

In a planetary configuration, the planet gearwheels are engaged into simultaneous meshing contacts with the sun and the ring gearwheels. Consequently, any manufacturing deviation induces heavy load unbalances between the planets and results in critical overloads.

A conventional way to solve this condition is to select a "floating" element, introducing an additional degree of freedom to allow a more homogeneous force distribution [19]. The sun or ring gearwheels are typically selected as floating elements in larger PGTs, while for smaller gearboxes, the carrier is also an option [20]. We chose a floating carrier, due to its simpler implementation and slimmer design.

#### B. Bearings

Bearings allow relative movement with small friction between parts subject to significant contact forces, particularly for small diameters. Bearings tend to be heavy, thus restricting their use to the strictly necessary amount and size is very convenient for an ultralight gearbox.

This reasoning motivated us to incorporate only two bearings into our prototype. They are shown in Fig. 9, connecting the central shaft to each of the ring gearwheels’ housings, which receive the highest torques. The diameter is increased on the side connecting to ring B to fit a slightly larger bearing because ring B typically has a lower ability than the grounded ring A to receive structural support from the device’s joint.

We also decided not to use bearings on the planet sets. Taking advantage of the relatively small forces on the carrier, we allowed the planets to rotate freely around their pins with a direct (grease lubricated) contact. This solution is frequently used in power-tool devices, where the porous structure of sintered materials, filled with lubricant, provides similar contact conditions to journal bearings.

#### C. Materials

FEM simulations confirmed the largest potential of the housings and carrier to use lightweight materials. This recommended separating the housings of the ring gearwheels into two parts: (i) a rim carrying the teeth, subject to high stresses for which steel allows smaller teeth sizes, therefore reducing size and weight, and (ii) the housings, for which a stress analysis opened the possibility to use plastic materials at the cost of a slightly reduced compactness.

The carrier was also split into two parts, (i) a structural frame for which aluminum provided the best balance between stress and compactness and (ii) two plastic support discs that are interference-pressed onto the frame.

Finally, the sun and the first planet stage are not subject to high stresses and can also be manufactured in plastic material.

#### D. Manufacturing Means

Our budget (750,00EUR) defined the quality-cost balance and conditioned the choice of the manufacturing means:

- 3D printed polyamide for the two ring housings and the disc support of the carrier
Commercial, injection-molded duracon-acetal gears for the sun and the first stage of the planet set
Catalog bearings for the Bearings A and B
CNC machined aluminium for the carrier frame
CNC machined C45 steel for the planet pins and the central shaft
CNC machined 42CrMo4 gear steel for the ring wheels and the second and third gearwheels of the planet set.

TABLE I: Main parameters of 3-stage PGT

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear Ratio</td>
<td>r</td>
<td>1 : 494</td>
</tr>
<tr>
<td>Repeatable Peak Torque (estimated) [Nm]</td>
<td>-</td>
<td>40</td>
</tr>
<tr>
<td>Diameter [mm]</td>
<td></td>
<td>57</td>
</tr>
<tr>
<td>Height [mm]</td>
<td></td>
<td>34</td>
</tr>
<tr>
<td>Width [mm]</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Weight [g]</td>
<td></td>
<td>130</td>
</tr>
<tr>
<td>Number of teeth (NOT) sun gearwheel</td>
<td>Z_S</td>
<td>18</td>
</tr>
<tr>
<td>NOT planet gearwheel, fist stage</td>
<td>Z_Pa</td>
<td>36</td>
</tr>
<tr>
<td>NOT planet gearwheel, second stage</td>
<td>Z_Pb</td>
<td>20</td>
</tr>
<tr>
<td>NOT grounded ring (A) gearwheel</td>
<td>Z_Ra</td>
<td>46</td>
</tr>
<tr>
<td>NOT output ring (B) gearwheel</td>
<td>Z_Rb</td>
<td>49</td>
</tr>
<tr>
<td>Module, first stage [mm]</td>
<td>m_S</td>
<td>0.5</td>
</tr>
<tr>
<td>Module, second stage [mm]</td>
<td>m_a</td>
<td>1.01</td>
</tr>
<tr>
<td>Module, third stage [mm]</td>
<td>m_b</td>
<td>0.94</td>
</tr>
</tbody>
</table>

IV. EXPERIMENTAL RESULTS

In this section the feasibility of the 3-stage Wolfrom will be shown and the trends predicted by the efficiency model discussed in section II-C will be validated.

The complete test setup is presented in Fig. 10 and consists of the Wolfrom PGT, two torque sensors, two motors (of which the one on the right is used as generator) and two optical encoders connected directly to the motor shafts. All elements were connected and synchronised using the EtherCAT protocol using TwinCAT. Detailed information of the motors and sensors used in this test setup is given in Table II.

In order to calculate the efficiency, by means of (18), the loss factors have to be determined first. Using [21], these can be calculated analytically when the geometries and friction coefficients of the gears are known. For the three meshings in the Wolfrom these are are listed in Table III.

Where \( \eta_x \) and \( \mu_x \) are the efficiency and friction coefficient of the meshing between gear \( x \) and its corresponding planet gear, respectively. With this information, the theoretical overall efficiency can be calculated and is equal to: \( \eta_{tot} = 26.08\% \).

As could be expected the overall efficiency is very low due to the very high gear ratio and relatively low meshing efficiencies (when optimised the latter can be as high as 99.7% [24]).

The experimental efficiency is determined by operating the gearbox at a constant operating condition, \( \omega_S = 500\) rpm and \( T_{Rb} = 3\) Nm, for five minutes. The average of the instantaneous input- and output power is than divided by each other to obtain the experimental efficiency. The results are given in Table IV.

TABLE II: Details of the test setup

<table>
<thead>
<tr>
<th>Part</th>
<th>Specific Name/Range/Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input motor</td>
<td>Maxon EC 387601 (120V)</td>
</tr>
<tr>
<td>Load motor</td>
<td>Maxon DC 454756 (200V)</td>
</tr>
<tr>
<td>Load Gearbox</td>
<td>Maxon GP 62A 110506 (139 : 1)</td>
</tr>
<tr>
<td>Input torque sensor ETH DRBK-2</td>
<td>(2 ± 0.01)Nm</td>
</tr>
<tr>
<td>Output torque sensor ETH DRBk-50</td>
<td>(50 ± 0.25)Nm</td>
</tr>
<tr>
<td>Optical encoders</td>
<td>Maxon HEID 5540 (500 counts per turn)</td>
</tr>
</tbody>
</table>

TABLE III: Properties of the prototype used for the verification of the efficiency model

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
<th>( \mu_x )</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_{tot} ) ( = 1 - \eta_L.S )</td>
<td>91.7%</td>
<td>( \mu_S )</td>
<td>0.46</td>
<td>[22]</td>
</tr>
<tr>
<td>( \eta_{Ra} ) ( = 1 - \eta_{L.Ra} )</td>
<td>98.5%</td>
<td>( \mu_{Ra} )</td>
<td>0.3</td>
<td>[23]</td>
</tr>
<tr>
<td>( \eta_{Rb} ) ( = 1 - \eta_{L.Rb} )</td>
<td>98.5%</td>
<td>( \mu_{Rb} )</td>
<td>0.3</td>
<td>[23]</td>
</tr>
</tbody>
</table>

TABLE IV: Experimental data

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Theoretical value</th>
<th>Experimental value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed ratio (( i ))</td>
<td>494.45</td>
<td>494.03</td>
</tr>
<tr>
<td>Efficiency (( \eta_{tot} ))</td>
<td>26.08%</td>
<td>21.22%</td>
</tr>
<tr>
<td>Torque ratio (( i_T ))</td>
<td>-</td>
<td>90.89</td>
</tr>
</tbody>
</table>

It can be seen that the experimental efficiency is close to the theoretical value of 26.08%, thereby confirming the trend theoretically derived in section II-C. The difference between the theoretical and experimental value can be explained by the fact that the efficiency model only takes into account the losses of the gear meshings while it is known that other sources of losses are present, among others bearing losses.

To position the novel concept with respect to other solutions a comparison is made with a standard Wolfrom and two Harmonic Drives –CSD 17 (100:1) and CSD 20 (160:1)– in Table V. The CSD 17 has a similar rated torque as our
prototype but only a maximum gear ratio of 100:1. Therefore, the bigger variant is given as well as it has a gear ratio of 160:1. To calculate the rated torque of the novel- and standard Wolfrom, the commercial software KISSsoft has been used. For the Harmonic Drive (HD) the catalog values are taken [3]. Since the efficiency is not constant for all operating conditions, it has been chosen to compare them for the operating conditions of at which we tested our prototype, i.e. an output torque of 3 Nm and input speed of 500 rpm.

It can be seen that the rated torque of the novel Wolfrom could be increased by 31% compared to a standard Wolfrom for the same diameter and efficiency. This due balancing the bending- and contact stresses. Moreover, the efficiency is similar to that of the HD while the speed ratio is tripled. It can also be noted that the weight of our prototype is very similar to the HD without housing (CSD-series).

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Novel Wolfrom</th>
<th>Standard Wolfrom</th>
<th>HD CSD-17-100</th>
<th>HD CSD-20-160</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed ratio</td>
<td>494.35</td>
<td>493.87</td>
<td>120</td>
<td>160</td>
</tr>
<tr>
<td>Efficiency [%]</td>
<td>21.22*</td>
<td>25.81</td>
<td>~32 (80)</td>
<td>~20 (78)</td>
</tr>
<tr>
<td>Rated Torque [Nm]</td>
<td>13.1</td>
<td>10.5</td>
<td>16</td>
<td>28</td>
</tr>
<tr>
<td>Weight [g]</td>
<td>130*</td>
<td>100**</td>
<td>130**</td>
<td></td>
</tr>
</tbody>
</table>

* Experimental data  
** Without housing 
(x): at rated conditions

V. CONCLUSION AND FUTURE WORK

In this paper, we introduce a novel Wolfrom topology. Splitting the input stage of a standard Wolfrom gearbox allows the engineer to optimize each gear meshing individually, compensating for the traditional efficiency limitations of these gearboxes. An efficiency model based on the meshing losses is derived, showing a good correlation with the results on a proof-of-concept prototype with a very high reduction ratio.

Additionally, a kinematic- and static analysis is performed to determine the weakest gearing element, based on the gear ratio and the quotient $Z_{pa}/Z_{s}$. The design engineer can use these to dimension all gears for minimum weight and inertia.

These advantages allow higher reduction ratios with the same or better efficiencies as state-of-the-art gearboxes. This principle can be used to build high torque-density actuators for applications requiring lightweight, efficient motorization.

Although the small input torque allows very thin gearwheels, the additional input stage increases slightly its axial dimension. For some applications, this could represent a disadvantage and was considered in the gearbox comparison in Fig. 1. Note also that Wolfrom gearboxes typically show moderately higher backlash than Cycloid and HDs, resulting in lower positioning accuracy.

Future work will consist of building a more sophisticated prototype incorporating tooth profile and lead modifications, and profile shift, to demonstrate our gearbox’s potential to achieve high ratios with good efficiency under more representative (higher) torque and speed levels. Integrating it in the drive-train of a mobile robot of our lab, we will also be able to assess its performance in positioning accuracy, compactness, noise, backdrivability and dynamic behavior.

REFERENCES

[22] DURACON POM Grade Catalog, Polyplastics Co., 2019, [PDF file].