

An Industrial Applicable Approach towards Design Optimization of a Reciprocating Mechanism: an emergency ventilator case study.

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Abstract—

Design optimization of mechanisms is a promising research area as it results in more energy-efficient machines without compromising performance. However, machine builders do not actually use the potential described in the literature, as these methods require too much theoretical analysis. Therefore, this paper proposes a novel industrial applicable approach that enables the design optimization of reciprocating mechanisms using CAD models.

The 3D multi-body software is used to perform motion simulations, from which the objective value samples can be extracted. In this paper, the considered objective value is the required torque, for a specific combination of design parameters, to fulfil the movement. Dedicated software can execute multiple motion simulations sequentially and interchange data between the different simulations, which automates the process of retrieving objective value samples. Therefore, without in-depth analytical design analysis, a machine designer can evaluate multiple designs at a low cost. Moreover, an optimal design that meets the objective can be found by implementing an optimization algorithm. In a case study of an emergency ventilator mechanism which considers three link lengths as design parameters (DP's), 39 CAD motion simulations allowed a reduction of the RMS torque of the mechanism by 57.2%.

I. INTRODUCTION

The energy consumption of industrial machinery is a topic of primary importance due to environmental and economic considerations [1]. The 45% share of electric motors in the global electric consumption [2] supports the statement that any energy-saving method should be investigated thoroughly. The methodology proposed within this paper applies to all mechanisms with a reciprocating movement of the end-effector or tooltip. Many recent studies [3] pay attention to minimizing the energy dissipation in the electric motor to reduce the consumed electrical energy. Moreover, reducing the energy losses in the motor lowers the probability that the motor can be overheated [4]. The link lengths in a mechanism can differ while fulfilling the same task, being the end-effector's Point-To-Point (PTP) displacement. Therefore, within this case, the geometry parameters $|OA|$, $|AB|$, and $|BC|$ of the emergency

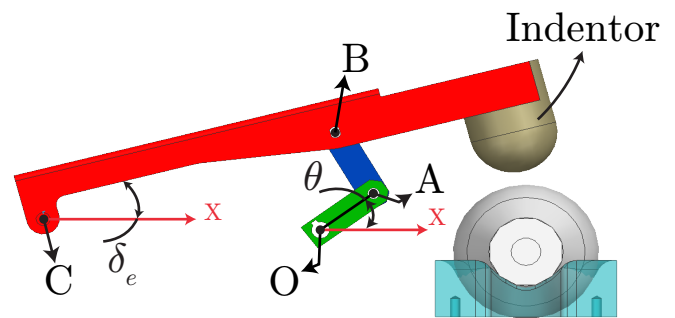


Fig. 1. The emergency ventilator as supplied by the machine builder, at the top. The considered design parameters $|OA|$, $|AB|$ and $|BC|$ within the CAD model of an emergency ventilator, at the bottom.

ventilator depicted in Fig. 1 can be considered as design parameters to be optimized, while maintaining the imposed movement of the indenter (being the end-effector). Design optimization of a PTP mechanism is one specific approach to reduce the energy consumption of electric machinery. As indicated in Fig. 2, changing the geometry parameters ($|OA|$, $|AB|$ and $|BC|$) can result in a lower RMS torque (T_{RMS}). For the envisaged high dynamical applications where friction is negligible, minimizing the T_{RMS} corresponds to reduced energy losses and overall energy consumption [5].

The influence of hardware replacement, within a machine, on energy consumption has recently attracted attention. As [3], [6] state, one should make components lighter and use more energy-efficient components (e.g. choosing an optimal gear ratio) to dissipate less energy. Moreover, [7] changed the concept of the mechanism to get a more efficient machine for a specific task.

An emergency ventilator is used as a proof of concept within this study. This mechanism was constructed during the first wave of the covid-19 pandemic by a non-profit organization [8]. Having continuous (24/7) electricity access is not obvious in low- and middle-income countries. Thus, having a mechanism that consumes a minimum of electric energy enabling the usage

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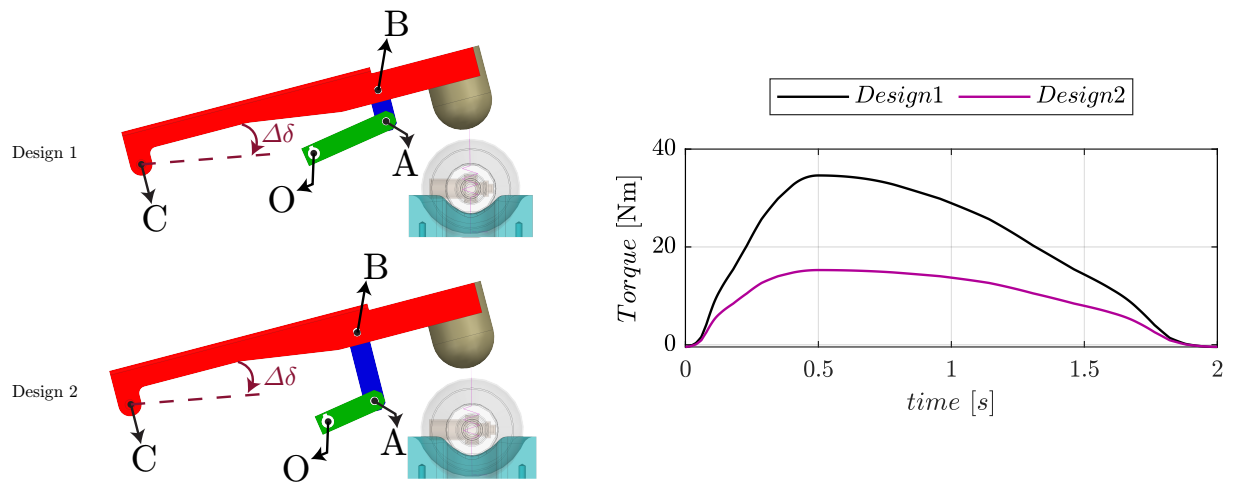


Fig. 2. Defining specific lengths for the mechanism links influences the required speed and torque to move the end-effector (red beam) over a range of $\Delta\delta$, driven from point O.

of batteries, is highly relevant. Therefore, the objective of this study, as stated in Equation (1), is to find the optimal design (being lengths $|OA|$, $|AB|$ and $|BC|$ in Fig. 1) leading to a minimal T_{RMS} for this PTP mechanism.

$$\begin{aligned} \min : & \quad T_{RMS}(\mathbf{X}) \\ \text{subject to:} & \quad \text{Feasible combination of } \mathbf{X} \\ & \quad x_i \in [x_{i\min}, x_{i\max}]; \quad x_i \in \mathbf{X} \end{aligned} \quad (1)$$

where

T_{RMS} is the objective function,

The feasibility of a certain design is the constraint evaluated within the CAD software,

\mathbf{X} is a vector, which contains the independent design parameters $|OA|$, $|AB|$ and $|BC|$,

$x_{i\min}$ and $x_{i\max}$ define the limits of each design variable x_i .

State-of-the-art design optimization methods derive the torque for a mechanism's movement analytically [9], which is mechanism specific and less convenient. However, the methodology described in this paper only needs CAD software, which ensures broad industrial applicability. That is because the industry heavily relies on 3D multi-body software to design a mechanism. The method introduced in this paper uses these CAD models to sample the objective value through motion simulations. Dedicated software [10] performs the automation of the process in which multiple simulations are run sequentially. Mechanism models replace prototyping, allowing computational evaluation of multiple designs with limited cost. In [4], [11], the machine's system properties are derived from CAD models to optimize the motor's motion profile towards minimal energy consumption. However, the present paper does not include the optimization of the motor's motion profile, yet focuses on the consequence of the component's geometry on energy consumption.

In the literature [12], the minimization of the driving torque is done by establishing dynamic equations of the system to

predict the dynamics. Moreover, [9], [12], [13] do not define the feasible search domain nor include it in searching for the optimal result. Indicating the feasibility of a particular design is essential as defects, giving infeasible designs, [14] can occur in synthesizing a mechanism. The optimization algorithms of [9], [12], [15] assure that the objective function converges towards a minimum, yet it is generally not guaranteed that the designed linkage will be feasible. Therefore, the necessary analysis should be added so that the optimal solution can fulfil the movement without issues.

Developing a reciprocal mechanism that follows the desired end-effector trajectory is a classical design problem that researchers extensively explore [16]. However, all methods in the literature [9], [12]–[16] use dynamic equations, which are case-specific and inconvenient for industrial applicability. Therefore, this paper aims to describe a workflow on optimizing PTP mechanisms through motion simulations and dedicated software that automates this methodology.

The proposed methodology that uses only CAD software without relying on any analytic derivation to minimize the T_{RMS} is novel within the state-of-the-art because:

- If the design of the mechanism is optimized in literature, the most common objective is to obtain a certain path for the end-effector. Such optimizations only require kinematic modelling [17], [18]. In this paper, on the other hand, we consider energy consumption as an objective. Therefore, dynamic models are required.
- The dynamics of a mechanism are, in the literature, often described using a mathematical equation of motion, with machine-specific parameters (e.g. mass, the center of gravity, external loads,...) [5], [19], [20]. However, when altering the lengths of the bars, the machine-specific parameters will change in a certain way. In literature [9], [12], the relationship between design parameters (such as $|OA|$, $|AB|$, and $|BC|$) and the machine-specific parameters are approximated and simplified to allow for computation-

ally efficient optimization. However, in this paper we avoid such approximations leading to inaccuracies by exactly modelling the mechanism in CAD software. As literature proves [21], the models of machines built by machine designers are very accurate. In that way, using these CAD simulations in an optimization algorithm give trustworthy results.

- Modelling the dynamics of the mechanism is a severe hurdle for machine builders, which makes the methods relying on mathematical equations inconvenient for industrial applicability. Therefore, the proposed methodology relies on CAD motion simulations to improve industrial applicability. Moreover, using CAD motion simulations makes the novel workflow scalable to more complex mechanisms.
- In [22] CAD simulation is included in an optimization loop. However, as [22] focuses on Finite Element Modelling (FEM), only one CAD motion simulation for each optimization routine iteration is required. This paper, however, introduces a novel workflow including multiple consecutive CAD simulations for PTP mechanisms. This approach's necessity is explained in section II.

This paper proposes the methodology to set up multiple motion simulations in a specific order and interchange information to optimize a reciprocating mechanism. Mechanical design of systems is mainly done in Computer-Aided Design (CAD) software. These CAD models include all required information (i.e., volume, mass, friction, damping, joints,...) to model the dynamics of a mechanism. This information is necessary to calculate the required torque of the mechanism through motion simulations. By driving the mechanism with the motion profile $\theta(t)$ at point O (Fig. 1), being the axis driven by a motor, the user can extract the necessary torque from the software (as in Fig. 2) to fulfil the prescribed movement $\delta(t)$ of the end-effector. The objective value is the RMS value of the torque profile $T_m(t)$, which is necessary to drive the mechanism fulfilling an imposed PTP motion ($\delta(t)$).

Hence, by calculating the T_{RMS} based on CAD simulations as elucidated in section II, the objective value for a particular design (i.e., specific values for the three design parameters $|OA|$, $|AB|$, and $|BC|$) is obtained. Be aware that changing the geometric parameters ($|OA|$, $|AB|$, and $|BC|$) influences the start- and end angle of the motor θ as the imposed end-effector movement $\delta(t)$ may not change. Therefore, a kinematic transformation is necessary to derive the motor profile $\theta(t)$ for a specific end-effector movement $\delta(t)$. However, performing a kinematic transformation for a design parameter combination ($|OA|$, $|AB|$, and $|BC|$) that results in an infeasible design is irrelevant. Thus, all designs have to pass a feasibility check before any calculations are performed on the design. After obtaining the motion profile of the motor ($\theta(t)$), for a specific feasible design parameter combination, it can be used as a driver of the mechanism in simulation to perform dynamic analysis and derive the required torque profile. The process described above is a sequence of motion simulations that are

automated to obtain the objective value for different feasible design parameter combinations ($|OA|$, $|AB|$ and $|BC|$). As the objective value, T_{RMS} for a specific mechanism design can be derived, an optimization algorithm can be used, as discussed in section III. This algorithm is necessary to minimize the T_{RMS} and thus optimize the mechanism. After optimization, an optimal combination of the design parameters $|OA|$, $|AB|$, and $|BC|$ is obtained that has a minimal T_{RMS} . Therefore, it consumes a minimal amount of energy, as shown by the results in section IV.

II. CAD MOTION SIMULATIONS

The validation case is clarified to make all the following more tangible. This mechanism, shown in Fig. 3, can ventilate a patient by pressing the indenter into the bag, which causes airflow towards the patient. The movement of the end-effector (indenter) from a starting angle δ_i towards an end angle δ_e is caused by moving point O over $\theta(t)$. In this paper, the machine designer only defined an end-effector (indenter) motion profile $\delta(t)$, resulting in a reciprocal movement between the positions δ_i and δ_e .

Fig. 3 presents the CAD model of the emergency ventilator and illustrates that the red beam, connected with the indenter (i.e., the end-effector), moves by rotating input link OA around point O. This is the point where an electric motor drives the mechanism. The red beam has two predefined angles: an angle δ_e that holds the mechanism in a position where the indenter touches the bag and an angle δ_i that corresponds to a position in which the air is compressed out of the bag. Within these CAD models, the design parameters $|OA|$, $|AB|$, and $|BC|$ of the emergency ventilator can be parameterized to simulate different designs with different corresponding torque profiles, as shown in Fig. 2.

A CAD motion simulation [23] can determine the necessary torque to drive the mechanism at point O only if the required position profile $\theta(t)$, at that point O, is known. However, the user solely defines the required motion profile of the end-effector, in this case, $\delta(t)$. Thus, a machine designer should determine the specific motor angles $\theta(t)$ to move the end-effector through the imposed motion profile $\delta(t)$. It should be noted that the kinematic transformation from $\delta(t)$ to $\theta(t)$ depends on the chosen design parameter combination $|OA|$, $|AB|$, and $|BC|$. Moreover, each evaluated design must be feasible to extract a representative objective value. Therefore, each selected design parameter combination ($|OA|$, $|AB|$ and $|BC|$) is analyzed by a sequence of three motion simulations executed automatically, as indicated in Fig. 4.

A. Motion Simulation 1: Feasibility Check

As a first step within the series of motion simulations, each combination of design parameters ($|OA|$, $|AB|$ and $|BC|$) should be checked on feasibility. As depicted in Fig. 5 (left), a design parameter combination can result in an infeasible mechanism in which the link OA' cannot be connected with link A''B at the highest position of its range of motion. Additionally,

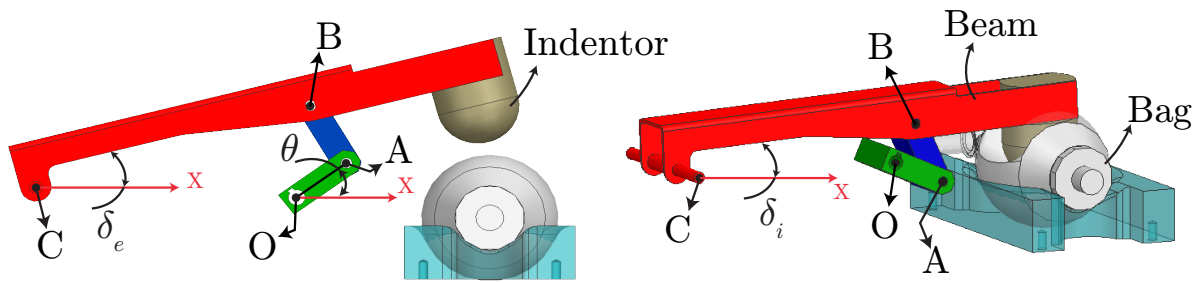


Fig. 3. The end-effector (indenter) requires a movement from δ_i to δ_e , which is performed by moving θ over a design-specific angle.

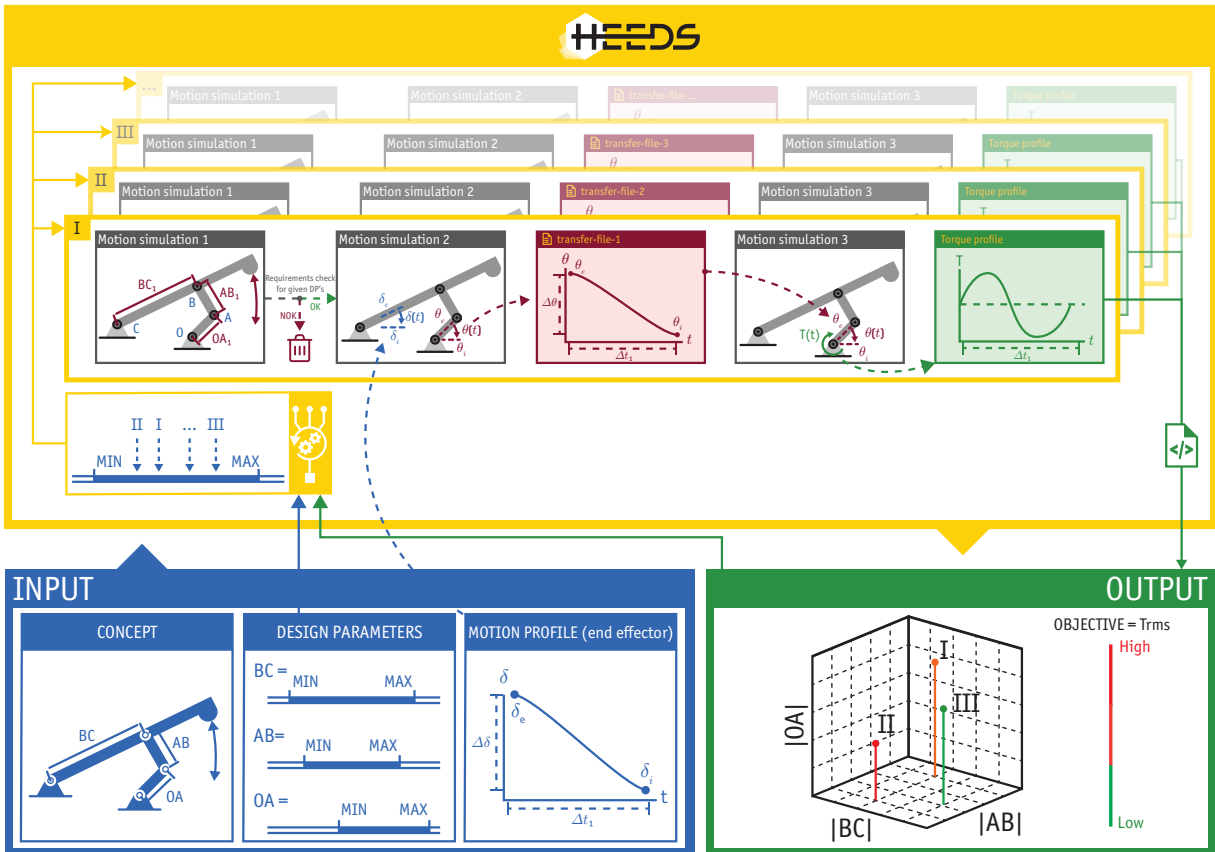


Fig. 4. The workflow for automated extraction of the necessary driving torque for different feasible mechanism designs. One can optimize the mechanism using an algorithm that chooses the design parameter combinations of what the objective value must be determined.

Fig. 5 (right) shows an infeasible design wherein the end-effector cannot reach the lowest desired position. Thus, a first simulation is required to check the assemblability of each new mechanism over the required range of motion.

The first motion simulation drives the CAD model from point C, the end-effector see Fig. 5, over the desired range of motion. The output of this simulation is either false or true, which means that the simulated design is respectively infeasible or feasible. Infeasible design parameter combinations are neglected and neither used in the following simulations, nor by an algorithm to choose a better design, as shown in Fig. 4. A specific range of motion can be a machine designer requirement for the machine, as it is in this case. The indenter has to move

further up, so there is enough clearance to remove or place the bag. As shown in Fig. 6, the range of motion is 2 degrees bigger than the actual $\delta(t)$ movement. However, when the range of motion is not explicitly desired the first simulation is removed, as the second simulation can give the same outcome.

B. Motion Simulation 2: Kinematic Transformation

A feasible design is provided for the second motion simulation. As indicated in Fig. 4, each design needs the complete motor's motion profile $\theta(t)$ to move the end-effector according to the imposed motion profile $\delta(t)$. This step is crucial as each design parameter combination requires another motion profile $\theta(t)$ at the motor (point O) to preserve the same end-

effector movement. It is possible to calculate the motion profile $\theta(t)$ analytically. However, deriving the kinematic equations is complex and can only be used for a particular mechanism. For this reason, machine builders use CAD motion simulation to perform complex calculations instead of manual analysis. The CAD model is driven from the end-effector (point C) with the desired motion profile $\delta(t)$. Because of the kinematic transformation that the CAD software executes on the mechanism, the user can extract the corresponding motor profile $\theta(t)$ for a specific design (Fig. 4). Subsequently, the motor profile is used in the next and last motion simulation.

C. Motion Simulation 3: Dynamic Analysis

As shown in Fig. 4, the last simulation determines the required driving torque of a specific design. The design that just passed through the previous simulation and the design-specific motor motion profile is provided in this third simulation. The CAD model drives the mechanism, as in real-life, from point O. As a consequence of the dynamic analysis the CAD software performs during such a simulation, one can extract the required torque the motor should provide to drive a specific design of the mechanism as desired. The T_{RMS} objective value for each design can be calculated based on the design-specific torque profile.

The complete sequential process with the three motion simulations makes it possible to extract the objective value for every feasible design. However, this workflow has to run automated to optimize a mechanism through algorithms. Therefore, the present paper utilizes HEEDS MDO [10] as commercial software to automate this workflow and has the most common optimizers integrated. As explained in section III, numerical optimization algorithms are used to optimize the mechanism.

III. DESIGN OPTIMIZATION

An optimization algorithm uses the obtained objective value for a specific design of the mechanism to create a new design parameter combination, possibly improving the objective value and getting closer to the optimal design with a minimal T_{RMS} . Yet, notice that a design with a lower T_{RMS} requires a higher maximal speed of the motor, as shown in Fig. 7. However, the increased motor speed stays within a realistic range, thus giving it no further focus.

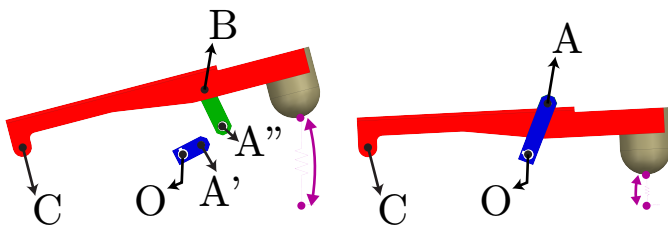


Fig. 5. On the left, an emergency ventilator design that cannot be assembled in the highest desired position of the complete range of motion. Another combination of design parameters results in a mechanism that cannot complete the desired range of motion, on the right.

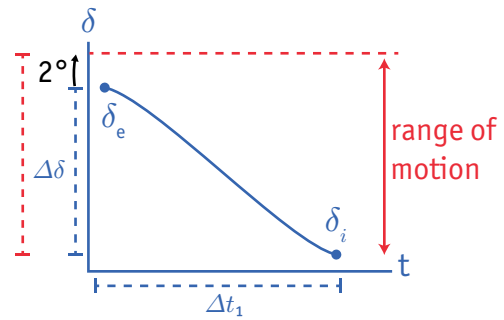


Fig. 6. The machine designer requires a difference between the range of motion and the movement $\delta(t)$ to improve the ease of use.

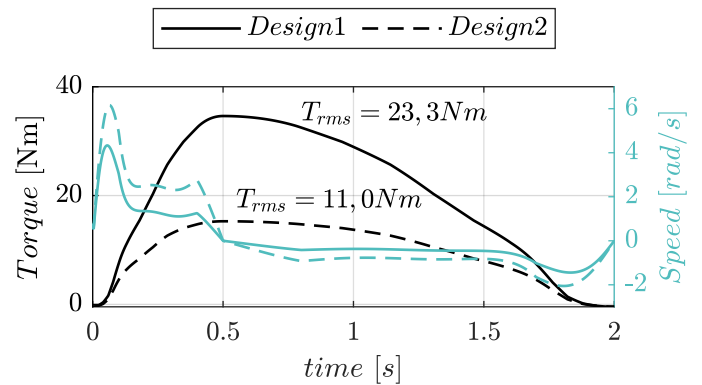


Fig. 7. A design with a lower T_{RMS} demands a higher maximal speed from the motor.

The optimization of the mechanism is an iterative method. Therefore, automating the sequence of motion simulations is a crucial step. The two most commonly used algorithms in design optimization [14] are Sequential Quadratic Programming (SQP) and the Genetic Algorithm (GA). Therefore, a comparative study is conducted between these two algorithms.

- Genetic algorithm:** A GA is an adaptive heuristic search method based on the evolution of genetics [24]. The algorithm starts with a population of 20 random designs (i.e. a generation), within our design optimization problem, the population are sets of design variables ($|OA|$, $|AB|$, and $|BC|$) randomly generated within the search space. Each individual design of the population is evaluated, through the sequence of motion simulations, resulting in an objective value as described in section II. Now the second generation has to evolve into a population where individuals have an improved objective value compared to the previous generation. This is what the algorithm strives for based on evolutionary principles. Therefore the GA uses three principles. The first principle is "selection", which transfers the best individuals from the previous generation to the following. This paper transfers the best 20% of each generation to the next generation. Secondly, the crossover principle is based on mixing two designs within the previous generation to create a descendant. At

last, the mutation principle will complete the population of the new generation by performing random adaptations (or so-called mutations) to individual design parameters [25], [26]. In this paper, each following generation is created by applying mutation on 40% of the previous generation and applying crossover on the other 40%.

- **Sequential quadratic programming:** The basic idea of Sequential Quadratic Programming is establishing an iterative procedure where the gradient of the quadratic model on the objective function in every design parameter combination leads the search for the optimal solution. For each design, the objective function decreases fastest if the following design goes in the direction of the negative gradient of the objective function. The literature [27] describes this algorithm as one of the most successful in solving nonlinear constrained optimization problems.

Thus, adopting the algorithms above on our mechanism will drive the process towards an optimal design parameter combination ($|OA|$, $|AB|$, and $|BC|$) for the emergency ventilator. The algorithm's number of evaluations must be limited as the sequence of motion simulations can be very time-consuming. However, taking the number of evaluations too low can lead to a poor result of the algorithm. Therefore, the algorithm can only stop when the objective value T_{RMS} converges to a minimum. However, the reached minimum does not guarantee that no better solution exists. This is a consequence of having an objective function with local minima, as in this case (see Table I). On the one hand, using the Genetic Algorithm reduces the risk of getting stuck in local minima, yet it cannot be assured [4], [25]. On the other hand, the result of the SQP algorithm is strongly influenced by the selected starting point of the algorithm. The algorithm starts from the given start point and follows the steepest negative gradient towards a minimum. This working principle indicates that the algorithm converges faster. However, it causes a great chance of getting stuck in a local optimum.

IV. RESULTS

The method described in section II is employed on the emergency ventilator, which optimizes the mechanism by using an algorithm that searches for a new design parameter combination ($|OA|$, $|AB|$ and $|BC|$), as described in section III. The first algorithm used in the methodology is the Genetic Algorithm, which found an optimal solution after 399 design evaluations, as indicated in Fig. 8. Within these evaluations, the algorithm chose some feasible and other infeasible designs. Each feasible design requires, on average, 1 minute and 25 seconds, while an infeasible design only takes 21 seconds. The time difference is a consequence of the methodology in which infeasible designs are detected in the first simulation and not passed on to the following simulation, as explained in section II. The Genetic Algorithm evaluates 272 feasible and 127 infeasible designs, giving a total of 399 designs (see Table I), to search for the optimal design requiring a calculation time of 5 hours and 40 minutes.

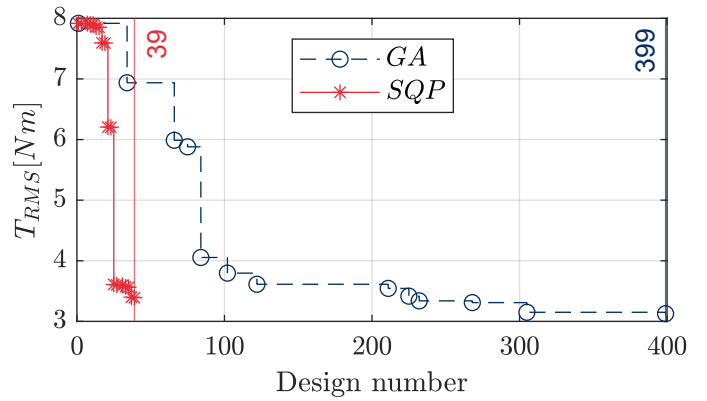


Fig. 8. Both algorithms require a different number of design evaluations to reach a minimal T_{RMS} value. Moreover, the optimal objective value slightly differs for both algorithms.

By contrast, the mechanism's optimal design through a gradient-based algorithm is found through a reasonable number of 39 feasible design evaluations, requiring 42 minutes of simulation time. It can be noticed that all designs chosen by the SQP algorithm are feasible designs, which is a consequence of our objective and the search method of the algorithm. The algorithm starts in a feasible design parameter combination ($|OA|$, $|AB|$ and $|BC|$) and chooses the following design, with an increment, in the direction of the steepest negative gradient of the objective function.

TABLE I
SAVING POTENTIAL ACHIEVED BY DESIGN OPTIMIZATION WITH STATE-OF-THE-ART OPTIMIZATION ALGORITHMS.

Design	$ OA $ [mm]	$ AB $ [mm]	$ BC $ [mm]	T_{rms} [Nm]	T_{max} [Nm]	T_{rms}	T_{max}	Number of evaluations
						savings	savings	
						[%]	[%]	
Original	53	65	282	7.91	13.26	-	-	-
GA	82.68	141.25	281.8	3.13	5.43	60.5	59.1	399
SQP	30	76.22	271.75	3.39	5.16	57.2	61.1	39

However, both algorithms lead to different optimal design suggestions, which are better than the original design suggested by a machine builder. The optimal objective value found with the SQP algorithm is slightly higher than the solution obtained with GA, as shown in Table I. Yet, the contrast between the two algorithms is noticeable in the required number of design evaluations to find an optimal design. In summary, Table I shows that GA could reduce the T_{RMS} by 60.5%, while SQP diminished the objective value by 57.2%. The optimal design found through a gradient-based method is quick, yet the obtained T_{RMS} value strongly depends on the combination of $|OA|$, $|AB|$, and $|BC|$, in which the algorithm starts searching (i.e. starting point). This reveals that by using the SQP algorithm, a risk is taken of having a sub-optimal design, which is a local optimum. In addition, both algorithms were able to lower the maximal torque the motor should deliver during the mechanism's movement, which means that the mechanism can operate with a smaller, and thus cheaper motor.

V. CONCLUSION

This study proposes a convenient and broad industrial applicable design optimization approach, in which CAD models are used as a basis. The workflow requires multiple motion simulations of the CAD model to extract the necessary torque to drive the designed mechanism. The methodology described in this paper does not demand any theoretical analysis of the mechanism, making it scalable and applicable to any complex PTP mechanism in the industry. The first motion simulation checks the feasibility of the proposed design parameter combination ($|OA|$, $|AB|$, and $|BC|$). Then only the feasible designs are passed to the second simulation, which performs the kinematic transformation to derive the design-specific motor profile for an imposed end-effector movement. At last, the derived motor profile is used in the last motion simulation to perform dynamic analysis and extract the necessary torque to drive the mechanism with the chosen design. The optimization algorithm uses the obtained objective value (T_{RMS}) from the torque profile to select an improved design evaluated by the sequence of motion simulations.

The results clearly show that the proposed method outperforms the arbitrary designs chosen by the machine builder and reveals an energy-saving potential of up to 60.5%. Moreover, the choice of an algorithm significantly influences the number of designs evaluated to find an optimal design for the mechanism. The gradient-based algorithm converged after 39 design evaluations, which benefits the method's applicability.

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