Abstract—This paper proposes a novel actuator that can generate desired tension by torque control to provide the necessary load for physical exercise. In order to reduce the required torque level of the motor, a spring is incorporated in this actuator design such that the torque (and thus tension) can be generated not only by the motor but also from the spring. The proposed mechanism is a type of Parallel Elastic Actuator (PEA), which consists of the motor, the spring, and the reduction gear. Dynamic model and analysis of PEA are developed in this paper to properly understand the working principle of PEA. The precise analysis of the proposed mechanism is conducted based on the developed model. The optimal characteristic of the spring and the gear ratio are determined to generate large torque while utilizing motor torque efficiently. In addition to this optimal design, the force controller is developed for the proposed actuator module, considering the dynamic characteristic of PEA. Finally, the performance of the proposed PEA and controller is verified through experiments.

I. INTRODUCTION

A. Background and Motivation

Actuators that can generate and control the desired torque have been developed aiming at various human-machine interactions [1]. Among these human-machine interaction systems, the home-based exercise has been receiving a lot of attention. Home exercise is more efficient than going to the gym in terms of time and money [2], [3]. Accordingly, exercise equipment is being developed utilizing torque/force-controlled actuators, which can be used for home training or exercise in non-gravity conditions for astronauts because they can provide various exercise loads.

Passive elastic equipment such as rubber bands is widely utilized thanks to its advantage in terms of price and size. However, the ability of this passive exercise equipment is limited in exerting large force, and it does not provide constant force. Therefore, motorized exercise equipment has been developed to overcome this issue. A limitation of this motorized exercise equipment is that the system tends to become large as the required exercise load level is high [4]–[6]. Meanwhile, the flywheel exercise device (FWED) was proposed [7] as a device that provides resistance, through the inertia of a spinning flywheel. Another advanced resistive exercise device (ARED) was proposed by NASA [8] which uses vacuum cylinders to provide resistance. These devices attempted to provide exercise load large enough while keeping the size as compact as possible.

This research proposes a novel actuator module that can generate force/torque large enough for physical exercise utilizing the electric motor. A Parallel Elastic Actuator (PEA) structure is employed for this actuator module such that the spring force/torque is also utilized to enhance the output force/torque while reducing the required motor torque. Moreover, the motor torque can be controlled in real-time to provide the various levels and types of exercise load. The PEA can compensate for insufficient motor torque by connecting elastic components in parallel. Because the parallel spring can provide part of the torque required to perform the task, lowering the required motor torques. The reduction of motor torques also allows for a smaller motor and gear ratio. Due to these reasons, it can increase the force that the equipment can produce while reducing the size and weight of the entire equipment [9], [10].

The wire-driven mechanism is also widely utilized for exercise equipment thanks to its advantage such as simplicity and variability. Since the wire-driven mechanism requires a small winch mechanism composed of a wire drum [11], it can be manufactured with low cost and low inertia of the system [12], [13]. In exercise application, wire-driven equipment is capable of various exercises, e.g, squat, bench press, deadlift, and so on. In most cases of wire-driven exercise systems, the load is generated by the weights to realize the isotonic exercise. Hence, it is necessary to prepare various types of weights to provide many weight levels for many users. Motorization can change this issue since it can provide the required load by employing force control.

In order to combine the advantages of the wire-driven system and motorization and also to keep the required motor torque small, PEA is designed and controlled in this research to provide this variable exercise load through tension control.

B. Contributions

The main purpose of this study is to design a PEA with optimization of mechanical parameters and controller design based on the dynamic analysis. As explained above, the motor torque and spring torque are combined in PEA, which can be exploited to reduce the required motor torque. In addition to this advantage, the optimization design method is derived in this paper to determine the optimal gear ratio and the optimal spring shape for the compact and lightweight design.
To design PEA, it is necessary to properly select a motor, a gear, and an elastic component. Since the target application is the small size wire-driven exercise equipment, high torque capacity is required for the actuator, while low stiffness is required for performance at the same time. Thus, high torque capacity with low stiffness is essential for the spring design. However, it is difficult to design a spring that can overcome the limitations in lowering stiffness while increasing torque capacity [14]. The spiral spring is adopted in the proposed PEA design since the spiral spring is known to have high torque capacity with large deformation compared with the helical torsion spring [15]. Moreover, the mechanical properties of the spring are optimally designed in this research to realize high torque capacity while keeping its volume as small as possible.

The selection of the gear also plays an important role in reducing the required motor torque while satisfying the required torque and velocity. The stiffness and the gear ratio are usually determined to minimize the peak power or energy consumption. Verstraten et al. designed elastic elements with the natural frequency to minimize the peak power and energy consumption [16]. Hollander et al. also designed elastic elements to minimize the peak power [17]. Edgar et al. designed the elastic elements to simultaneously minimize the peak power and energy consumption [18]. The optimal selection of the gear ratio in this paper is determined to minimize the input motor torque, taking into consideration the repetitive load pattern required for the physical exercise.

Lastly, this paper investigates the dynamic characteristic of PEA, and the force/tension control algorithm is designed taking into consideration the dynamic analysis. Since the motor and spring are connected in parallel in PEA, the spring connected in parallel must be considered in order to generate the desired torque. Plooij et al. and Häufle et al. proposed a clutch mechanism [9], [19] to control the engagement of the spring torque. However, this approach is suitable for applications that require instantaneous power as the trajectory is repeated, such as the exoskeleton. For physical exercise, this approach may not be suitable because it has to continuously generate large force. Also, the electronic clutch with high permissible torque is very large and heavy. Toxiri et al. used a torque sensor [10] to directly measure the total torque output by the motor and the spring and feedback it. This approach may achieve accurate torque control, but it does not fully exploit the advantage of PEA. The force/torque output of PEA is discussed in this paper, and a control algorithm is proposed to control the output force/torque of PEA.

In summary, the main contributions of this study are to design a compact PEA by optimizing the reduction gear ratio and the spiral spring and conducting dynamic analysis of PEA to understand and design high-performance force control. This paper is organized as follows. The dynamic characteristic of PEA is analyzed and then a PEA is designed in Sec. II with the optimal selection of the gear ratio and the optimal design process of the spring. Actual design is also discussed in Sec. III. Finally, the controller design and its performance verification are presented in Sec. IV. Sec. V concludes the research.

II. DYNAMIC ANALYSIS AND OPTIMIZATION OF PEA

A. PEA Developed for Physical Exercise

The ultimate goal of this study is to develop an actuator to provide various loads for physical exercise through the wire-driven mechanism. Fig. 1 illustrates the exercise procedure using the proposed PEA, where the user exercises by repetitively pulling and releasing the wire that is connected to the proposed PEA. By applying this operation principle, various exercises are available such as arm curl, seated row, and so on. To allow for various exercises and exercise postures, adjusting the point from which the spring deformation starts. To cope with this issue, a clutch mechanism is incorporated in the proposed actuator module such that the user can engage or disengage the force from the spring according to his/her preferred starting position. The slot and saw-tooth structure are adopted to engage/disengage the wire drum from the shaft. In addition to this, an auxiliary spring is connected to the wire drum such that it can wind the wire when the user releases the wire even when the main spring is disengaged. The overall design of the actuator considering these points is illustrated in Fig. 2a. The size of the entire actuator module is as follows: diameter and length are 120 mm and 150 mm, respectively, and the weight is 2.8 kg. The information on the components utilized in the proposed PEA module is shown in Table I.

B. Consideration for High Performance PEA

However, the parallel mechanism can cause undesired coupling, and the range of motion (ROM) of the body is limited by the spring, which can be considered a disadvantage of PEA. This is why many previous PEA designs have employed the clutch mechanism; the spring needs to be detached from the main body to extend the ROM and remove the undesired coupling. Nevertheless, PEA has the advantage particularly when the
desired ROM is limited and the desired motions are repetitive, e.g., suspension of vehicles and physical exercise.

To fully take advantage of PEA, several technical issues are to be addressed. Dynamic modeling of PEA plays a significant role in addressing these issues. The PEA needs to be designed in an optimal way taking into consideration its dynamic characteristics. The spring characteristics and the motor specification as well as the gear ratio can be optimally determined when its dynamic characteristics and required motion/force are known. As the ROM is limited in PEA, the gear ratio can be optimally selected in such a way the required motor torque and power are minimized.

Moreover, the coupling of PEA can be efficiently utilized when the dynamics are properly understood. Detailed analysis of PEA dynamics can clarify how the coupling affects the PEA output force. The control algorithm can be designed to achieve high performance and effective force control of PEA based on the dynamic model.

This paper proposes solutions to these issues by presenting the dynamic analysis of PEA and the optimal design process of PEA as well as control algorithm design based on the dynamic model.

C. Dynamic Model of PEA

Fig. 2b represents the schematic diagram of the proposed PEA. Since the PEA consists of a motor and a spring that is connected in parallel, the equation of the motor angle motion is defined as follows.

\[ \tau_m = J\ddot{\theta}_m + B\dot{\theta}_m + KN^{-2}\theta_m + N^{-1}\tau_{out}, \]  (1)

where \( J \) is the sum of the motor inertia, the gear inertia and the wire drum inertia, \( B \) is the damping coefficient, \( K \) is the stiffness of the spring, and \( N \) is the gear ratio. \( \theta_m \) is the angular position of the motor, while \( \tau_{out} \) is the output torque to the wire drum. As a reduction gear is employed, the relationship between the motor angular position and the wire drum angular position \( \theta_i \) is given as follows.

\[ \theta_m = N\theta_i \]  (2)

Based on this, the dynamics of (1) can be rewritten in terms of the wire drum motion as follows.

\[ \tau_m = JN\ddot{\theta}_i + BN\dot{\theta}_i + KN^{-1}\theta_i + N^{-1}\tau_{out} \]  (3)

D. Optimal Selection of Gear Ratio

Taking into consideration the derived dynamics, the gear ratio can be optimally selected. To this end, the pattern of the load angular position and the load torque trajectory is assumed to be given. This assumption is reasonable enough considering the point that there is a pattern in the exercise motion. A sinusoidal pattern is adopted for the exercise motion pattern and the load torque trajectories as the exercise motion is mostly periodic.

The objective of the optimization is to minimize the required motor torque, and thus we define the objective function and constraints as follows.

\[
\begin{align*}
\text{minimize} & \quad \tau_m(N) = JN\ddot{\theta}_i + BN\dot{\theta}_i + KN^{-1}\theta_i + N^{-1}\tau_{out} \\
\text{subject to} & \quad \theta_i(t) = \theta_{d_{\text{max}}} \sin(\omega t), \\
& \quad \tau_{out}(t) = \tau_{d_{\text{max}}} \sin(\omega t),
\end{align*}
\]

where \( \tau_{d_{\text{max}}} \) is the desired maximum exercise load, \( \theta_{d_{\text{max}}} \) is the desired maximum exercise motion amplitude, and \( \omega \) is the frequency of the exercise motion.

The exercise motion \( \theta_i(t) \) and the required exercise load \( \tau_{out}(t) \) are assumed to be given as follows.

\[
\begin{align*}
\theta_i(t) &= \theta_{d_{\text{max}}} \sin(\omega t) \quad (4) \\
\tau_{out}(t) &= \tau_{d_{\text{max}}} \sin(\omega t) \quad (5)
\end{align*}
\]

Then, substituting (4), (5) into (3), the peak time \( t_m \) at which the motor torque to realize the required motion and load becomes the maximum is obtained by calculating the point where \( \partial\tau_m/\partial t = 0 \).

\[
\sqrt{\alpha^2 + \beta^2} \sin(\omega t + \phi) = 0 \quad (6)
\]

\[
t_m = (\pi - \phi)/\omega, \quad (7)
\]

where \( \alpha, \beta \) and \( \phi \) represent as follows.

\[
\begin{align*}
\alpha &= -BN\theta_{d_{\text{max}}} \omega^2 \\
\beta &= -JN\theta_{d_{\text{max}}} \omega^3 + (K\theta_{d_{\text{max}}} + \tau_{d_{\text{max}}})N^{-1}\omega \\
\tan(\phi) &= \beta/\alpha
\end{align*}
\]

Then, the maximum motor torque at \( t_m \) is derived as follows.

\[
\tau_{m_{\text{max}}} = \sqrt{\gamma N^2 + \delta N^{-2} - \varepsilon}, \quad (8)
\]
where \( \gamma, \delta \) and \( \varepsilon \) represent as follows.

\[
\gamma = (J\theta_{\text{max}}^2 \omega^2 + (B\theta_{\text{max}} \omega)^2) \\
\delta = (K\theta_{\text{max}}^2 + \tau_{\text{max}}^2)^2 \\
\varepsilon = 2J\theta_{\text{max}}(K\theta_{\text{max}} + \tau_{\text{max}})\omega^2
\]

Finally the optimal gear ratio \( N_{\text{opt}} \) which minimizes the maximum required motor torque can be obtained using the partial derivative \( \partial \theta_{\text{max}} / \partial N = 0 \).

\[
N_{\text{opt}} = \sqrt{\delta / \gamma}
\]  

Notice that the same optimization process can be applied even when there is a phase difference between the exercise motion and the required load pattern.

### E. Analysis of Spiral Spring

The volume of the spring also needs to be minimized to design the PEA as small as possible, and the optimization of the spring shape can be conducted utilizing the property of the spiral spring. At first, the stiffness \( K_s \) of the spiral spring is determined as the division of the torque and the deformation. The deformation of the spiral spring is approximated using the bending stress model, which can be computed by the following formulas [3].

\[
K_s = \frac{EbT^3}{12L}, \\
S = \frac{6\tau}{bT^2},
\]

where \( K_s \) is the stiffness, \( S \) is the stress, \( E \) is Young’s modulus, \( b \) is the width of spring strip, \( T \) is the thickness of a spiral spring, \( L \) is the length of the strip, and \( \tau \) is the moment applied on the spring. Therefore, the maximum torque and deformation of the spiral spring are determined when the material and spring parameters in (10) and (11) are determined.

Meanwhile, the maximum compression angle of the spiral spring is also determined kinematically as follows.

\[
\theta_{\text{max}} = \frac{\pi(\sqrt{A^2 + 1.27LT} - A)}{T} - \frac{4L}{OD + A},
\]

where \( A \) is the arbor diameter, \( OD \) is the outer diameter. If the deformation exceeds this maximum compression angle, non-linearity occurs due to friction. Therefore, maximum deformation should satisfy the following inequality.

\[
\theta_{\text{max}} \leq \frac{\tau_{\text{max}}}{K_s} 
\]

### F. Optimal Design of Spiral Spring

As the objective of the spring shape optimization is to minimize the spring volume for compact actuator design, we define the objective function and the constraints using (10) and (11) as follow.

\[
\begin{align*}
\text{minimize} & \quad V_s(T) = \pi \{OD(T)/2\}^2 b(T) \\
\text{subject to} & \quad S \geq \frac{6\tau_{\text{max}}}{bT^2}, \\
& \quad \frac{E\theta_{\text{max}}}{2S} - T, \\
& \quad OD \geq \frac{4L}{\pi(\sqrt{A^2 + 1.27LT} - A)/T - \theta_{\text{max}} - A}.
\end{align*}
\]

The relationships between the spring properties \( b, L, OD \), and \( T \) are obtained from these constraints as follows.

\[
\begin{align*}
b & \geq \frac{6\tau_{\text{max}}}{ST^2}, \\
L & \geq \frac{E\theta_{\text{max}}}{2S} T, \\
OD & \geq \frac{4L}{\pi(\sqrt{A^2 + 1.27LT} - A)/T - \theta_{\text{max}} - A}.
\end{align*}
\]

Notice that all the properties of the spring, the width \( b \), the length \( L \), and the outer diameter \( OD \) are the functions of the thickness \( T \). In other words, the minimum \( b(T) \) and \( OD(T) \) can be obtained as follows, which can minimize the spring volume when the thickness \( T \) is given.

\[
\begin{align*}
b_{\text{min}}(T) & = \frac{6\tau_{\text{max}}}{ST^2}, \\
OD_{\text{min}}(T) & = \frac{4E\theta_{\text{max}}T^2}{\pi(\sqrt{4S^2A^2 + 1.27E\theta_{\text{max}}^2T^2} - 2S(\pi A - \theta_{\text{max}}T))} - A.
\end{align*}
\]

Notice that the minimum values of \( b_{\text{min}} \) and \( OD_{\text{min}} \) decrease as \( T \) increases. This relationship reveals that the volume of the spring \( V_s(T) \) decreases as \( T \) increases, and there is no the optimal thickness which satisfies \( \partial V_s(T) / \partial T = 0 \). The thickness \( T \), however, is usually determined based on the practical constraint such as manufacturing constraints.

In summary, the proposed optimal design process of PEA can be illustrated as in Fig. 3.

### III. DEVELOPMENT OF PEA WITH OPTIMAL GEAR RATIO AND SPRING PARAMETER

#### A. Optimal Design of Spring

For the proposed PEA design, the maximum exercise load and range of motion are set to 10 Nm and 12.57 rad, respec-
IV. Torque Controller Design and Verification

A. PEA Output Torque Controller

One of the main purposes of our research is to generate the desired exercise load. To this end, a feedback force controller is designed to control the exercise load the user feels. The block diagram of the proposed force controller is illustrated in Fig. 6. In the proposed controller, the estimated output torque of the PEA is defined as (19) based on the PEA dynamic model in (1) and Fig. 2b.

\[
\hat{\tau}_{out} = N\tau_m - KN^{-1}\theta_m
\]  

(19)

Proportional-Integral (PI) controller is designed using this output force feedback to realize the desired exercise load \(\tau_d\).

\[
\tau_m^{PI} = K_p(\tau_d - \hat{\tau}_{out}) + K_i \int_0^{\tau_d}(\tau_d - \hat{\tau}_{out})dt,
\]  

(20)

where \(K_p\) and \(K_i\) are the P gain and the I gain, respectively.

B. Force Control Verification

The force control performance confirmed that the sum of elastic and actuator torque follows desired torque well. In other words, the goals of the experiment are, 1) to verify whether the spring can assist the insufficient torque of the motor and 2) to verify that the output torque follows the desired torque well.

In the first experiment, the desired torque is set constant, and the user conducts the exercise by pulling and releasing the wire. The actual output torque is measured by the load cell, and the spring torque and the motor torque during the exercise are plotted in Fig. 7. Notice that the torque output estimated as in (19) shows similar behavior as the desired constant torque. The difference between the two can be attributed to the feedback control error and the ignored dynamic characteristic of the inertia and the damping. Another interesting point in this
experiment is that insufficient motor torque is compensated for by the spring force. Notice that the desired torque is realized by the summation of the motor torque and the spring torque.

In the second experiment, it is shown that the output torque of the proposed PEA can be well-controlled by the feedback of the estimated output torque. In the experiment, the user pulls the wire such that the wire drum angle varies from 1 rad to 4 rad and maintains the angle to verify the static characteristic. The investigate the performance of the output torque feedback controller, the PI controller is implemented and the results are analyzed. The desired torque level is set to 1 Nm, and the actual output torque is measured by the load cell and compared with the desired torque level. The experimental results in Fig. 8 verify that the output torque of the proposed PEA (the red dotted line) can be controlled using the estimation feedback control even with different spring deformation level. The results show that the feedback of the output torque estimation performs as expected with PI controllers. However, since dynamics characteristics such as inertia and damping term are not considered when configuring the controller, overshoot occurs significantly in the section where the angle changes.

V. CONCLUSION

This paper proposed a novel parallel elastic actuator that can generate force for physical exercise. The PEA mechanism can compensate for insufficient motor torque by connecting the spring in parallel. This characteristic allows a smaller motor to be used, resulting in a reduction in the size and weight of the actuator. Although a PEA has the disadvantage that ROM is limited, it works as an advantage in physical exercises that require limited ROM and repeated motions. Therefore, it is suitable for exercise applications.

To maximize these strengths and achieve research goals, optimization methodology and controller design based on dynamics were proposed. The optimization contributes to designing a compact and lightweight design actuator, and the controller design contributes to generating the desired exercise load for the PEA to function as exercise equipment.

REFERENCES