Abstract—This paper addresses the quasi linear parameter-varying (Quasi-LPV) Luenberger Interconnected Fuzzy (QLIF) observer to estimate simultaneously both longitudinal and lateral vehicle dynamics. In a different manner from the commonplace state-of-the-art of vehicle state observers that consider single driving motion, the proposed approach considers the coupled dynamics with tire-ground interaction to estimate the most important states while reducing the complexity related to the observability and conservatism. This consideration leads to a nonlinear parameter-dependent interconnected model with unmeasured premise variables. Thereafter, the Takagi-Sugeno (TS) fuzzy form is considered to deal with the nonlinearities of the vehicle forward speed and the slip velocity at the front and rear tires. The concept of “Input to State Stability (ISS)” is exploited using Lyapunov stability arguments to guarantee the boundlessness of the estimation errors. Finally, the performances and effectiveness of the suggested approach are evaluated through hardware experiments performed with the well-known SHERPA car simulator under real-world driving situations.

I. INTRODUCTION

The involvement of autonomous ground vehicles in the daily life transportation has attracted the attention of industrial and research laboratories to face the new arising challenges. It is well recognized in the automotive research community that the knowledge of the real-time pertinent vehicles states can be extremely valuable for safety applications [1]. This topic is one of our research interest which intends to develop driver assistance systems starting from a minimum set of vehicle self-integrated sensors [2]. Recently, many studies on the estimation of the tire forces have been introduced to improve the vehicle performance in safety and comfort terms. A sliding mode approach is used in [3] for an adaptive estimation of the vehicle-road interaction. In [4], an adaptive neural network nonlinear observer is proposed to estimate the vehicle longitudinal forces. Various versions of unknown input observers were studied to reconstruct the vehicle lateral dynamics such as proportional-integral observers in [5] and sliding mode observers in [6]. In [7], [8], the simultaneous estimation of lateral dynamics and driver's torque is proposed using either LPV techniques or the Takagi-Sugeno transformation. [9] deals with both a cooperative shift control of drive motor together with dynamic torque observer of a parallel hybrid electric vehicle. In [10], [11], non-linear unknown input observers are applied to lateral dynamics on banked or slop roads. In [12], adaptive observer was presented as a convenient approach to deal with both dynamics states and parameters estimation. Further, in [13], a double closed-loop cascade control architecture for a brake system is proposed with an experimental validation demonstrating excellent tracking performance and robustness. Almost references, the estimation of the vehicle dynamics is done by considering restrictive assumptions as the independent behavior and, tire-road contact has often been neglected. Also, the motion coupling of the lateral and longitudinal dynamics of land vehicle have not received much attention in the literature. In this scope, a Quasi-LPV Luenberger Interconnected Fuzzy Observer (QLIF) for coupled nonlinear vehicle dynamics is synthesized. This interconnection scheme overcomes the mutual dependence problem and reduces the conservatism. The main contributions are summarized in the following items:

- Reconsider the Luenberger observer synthesis for an interconnected system where the varying vehicle forward speed and the slip velocity at the front and rear tires are considered immeasurable.
- The influence of these immeasurable premise variables on the state estimation error is minimized according to the ISS concept using Lyapunov theory.
- The QLIF observer structure aims to reduce also the conservatism problem related to the number of immeasurable varying parameters, which reduces the number of linear matrix inequalities (LMIs).
- Observer effectiveness is highlighted using SHERPA car simulator under ideal conditions, then, robustness test is performed with respect to the measurement noises and parameters uncertainties.

The document is organized as follow: Sec II deals with the lateral and longitudinal models with tire-ground forces dynamics; while the Sec III presents the TS fuzzy form. Sec IV presents the design of QLIF observer. Sec V is devoted to the simulations and analysis of the results. Finally, the conclusion is given in the last section.

II. COMPLETE INTERCONNECTED VEHICLE DYNAMICS

Ground vehicles are complex systems made up of several interconnected subsystems such as braking, suspension, steer-
ing, etc. Therein, we are concerned by a nonlinear coupling vehicle model with 9-DoF (nine degrees of freedom), which includes longitudinal, lateral, yaw motion, wheels rotational movements and the tire forces dynamics.

**A. Vehicle Lateral Dynamics Description**

The vehicle lateral dynamics is represented as a bicycle model Fig. 1, which have only planar motion parallel to the road’s surface [1]. We assume that the vertical, pitch, and roll dynamics are neglected. The body-fixed yaw motion $r = \psi$ and the lateral displacement $y$, with the following equations

$$
\begin{align*}
\dot{m}v_y &= 2(F_{yf} + F_{yr}) - mv_yr \\
I_z\dot{\psi} &= 2(l_fF_{yf} - l_yF_{yr})
\end{align*}
$$

(1)

Fig. 1: Bicycle Model.

**B. Mathematical Model of Longitudinal Vehicle Dynamics**

To consider the vehicle longitudinal dynamics, the traction and braking motion are modeled under the wheel’s rotational motion $\omega_f$, and added to the vehicle lateral one.

$$
\begin{align*}
\dot{m}v_x &= 2(F_{xf} + F_{xr}) - F_x + mv_yr \\
i_f\dot{\omega}_f &= -2RF_xf + T_f + B_f \\
i_f\dot{\omega}_r &= -2RF_xr + B_r
\end{align*}
$$

(2)

- $F_x = C_Dv_x^2$: aerodynamic force and $C_D$: drag coefficient.
- $B_f, B_r$: braking torques applied to the front and rear tires, $T_f$ is the engine torque applied on front wheels.

**C. Tire Forces Dynamics**

The Pacejka model is the most widespread used for modeling tires forces including tire saturation [1]

$$
F_i(v) = D_i\sin(C_itan^{-1}(B_i(1-E_i)v_i + E_i\tan^{-1}(B_iv_i)))
$$

(3)

- $i = \{r, f\}$ denotes rear and front of the vehicle;
- $D_i, C_i, B_i$ and $E_i$: pneumatic intrinsic characteristics.
- $v$ is a generic variable which corresponds to the side-slip angle $\alpha$ or the longitudinal slip ratio $\lambda$.

$$
\alpha_f = \delta - \frac{v_y + l_f r}{v_x} \quad \text{and} \quad \alpha_r = -\frac{v_y - l_f r}{v_x}
$$

$$
\lambda_f = \frac{(R\omega_r - v_x)}{\max(R\omega_f, v_x)} \quad \text{and} \quad \lambda_r = \frac{(R\omega_f - v_x)}{\max(R\omega_r, v_x)}
$$

(4)

In order to quantify the proportion of the sliding to the wheel rolling movement and to switch between braking and traction motions, the nonlinear slip velocity (denoted $\rho$) with $i \in \{r, f\}$ is considered as a varying parameter and the switching signal is expressed as

$$
\rho(t) = \begin{cases} 
\rho_i = \frac{1}{\omega_R} \quad \lambda_i > 0 & \text{if Traction: } v_x < \omega_R \\
\rho_i = \frac{1}{\omega_R} \quad \lambda_i < 0 & \text{if Braking: } v_x > \omega_R 
\end{cases}
$$

(5)

Then, $\lambda_i = (R\omega_i - v_x)\rho_i$, $i \in \{r, f\}$. Consequently, we consider that the $\rho_i$ are assumed to be unknown but bounded with a prior known bounds. The previous Pacejka formula or its corresponding linear form describes only the static behavior of the pneumatic efforts. However, due to its elastic deformation, a transient behavior occurs. Almost literature includes a first order low-pass filter to the model, known as tire relaxation $\sigma$

$$
\begin{align*}
\sigma & = -C_\alpha \alpha + F_y^0 \\
\sigma & = -C_\lambda \lambda + F_x^0
\end{align*}
$$

(6)

$F_x^0, F_y^0$: steady-state value of the longitudinal and lateral forces obtained by applying the magic formula (3) or the linear form.

**III. LPV REPRESENTATION OF VEHICLE SYSTEM**

In this work, the state models of the lateral and the rectilinear motions will be considered variable. Thereafter, the systems (1), (6) and (2) are transformed into a Quasi-LPV interconnected system modelled by the following two sub-state models

$$
\begin{align*}
\sum_1: & \quad \dot{\xi}_1 = \tilde{A}(\xi_1(t))\xi_1(t) + \tilde{B}\delta_B(t) + \tilde{D}(\xi_1)\xi_2(t) \\
\sum_2: & \quad \dot{\xi}_2 = \tilde{A}(\xi_2(t))\xi_2(t) + \tilde{B}\delta_B(t) + \tilde{D}(\xi_2)\xi_1(t)
\end{align*}
$$

(7a, 7b)

where $\xi_1(t)$ refers to $[v_x, \omega_f, \omega_r, F_x, F_y, F_{xf}, F_{yr}]^T$, and $\xi_2(t) = [v_y, r, F_{yf}, F_{yr}]^T$ represent respectively the state vectors for the longitudinal and the lateral dynamics, the control inputs of subsystems ($\sum_1$) and ($\sum_2$). $\omega_B = [B_f + T_f, B_f]^T$ and $\delta_B = \delta$. Also, $\tilde{y} = [\alpha_f, \omega_r]^T$ and $\bar{y} = [r, \alpha_f]^T$ are the output vector for each model. All variables are defined in Table II. The quasi-LPV form of the interconnected dynamics (7a-7b) with its $q$ varying parameters is exactly rewritten as a compact TS representation [14] with $r_q = 2^q$ linear sub-models weighted by membership functions $\mu_i(\tilde{\theta})$. These latter satisfy the convex-sum property

$$
0 \leq \mu_i(\tilde{\theta}) \leq 1 \quad \text{and} \quad \sum_{i=1}^{r_q} \mu_i(\tilde{\theta}) = 1
$$

(8)

where $\tilde{\theta}_j(t)$ (with $j = \{x, y\}$) is called premise variables vector, for the longitudinal model: $\tilde{\theta}_1 = v_x$, $\tilde{\theta}_2 = \omega_f$, $\tilde{\theta}_3 = \rho_f$, and for the lateral model: $\tilde{\theta}_4 = v_x$, $\tilde{\theta}_5 = \psi$, $\tilde{\theta}_6 = \frac{1}{\omega_R}$.

The bounds of the premise variables are given by

$$
\begin{align*}
\tilde{\theta}_1^{\min} & \leq \tilde{\theta}_1 \leq \tilde{\theta}_1^{\max} \\
\tilde{\theta}_2^{\min} & \leq \tilde{\theta}_2 \leq \tilde{\theta}_2^{\max} \\
\tilde{\theta}_3^{\min} & \leq \tilde{\theta}_3 \leq \tilde{\theta}_3^{\max} \\
\tilde{\theta}_4^{\min} & \leq \tilde{\theta}_4 \leq \tilde{\theta}_4^{\max} \\
\tilde{\theta}_5^{\min} & \leq \tilde{\theta}_5 \leq \tilde{\theta}_5^{\max} \\
\tilde{\theta}_6^{\min} & \leq \tilde{\theta}_6 \leq \tilde{\theta}_6^{\max}
\end{align*}
$$

(9)

Membership functions $\mu_i$ are computed as follows:

$$
\begin{align*}
\mu_1 = \eta_{11} \eta_{21} \eta_{31}, \quad \mu_2 = \eta_{12} \eta_{21} \eta_{31}, \quad \mu_3 = \eta_{11} \eta_{22} \eta_{31}, \\
\mu_4 = \eta_{12} \eta_{22} \eta_{31}, \quad \mu_5 = \eta_{11} \eta_{21} \eta_{32}, \quad \mu_6 = \eta_{12} \eta_{21} \eta_{32}, \\
\mu_7 = \eta_{11} \eta_{22} \eta_{32}, \quad \mu_8 = \eta_{12} \eta_{22} \eta_{32},
\end{align*}
$$

(10)
When the number of varying parameters increases the number of LMI related to the induced sub-models increases, which leads to computational complexity solving of the observer. To avoid this inconvenience, we express the system as an interconnected system to have less number of vertices and to reduce the conservatism. Considering the interlinked models with \( \theta_s \) and \( \theta_r \) separately taken lead to \( r_q = 8 \) for each one. However, if we consider \( \theta = \{ v_s, p_f, p_r, \frac{1}{v_s} \} \) is the global vector of the full system with \( r_q = 32 \) sub-models.

Before designing the QLIF observer, the state representations (7a,7b) are rewritten in the equivalent TS form

\[
\begin{align*}
\dot{\xi} &= \begin{bmatrix} \bar{A}_\mu & 0 \\ \bar{C} \end{bmatrix} \xi + \begin{bmatrix} \bar{B}_\mu & 0 \\ 0 & \bar{D}_\mu \end{bmatrix} u + \begin{bmatrix} 0 & \bar{D}_\mu \\ 0 & 0 \end{bmatrix} \xi \\
y &= \begin{bmatrix} \bar{C} \\ 0 \end{bmatrix} \xi, \quad r_q = 8
\end{align*}
\]

where:

\[
\begin{align*}
\bar{A}_\mu &= \sum_{i=1}^{r_q} \bar{A}_i(t) \bar{A}_i, \\
\bar{B}_\mu &= \sum_{i=1}^{r_q} \bar{A}_i(t) \bar{B}_i, \\
\bar{D}_\mu &= \sum_{i=1}^{r_q} \bar{A}_i(t) \bar{D}_i
\end{align*}
\]

where matrices \( \bar{A}_i, \bar{A}_i, \bar{B}_i, \bar{B}_i, \bar{D}_i \) and \( \bar{D}_i \) are constant for all \( i \in [1, ..., r_q] \) and \( r_q = 2^q, q = 3 \) for each sub-model.

\[
\sum_{\xi_i} \begin{cases}
\dot{\xi}_1 = \bar{A}_1 \bar{y}_1(t) + \bar{D}_1 \bar{y}_2(t) + \bar{B}_1 u(t) + L_\mu (\bar{y} - \bar{y}) \\
\dot{\bar{y}}(t) = C_\mu \bar{y}
\end{cases}
\]

With \( \xi(t) = [\hat{x}_1(t), \hat{x}_2(t)]^T \) are the estimated states vector and \( \bar{y}(t) = [\hat{y}(t), \hat{y}(t)]^T \) are the outputs vectors. \( \mu \) is the estimated weighting functions. The observer’s matrices \( \bar{L}_\mu \) and \( \bar{L}_\mu \) are parameter varying gains and have the same quasi-LPV form as the matrices \( \bar{A}_\mu \) and \( \bar{A}_\mu \), i.e.

\[
\bar{L}_\mu = \sum_{i=1}^{r_q} \mu_i(\hat{\theta}_i) L_{\mu_i} \quad \text{and} \quad \bar{L}_\mu = \sum_{i=1}^{r_q} \mu_i(\hat{\theta}_i) L_{\mu_i}
\]

The observer design procedure aims to determine the aforementioned observer’s matrices. Let’s denote the observers errors \( \hat{e}_x(t) = [\hat{e}_1(t), \hat{e}_2(t)]^T = [\hat{x}_1 - \hat{x}_1, \hat{x}_2 - \hat{x}_2]^T \). According to the observer (12a and 12b) and the system dynamics (11), the estimation errors obey the differential equation

\[
\dot{\hat{e}}_x = \begin{bmatrix} \hat{\Phi}_\mu & \hat{D}_\mu \\ \hat{D}_\mu & \hat{\Phi}_\mu \end{bmatrix} \hat{e}_x + \Delta \xi_1 + \Delta \xi_2
\]

where

\[
\begin{align*}
\hat{\Phi}_\mu &= \bar{A}_1 - \bar{B}_1 \bar{C}, \\
\hat{D}_\mu &= \bar{B}_1 \bar{C}, \\
\Delta \xi_1 &= (\bar{A}_1 - \bar{D}_1) \xi_1 + (\bar{B}_1 - \bar{D}_1) u(t) + (\bar{D}_1 - \bar{D}_1) \xi_2 \\
\Delta \xi_2 &= (\bar{A}_1 - \bar{D}_1) \xi_2 + (\bar{B}_1 - \bar{D}_1) u(t) + (\bar{D}_1 - \bar{D}_1) \xi_1
\end{align*}
\]

**Remark 1.** According to the assumption 1, and since the weighting functions \( \mu_i \) are positive and convex, then \( \Delta \xi_1(t) \) are also bounded. Hence, the estimation problem is reduced to determine the observer gains \( \hat{L}_i \), \( L_i \) such that the estimation errors \( \hat{e}_x(t) \) have an asymptotic convergence towards zero if \( \Delta \xi_1(t) = 0 \), and to ensure an ISS property when \( \Delta \xi_1(t) \neq 0 \).

**Definition 1.** [15] The state estimation error dynamics verifies the ISS if there exists a \( \mathcal{K} \) function \( f_1 : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R} \), a \( \mathcal{K} \) function \( f_2 : \mathbb{R} \rightarrow \mathbb{R} \) such that for each input \( \xi(t) \) satisfying \( \| \Delta(t) \|_\infty < \infty \) and each initial conditions \( e(0) \), the trajectory of the error associated to \( e(0) \) and \( \Delta(t) \) satisfies

\[
\| e(t) \|_2 \leq f_1 (\| e(0) \|, t) + f_2 (\| \Delta(t) \|_\infty)
\]

**Lemma 1.** [16] Consider \( S \) and \( R \) matrices with appropriate dimensions. For every matrix \( A > 0 \), the property holds

\[
S^T R + R^T S \leq S^T A S + R^T A^{-1} R
\]

**B. Stability & Convergence Analysis of the QLIF Observer**

The theorem 1 ensures the estimation of the state vectors.

**Theorem 1.** Given the varying parameters dependent matrices \( \bar{L}_\mu \) and \( \bar{L}_\mu \), if there exist two symmetric positive definite matrices \( P \) and \( Q \), two diagonal positive definite matrices \( \Omega_1 \) and \( \Omega_2 \), two positive matrices \( \bar{R}_1 \) and \( \bar{R}_2 \), given a positive scalars \( \eta_1, \eta_2 \), \( \alpha \) and \( a \in [0,1] \) and gains matrices \( \bar{K}_i \) and \( \bar{K}_i, \quad i = 1, ..., r_q \) solutions of the following LMI problem
The observer gains are $\tilde{L}_i = P^{-1} \hat{K}_i$, and $\tilde{L}_q = Q^{-1} \hat{K}_q$. Hence, the estimation error has an ISS property with respect to $\Delta \xi$, and converges to an origin-centered ball region.

To prove this theorem, let’s consider the following Piecewise Quadratic Lyapunov function

$$ V(e) = \begin{bmatrix} e_1^T & e_2^T \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} e_1^T & e_2^T \end{bmatrix}, P = P^T, Q = Q^T > 0 $$

Taking the derivative of 20 along the trajectory of the estimation error dynamics, one deduces

$$ \dot{V}(e) = (\Phi_0 e_1 + D_1 e_2^T)^T P e_1 + e_1^T P (\Phi_0 e_1 + D_1 e_2^T) + e_2^T Q (\Phi_0 e_1 + D_1 e_2^T) + e_2^T Q \Delta \xi_1 - \Delta \xi_1^T e_2 + e_2^T \left(Q \Delta \xi_1 + \Delta \xi_2^T Q e_2\right) $$

Considering $\tilde{\Gamma}_\mu = \Phi_0^T P + P \Phi_0$, $\tilde{\Gamma}_\nu = \Phi_0^T Q + Q \Phi_0$, and applying Lemma (1), inequality (21) yields

$$ \dot{V}(e) < e_1^T (\tilde{\Gamma}_\mu + P \tilde{\Gamma}_\mu \tilde{\Gamma}_\nu P + P \tilde{\Gamma}_\nu P + \tilde{\Gamma}_\nu P) e_1 + e_2^T Q \Delta \xi_1 = \tilde{\Gamma}_\mu \Delta \xi_1 + \tilde{\Gamma}_\nu P \Delta \xi_1 + \tilde{\Gamma}_\nu P \Delta \xi_1 $$

Finally, the gains of the QLIF observer are computed from the min and max eigenvalues of the matrix $\mathcal{Q}$. Under this condition, the state estimation error is reduced to

$$ \|e_\xi(t)\|_2 \leq \sqrt{\frac{\lambda_{\max}(\mathcal{Q})}{\lambda_{\min}(\mathcal{Q})}} \|e_\xi(0)\|_2 e^{-\bar{\gamma}t} + \sqrt{\frac{1}{\alpha} \|\Delta_\xi(t)\|_\infty} $$

Hence, when $t \to \infty$ the exponential converge to zero, implies

$$ \lim_{t \to \infty} \|e_\xi(t)\|_2 < \sqrt{\frac{\lambda_{\max}(\mathcal{Q})}{\lambda_{\min}(\mathcal{Q})}} \|\Delta_\xi(t)\|_\infty $$

which is transformed easily into

$$ (\alpha \eta_1)^2 I - P^T P > 0, \quad (\alpha \eta_2)^2 I - Q^T Q > 0 $$

By applying the Schur’s complement lemma [16], inequality (30) can be written as the LMI constraint (19c). Always in the purpose of minimizing the quantities $\eta_1$ and $\eta_2$, in theorem 1, the chosen objective function is a linear combination $\min \alpha \eta_1 + (1 - \alpha) \eta_2$. Using the condition $\Psi < 0$ in (26) and replacing (23) lead to the optimization problem

$$ \tilde{\Gamma}_\mu + P \tilde{\Gamma}_\mu \tilde{\Gamma}_\nu P + P \tilde{\Gamma}_\nu P + \tilde{\Gamma}_\nu P \eta < 0 $$

The two matrix inequalities are connected by $\mathcal{F}_1$ and $\mathcal{F}_2$. Using Schur’s complement Lemma [16], inequalities (31a) and (31b) yield to

$$ \tilde{\Gamma}_\mu + \tilde{\Gamma}_\nu P \tilde{\Gamma}_\nu Q + \tilde{\Gamma}_\nu P \eta < 0 $$

By using the convex sum propriety, the definitions of the matrices $\tilde{\Gamma}_\mu$ and $\tilde{\Gamma}_\mu$ and change of variables ($\hat{K}_i = P L_i$, $\hat{K}_q = Q \bar{L}_i$) and ($\Omega_1 = \mathcal{Q}^{-1}_1$, $\Omega_2 = \mathcal{Q}^{-1}_2$) and ($R_1 = P F_1$, $R_2 = Q F_2$). Finally, the gains of the QLIF observer are computed from the LMI conditions given in theorem 1 and the proof is complete. To achieve the observer synthesis, the gains are given by

$$ \tilde{L}_\mu = P^{-1} \hat{K}_i \quad \text{and} \quad \tilde{L}_q = Q^{-1} \hat{K}_q $$

V. EXPERIMENTAL RESULTS AND DISCUSSIONS

In this section, an assessment of the QLIF observer performance is presented using hardware experiments under real-world driving maneuvers. To this end, the dynamic driving SHERPA (French Acronym for "Simulateur Hybride d’Etude et de Recherche de PSA Peugeot Citroen pour l’Automobile") LAMIH simulator is used. It includes a full car mock-up Peugeot 206 vehicle installed on a six-DoF Stewart platform.
the setup is placed in front of visual feedback displays of 240 deg wide projection screen offering a panoramic view, presented in Fig. 3. Therewith, the setup can accommodate real automotive equipment interfaced with the software layer [7], [17]. Usually, the SHERPA simulator is used in co-simulation with the SCANeR automotive software and Matlab/Simulink.

A. Observer Evaluation in Real-World Conditions

The real-world test scenario was performed on a Satory test track similar to the one located in the city of Versailles-Satory, 20 km west of Paris, France. This test track is an urban scenic road performed in accordance with a real regular riding condition and good environmental conditions. It allows to highlight the observer performance by covering a broad spectrum of the vehicle dynamics within and beyond its linearization domain. As depicted in Fig. 4 and road curvature Fig. 4, the road track is composed of straight lines followed by several curved profiles including narrow turns and big bends. This configuration allows to solicit the lateral and longitudinal dynamics. The simulations include an acceleration and braking phases, also the vehicle undergoes lateral motion with variable speed. Hence, the vehicle system requires: the two braking torques on both front and rear wheels applied to reduce the forward speed, the traction torque $T_f$ applied on the front wheel, involving also a medium hard steering angle input applied on the lateral model, represented in Fig. 5.

In Fig. 6, estimated yaw rate and angular velocities profiles are compared to their respective measurements provided by the car simulator software. Since these states variables are measured and used in the observer design, the state estimation demonstrates a finite-time estimation convergence.

For validation, the estimation of unmeasured states ($F_{yi}$, $F_{yi}$) are used to reconstruct the lateral and longitudinal accelerations at the center of gravity $CoG$ by using the two equations: $\dot{a}_y = \frac{(\hat{F}_y + F_{yi})}{m}$ and $\dot{a}_x = \frac{(\hat{F}_x + F_{xi})}{m}$.

Fig. 5: Braking, traction torques and steering angle inputs.

Fig. 6: Actual (red) and observer estimation (dashed blue).

Fig. 7: Estimation performance: unmeasured states.

Fig. 8: Validation performance for Satory test track.
B. Observer Robustness to noise & parameters uncertainties

To assess the observer in the presence of noise, we assume a centered and random noises with $5 - 10\%$ of the maximal measured values. The same digital database of the Satory test track presented in Section V, Fig. 4 is considered. Then, the observer sensitivity is tested against parameters variation. Hence, we consider that the vehicle mass with driver has undergone a variation of $\pm 300\text{kg}$ on the design values. The effect of the over or the underweight and the noise influence are evaluated using statistics indexes. The results of the estimation are compared with their counterparts by means of the root mean square (RMSE$_{\text{E}}$), mean square percentage (MSE$_{\text{E}}$) and normalized mean square errors (NMSE$_{\text{E}}$) in Table I.

\[
\text{MSE}_E = \frac{\sum (y - \hat{y})^2}{N}, \quad \text{NMSE}_E = 100 \left( \frac{\sum (y - \hat{y})^2}{\sum (y - \text{mean}(y))^2} \right), \quad \text{RMSE}_E = \sqrt{\frac{1}{N} \sum (y - \hat{y})^2}
\]

From Table I, the observer gives the better estimation for the nominal case where the maximal values of (MSE$_{\text{E}}$, RMSE$_{\text{E}}$) are the lowest and NMSE$_{\text{E}}$ the largest. As expected, the estimation errors increase when the vehicle mass changes and becomes more important in the case of noise effect with a maximum degradation of (10\%). However, even with noise effect or vehicle’s mass variation, these errors are always lower than MSE$_{\text{E}} < 9\%$. Indeed, the deviations amplitude of the errors is quantified with lower than RMSE$_{\text{E}} < 12.66\%$ for mass variation and RMSE$_{\text{E}} < 14\%$ for noisy case. Hence, the QLIF observer is robust enough to handle the noisy case and parameters uncertainties.

**TABLE II: System’s parameters description.**

| $v$, $v_1$ | Lateral and forward velocities ($\text{m/s}^{-1}$) |
| $\omega$, $\omega_1$ | Angular velocities of the front and rear wheels ($\text{rad/s}^{-1}$) |
| $m$, $m_1$ | Vehicle mass (kg), inertia about the z-axis (kg m$^2$) |
| $C_a$, $C_i$ | Cornering and longitudinal stiffness parameters ($\text{N rad}^{-1}$) |
| $l_r$, $l_f$ | Wheels moment of inertia (kg m$^2$) |
| $R$ | Rolling radius (m) |
| $l_c$, $l_f$ | Distances between the C.G. and front and rear axles (m) |

VI. CONCLUSION

This paper presents a QLIF observer synthesis for simultaneous estimation of the lateral and longitudinal vehicle dynamics. The outlined observer is designed considering Quasi-LPV vehicle model taking into account real constraints such as the variations in the forward speed and the slip velocities during the interconnected-sub observers design. The result is formalized using Lyapunov theory where the observer’s gains are computed by resolving an optimization problem in form of a set of LMIs aiming to minimize the estimation error bound based on ISS propriety. The observer was validated in nominal case with ideal sensors, also under noise consideration and parametric uncertainties. Thereafter, error quantification was proposed to evaluate the goodness and the robustness of the estimation scheme. For future works, the observer will be validated experimentally in real-time using the DS7 vehicle which is being instrumented with the various sensors required to measure the longitudinal and lateral dynamics.

**REFERENCES**


**TABLE I: Case (A) represents the observer evaluation, robustness to vehicle mass in: case (B): $M^* = M + 300$, case (C): $M^* = M - 300$ and case (D) considers the noise effect for the Satory Track Test.**

<table>
<thead>
<tr>
<th>$r$</th>
<th>$\omega$</th>
<th>$\omega_1$</th>
<th>$\omega_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{MSE}$</td>
<td>$\text{RMS}$</td>
<td>$\text{NMSE}$</td>
<td>$\text{RMS}$</td>
</tr>
</tbody>
</table>
| $\text{A}$ | $0.062$ | $2.33$ | $95.54$ | $3.12$ | $10.52$ | $96.56$ | $4.29$ | $11.78$ | $94.31$ | $0.031$ | $1.76$ | $99.98$ | $2.79$ | $6.69$ | $89.53$
| $\text{B}$ | $0.064$ | $2.47$ | $95.58$ | $3.49$ | $11.69$ | $94.69$ | $5.91$ | $12.33$ | $90.95$ | $2.29$ | $5.12$ | $98.97$ | $2.67$ | $6.32$ | $83.84$
| $\text{C}$ | $0.065$ | $2.56$ | $95.49$ | $3.50$ | $11.71$ | $95.78$ | $4.35$ | $12.66$ | $93.62$ | $2.92$ | $7.09$ | $97.86$ | $3.01$ | $7.41$ | $82.71$
| $\text{D}$ | $0.23$ | $4.29$ | $89.09$ | $5.62$ | $13.71$ | $89.45$ | $8.25$ | $13.26$ | $86.02$ | $3.26$ | $8.57$ | $89.98$ | $4.42$ | $8.76$ | $80.71$