Vibration reduction of flexible rope-driven mobile robot for safe façade operation*

Myoungjae Seo  
School of Mechanical  
Engineering  
Hanyang  
University  
Seoul, South Korea  
mjseo0616@hanyang.ac.kr

Sungkeun Yoo  
School of  
Mechanical and Aerospace  
Engineering  
Hanyang  
University  
Seoul, South Korea  
skyoo@rodel.snu.ac.kr

Myeongjin Choi  
School of Mechanical  
Engineering  
Hanyang  
University  
Seoul, South Korea  
aud0109@hanyang.ac.kr

Joohyun Oh  
School of Mechanical  
Engineering  
Hanyang  
University  
Seoul, South Korea  
oju0809@hanyang.ac.kr

Hwa Soo Kim  
Department of Mechanical  
System Engineering  
Kyonggi  
University  
Suwon, South Korea  
hskim94@kgu.ac.kr

TaeWon Seo  
School of Mechanical  
Engineering  
Hanyang  
University  
Seoul, South Korea  
taewonso@hanyang.ac.kr

Abstract—In recent years, cable-driven-parallel robots (CDPRs) have been studied for façade operations. There are various types of CDPRs; however, under-constrained CDPRs are capable of wider operating in façade workspaces than over-constrained CDPRs. Therefore, in this study, a dual ascender robot (DAR) was used for façade operations. Herein, two suggestions for safe façade operations are presented. First, a flexible nylon fiber rope was modeled such that the vibration direction, natural frequency, and damping ratio of the DAR could be converted through a Jacobian matrix and modal decomposition from the rope model. Second, input shaping control was applied to reduce vibrations, based on the vibration model of a DAR using the rope model. Modal decomposition was verified using a verification experiment, and the effect of input shaping was evaluated by comparing the w/ input shaping and w/o input shaping experiments. w/input shaping case was shown about 48% reducing robot vibration and about 35% shortening settling time compare with w/o input shaping case.

Keywords—Cable-driven parallel Robots, Mechanism design, Rope modeling, Modal decomposition, Input shaping

I. INTRODUCTION

Currently, there has been an increase in the demand for façade operations during building maintenance owing to an increase in the number of high-rise buildings. However, such façade operations involve a risk of falling, as workers are suspended at great heights using climbing ropes. This risk can be circumvented by using robots to replace these workers. [1] Cable-driven-parallel robots (CDPRs), which have the advantage of a relatively large workspace, have been studied for such façade operations. They have been widely employed in several applications, ranging from moving objects at high speeds to lifting heavy objects. The movement of CDPRs is related to the number of cables connected to it; this number of cables can be classified as redundant or under-strained, depending on the number of actuators.

Redundant CDPRs are connected to more than \( n+1 \) cables in a space with \( n \) degrees of freedom (DOF). In such cases, more than \( n+1 \) cables are subjected to a high tension at all times in the particular workspace, which significantly reduces the vibrations in the cables [2,3,22,23,26] but rather high tension should be avoided [4]. Therefore, redundant cables enable quicker movement of CDPRs due to the significantly reduced vibration; however, the workspace in such cases is limited, and it is also difficult to maintain the high tension. For instance, FALCON-7 [5,6], comprising seven cables, and WARP [7], comprising eight cables, could move at high speeds within a limited workspace. NIST ROBOCRANCE [8] achieved stable control even if the anchor point of the cable was shaken. As redundant CDPRs require many actuators for control, the control needs to be complex, and the workspace necessary to guarantee high tension in all the cables is limited.

In the case of under-constrained CDPRs, less than \( n-1 \) cables are used in a workspace with \( n \) DOF. Here, the uncontrolled movement is as much as the difference between the DOF and the number of actuators. Uncontrolled movement of under-constrained CDPRs results in vibration, which makes it difficult to control their position. Therefore, it is dangerous if such CDPRs move at high speeds or hoist heavy objects. To safely control under-constrained CDPRs, it is necessary to predict their vibration. Previous studies have predicted such vibration and analyzed the stability of CDPRs [9,10,11,24]. Despite these issues, under-constrained CDPRs offer a wider workspace as well as simple control, as compared to redundant CDPRs. Therefore, they are more suitable for façade operations, which involve working on large workspaces.

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Facade operations require a considerable selection of cables to operate over large workspaces. However, a special winch is essential for these operations because the cables used in most studies are steel cables, which are heavy and rigid. This study employs a dual ascender robot (DAR) moving vertically and horizontally using two nylon fiber ropes [12]. Nylon fiber ropes are primarily used for rock climbing or cleaning facades. Using these flexible fiber ropes requires methods to reduce vibration, because the vibration of n-1 ropes as well as their uncontrolled movement makes it more difficult to control the position of the CDPRs. Consequently, several studies have focused on reducing these vibrations [11,13,21,25].

This paper proposes the vibration reduction control of a DAR that is connected to a flexible fiber rope for safe facade operations. To reduce the vibration of mobile robots using flexible fiber ropes, two factors must be considered. The first is the vibration generated by kinematic properties of the under-constrained CDPR, which can be expressed via CDPR modeling using a Jacobian matrix; this has been conducted in previous studies. The second factor is the vibration caused by the properties of flexible ropes. The rope was modeled based on a preliminary experiment analyzing its characteristics and reflecting them in the kinematic model. The vibrations of the ropes were divided along two directions, and the size of the natural frequency through modal decomposition. This study presents a method for predicting the vibrations in mobile robots by expressing the natural frequency and damping ratio of these vibrations through a Jacobian matrix and modal decomposition employing physical values of two flexible ropes. This ultimately enables safe facade operations by using the input shaping control method. To predict the vibration in a flexible fiber rope, its characteristics are first determined via preliminary experiments. For this purpose, the rope was modeled as a simple model comprising springs and dampers connected in parallel. The elastic coefficient (k) and damping coefficient (c) along the length were determined based on tensile tests of the rope. Input shaping was used as the control method to reduce vibrations in the rope models [14-18]. A robot connected to two flexible fiber ropes on the facade was considered to verify the effect of reducing vibrations by using input shaping.

The rest of the paper is as follows. Section II, addresses the mechanical design of the DAR. Section III, proceed mathematical modeling of the DAR’s movement and use the modal decomposition to identify the vibration of the robot. Section IV, identifies the rope characteristics that move the DAR through preliminary experiment. Section V, experiment is conducted to verify the vibration characteristics of the DAR predicted by modal decomposition and to use a control method called input shaping to reduce the vibration of the robot. Finally, in section VI concludes by analyzing the experimental results.

II. MECHANICAL DESIGN OF 2-DOF DAR

The DAR is a mobile robot developed for facade operations as shown in Fig. 1 (a); it comprises a measuring mechanism and two ascenders as shown in Fig. 1 (b). First, the Ascender is a mechanical device that can wind and unwind the nylon rope to make the DAR to move the facade 2-DOF. Nylon rope used in the Ascender is fixed to the facade by anchor and there is no fixed point inside the ascender as shown in Fig.1 (d). Therefore, it is essential to maximize the friction force during this winding of the nylon ropes. The ascender is consisted with four friction pulleys, guide rollers, pressure rollers, and two differential gear mechanisms for maximizing the friction force. Next, there are two main types of measuring mechanisms: angle measurement of joint and rope, and length change measurement of rope. Both measuring mechanisms use the incremental encoder. In the measuring mechanism, the gap between the rotating axes that the angle measurement part of joint and rope is 7cm. These gap is considerably smaller than that of the entire DAR. However, small gap design is deeply related to robot rotation. This design helps to isolate the degree of freedom of the ascender and the rotation of the robot's body as much as possible and the vibration of the robot is also simplified as the pendulum.

The position of the DAR is estimated by measuring the length and angle of the rope, as well as the angle of the robot's body. Therefore, a rotary incremental encoder (angle), shown in Fig. 1 (b), is designed to rotate freely according to the movement of the rope, making it possible to measure the angle of the rope provided the initial angle is specified. For the length of the rope, a rotary incremental encoder (length), shown in Fig. 1 (c), is connected to a urethane-fused roller, making it possible to measure the length of the rope provided the initial value is specified. The absolute angle measurement of the robot body employs an inertial measurement unit (IMU) sensor with a 3-axis gyroscope, 3-axis acceleration sensor, and 3-axis geomagnetic sensor. The IMU sensor compensates for the angular error originating from the robust attitude algorithm timeout provided by the manufacturer, in order to compensate for the characteristics affected by vibration. Finally, the load cell, shown in Fig. 1 (c), is used for measuring the tension of the rope;
it is installed to detect the moment required for safety if the robot loses tension on reaching a singular position or if the rope is broken.

III. DAR MODELING AND MODAL DECOMPOSITION

The DAR is a mobile robot moving 2-DOF due to the pulling action of two flexible nylon fiber ropes. Therefore, the vibration applied to the robot body can be classified into two types: kinematic and rope vibrations. In this section, the vibration of the DAR is analyzed that generated by the flexible rope using kinematic models. The kinematic model of the robot utilizing elastic massless two ropes is presented in Fig. 2. The position vectors of the moving robot are as follows:

\[
\begin{align*}
\overrightarrow{p} &= \overrightarrow{A}_i - \overrightarrow{q}_i - R_x \overrightarrow{x}_i, \quad i = 1, 2 \\
x &= \overrightarrow{p} = [x \ y]^T, \quad \overrightarrow{q} = [\dot{x} \ \dot{y}]^T, \quad \overrightarrow{J} = [-\overrightarrow{u}_1^T \ -\overrightarrow{u}_2^T]^T
\end{align*}
\]

where \(\overrightarrow{p}\) is the position vector of DAR, \(\overrightarrow{A}_i\) is the anchor position vector, \(\overrightarrow{q}_i\) is the rope vector, \(R_x\) is the robot's rotation matrix, \(\overrightarrow{x}_i\) is the revolute joint vector from the body frame origin \((o)\), \(\overrightarrow{q} = [q_1 \ q_2]^T\), is joint velocity vector and \(J\) is the Jacobian matrix between the Cartesian space \((\text{global frame } O)\) and joint space. The joint space refers to the rotary coordinate system based on each joint. Therefore, the Jacobian matrix \(J\) converts the velocity vector (\(\dot{x}\)) of the DAR to the joint velocity vector \((\dot{\overrightarrow{q}})\); it is represented by the unit vector of the rope \((\overrightarrow{u}_i)\), as shown in (2).

Assuming that the robot is in an immobile equilibrium and that the Jacobian matrix is constant for an infinitesimal displacement, the velocity in the joint space and the speed in the Cartesian space are proportional to the Jacobian matrix, according to (3). Furthermore, the relationship between displacement in the Cartesian space \((\overrightarrow{x})\) and that in the joint space \((\overrightarrow{q})\) is expressed as (4).

\[
\begin{align*}
\delta \overrightarrow{x} &= J \delta \overrightarrow{q} \\
\delta \overrightarrow{x} &= J \delta \overrightarrow{q}
\end{align*}
\]

Assuming a steady state wherein the tension of the rope \((T = [T_1 \ T_2]^T)\) and the gravity acting on the robot \((\overrightarrow{g})\) are balanced, the effect of gravity is nullified. Then, the change in tension is represented as \(T'\). Therefore, the external force can be expressed as follows:

\[
F = -J^T T'
= -J^T K \delta \overrightarrow{x} - J^T C \delta \dot{x}
\]

\[
M \delta \ddot{x} + C \delta \dot{x} + K \delta \overrightarrow{x} = 0
\]

where \(T' = K \delta \overrightarrow{q} + C \delta \dot{\overrightarrow{q}} + r \delta \overrightarrow{q} + r \delta \overrightarrow{q}; \quad \overrightarrow{r} = [r_1 \ r_2]^T\) is the modal vector, and \(E\) is the mass-normalized mode shape matrix. The solution of the state equation in modal space and the general solution are as follows:

\[
\begin{align*}
\dot{\overrightarrow{E}}^T M \dot{\overrightarrow{E}} + \overrightarrow{E}^T (\beta/\alpha) K \dot{\overrightarrow{E}} + \overrightarrow{E}^T K \overrightarrow{E} &= 0 \\
\dot{\overrightarrow{r}} + \beta / \alpha \overrightarrow{r} + \Delta \overrightarrow{r} &= 0 \\
\dot{\overrightarrow{r}} + 2 \xi \omega_n \overrightarrow{r} + \omega_n^2 \overrightarrow{r} &= 0
\end{align*}
\]

where \(\Delta = \overrightarrow{E}^T K \overrightarrow{E}; \quad \overrightarrow{r} = \overrightarrow{r}_1 \cos(\omega_n t) + \overrightarrow{r}_2 \sin(\omega_n t)\) are natural frequency of rope hanged on robot according to robot's position. Modal decomposition can be used to predict the direction of \(\omega_1\) and \(\omega_2\) in the modal space. Fig. 3 presents the direction of the modal decomposition in the workspace \((2.5 \text{ m} \times 10 \text{ m})\); \(\omega_1\) and \(\omega_2\) are classified according to the size. The higher the workspace, the greater is the natural frequency. The direction of modal decomposition is formed vertically and horizontally in the workspace, except for the high portions. In the case of vertical vibration, as gravity acts constantly, the vibration of the robot disappears quickly, unlike the modal decomposition. However, in the case of horizontal vibrations,
the tendency of modal decomposition is observed because there is no external force such as gravity. This indicates that $\omega_2$ can be used to reduce vibrations in the workspace.

IV. PRELIMINARY EXPERIMENTS FOR DETERMINING ROPE CHARACTERISTICS

Several methods to reduce the vibrations in robots employing flexible ropes have been proposed. In redundant CDPRs, the damping ratio is high because of the high tension maintained within the limited workspace. Moreover, vibrations are significantly reduced on increasing the $k$ and $c$ values of the cable. Under-constrained CDPRs are directly affected by the characteristics of the rope. Therefore, if the characteristics of the rope can be predicted, it can be used for safe façade operation. In this paper, the rope is modeled as a simple model with springs and dampers connected in parallel. A preliminary experiment was conducted to identify the characteristics of the rope.

A. Rope modeling

A commercial nylon (polyamide) fiber rope with a diameter of 6 mm, which is primarily used for rock climbing and facade operations, was used in this study. The properties of this rope vary depending on its length and can be evaluated using (12) [19].

$$k = \frac{ec}{l} \tag{12}$$

where $k$ is the spring coefficient, $l$ is the length of the rope, and $ec$ is the constant obtained from the experiment. Although previous studies have modeled this type of nylon fiber rope [19], modeling the fibers of this rope is difficult because several strands are twisted spirally with each other; consequently, when subjected to a force, the physical properties of nylon as well as its geometrical characteristics are altered. Therefore, assuming that the nylon fiber rope does not change with respect to time, the rope can be expressed as a simple model where the spring and damper are connected in parallel, as shown in Fig. 4. This model was analyzed using preliminary experiments and analyses.

![Fig. 4. CDPR model with 2 flexible rope and rope model comprising a spring and a damper in parallel](image)

B. Experiment and analysis

The test bench in Fig. 5 can be divided into two parts. First, a measurement structure consisting of a load cell and linear encoder was used to measure the tension and vertical position of the rope, respectively. Additionally, weights (initial mass) can be attached to create a spring-damper system of the rope in order to account for the robot’s mass. Second, the test bench involves an electromagnet and a transport structure. The electromagnet can generate a tensile force (additional weight) of up to 250 kgf. The transport structure is driven by a motor at the bottom of the test bench, which can generate a tensile force of up to 90 kgf.

![Fig. 5. Preliminary experiment testbench](image)

The initial mass ($m$) is the sum of the masses of the structure and the weights. An additional weight was applied consistently with the tensile force applied by the electromagnet. Thereafter, an initial position ($x_0$) of the spring-damper system was considered; the tensile force was removed at this position. Thus, the transient response to the initial position ($x_0$) of the spring-damper system with an initial mass was analyzed.

The state equation of the spring-damper system for the initial displacement ($x_0$) can be expressed as (13): $\omega_{n,rope}$ and $\zeta_{rope}$ are the natural frequency and damping coefficients of the rope, respectively, (14) is the solution of the state equation expressed in (13). $\omega_d = \omega_n \sqrt{1 - \zeta_{rope}}$, $A$ is a function of the initial position $x_0$, and $\psi = \pi/4$.

$$\ddot{x} + 2\zeta_{rope}\omega_{n,rope}\dot{x} + \omega_{n,rope}^2x = 0 \tag{13}$$

$$x = A(x_0) \exp(-\zeta_{rope}\omega_{n,rope}t) \sin(\omega_d t + \psi) \tag{14}$$

Table I and Fig. 6 present the results of the experiment using a 5-m rope. The results are fit using the curve fitting function in MATLAB. It is calculated the $ec$ value from the values of $\zeta_{rope}$ and $\omega_{n,rope}$. Consequently, $ec$ and $\zeta_{rope}$ remain constant throughout the length of the rope. The relationship between the length of the rope and the spring coefficient $k$ ($= \omega_{n,rope}^2$) is expressed as (12). The properties of the nylon fiber rope were predictable along its length. The spring coefficient $k$ of the two flexible ropes connected to the robot with respect to the robot’s position can be calculated in real-time using the length of the rope and inverse kinematics.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>RESULTS OF THE PRELIMINARY EXPERIMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial mass (30 kg)</td>
<td>Rope length = 5 m, Added Weight = 10 kg</td>
</tr>
<tr>
<td>Average (1st, 2nd, 3rd impacts)</td>
<td>Average (1st, 2nd, 3rd impacts)</td>
</tr>
<tr>
<td>$\omega_{n,rope}(\text{rad/s})$</td>
<td>13.3088</td>
</tr>
</tbody>
</table>
The distance between (0,0) and (2.5,0), where the rope is fixed at 2.5 m and a height of 10.05 m, is illustrated in Fig. 8 (a). Fig. 8 (b) shows the position of the robot along the x-direction; the robot moves to a height of 1.41 m, as measured by the test bench. The result of transforming the plot in Fig. 8 (b) from the Cartesian space (x, y) to the modal space (r₁, r₂) by using (14) is presented in Fig. 8 (c, d).

### TABLE II

<table>
<thead>
<tr>
<th>Producer</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brushless DC motor (BLDC)</td>
<td>Maxon RE50 gear ratio (181:1)</td>
</tr>
<tr>
<td>Motor diver</td>
<td>Maxon EPOS2 70/10</td>
</tr>
<tr>
<td>Controller</td>
<td>National instrument cRio-9039</td>
</tr>
<tr>
<td>Incremental Encoder (Rope angle)</td>
<td>Autonics E3OS4-360-3-N-24</td>
</tr>
<tr>
<td>Incremental Encoder (Rope length)</td>
<td>Autonics E3OS4-360-3-N-24</td>
</tr>
<tr>
<td>IMU sensor</td>
<td>E2box EBIMU-9DOFV5</td>
</tr>
<tr>
<td>Loadcell (Rope tension)</td>
<td>CAS SB-20L</td>
</tr>
</tbody>
</table>

Vibration via modal decomposition is expressed by the r₁ and r₂ axes in the modal space. Fig. 8 (c) and Fig. 8 (d) represent the r₁ and r₂ axes in the modal space, respectively. Therefore, Fig. 8 (c) and (d) indicate that the vibration is detached through modal decomposition. Compared to the vibration of the r₁ axis, shown in Fig. 8 (c), the r₂ axis vibration in Fig. 8 (d) exhibits a similar amplitude and period compared with the x-direction vibration. it also allows us to ensure that the x-direction vibrations applied to the robot are similar to those of the r₂ axis in the modal space and that the modal decomposition is conducted appropriately.

The general solution obtained via modeling and modal decomposition is expressed in (14). Therefore, the r₁ axis vibration in Fig. 8 (d) can be fitted to (14) as shown in Fig. 8 (e, f) using the curve fitting function in MATLAB. The results thus obtained are listed in Table III. Moreover, the R-square value of the fitting confirmed that the verification and experimental values were well correlated.

The model discussed in Section III was verified by comparing the modeling and verification experimental results in Table IV. The model employed ωₘ, r₀pe and ζ₉₀pe obtained from the preliminary experiment in order to predict the vibration of the robot using two ropes, through the Jacobian matrix J and modal decomposition. Therefore, the mass-normalized modal shape matrix E can be obtained based on the robot's initial position. Table I and (11) provides the damping ratio ξ and damped frequency T_d required to generate the trajectory of input shaping. Additionally, the verification experiment data were obtained by analyzing r₂ when the DAR stops at the initial position. It can be observed that the modeling’s natural frequency ωₘ, damped frequency ω_d, and damped period T_d shown in Table IV are considerably similar to those of the verification experiment.
However, the values of the damping ratio obtained in the preliminary test and those from the verification experiment are different. This difference is due to the change in the mass, inertia of the rope, damping due to rotation of the joint, and length of the rope as the DAR moves along the x-direction. Despite these factors, it can be confirmed that the vibration reduction can be achieved via input shaping because the damped period is more significant than the damping ratio during input shaping.

### TABLE III

<table>
<thead>
<tr>
<th>Ex.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi \omega_i$</td>
<td>0.30</td>
<td>0.31</td>
<td>0.34</td>
<td>0.34</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>$\omega_{i\text{d}}$</td>
<td>2.17</td>
<td>2.12</td>
<td>2.15</td>
<td>2.17</td>
<td>2.16</td>
<td>2.16</td>
</tr>
<tr>
<td>R-square</td>
<td>0.99</td>
<td>0.99</td>
<td>0.97</td>
<td>0.99</td>
<td>0.99</td>
<td>0.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ex.</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi \omega_i$</td>
<td>0.30</td>
<td>0.30</td>
<td>0.34</td>
<td>0.32</td>
<td>0.31</td>
<td>0.32</td>
</tr>
<tr>
<td>$\omega_{i\text{d}}$</td>
<td>2.21</td>
<td>2.14</td>
<td>2.20</td>
<td>2.17</td>
<td>2.18</td>
<td>2.18</td>
</tr>
<tr>
<td>R-square</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.97</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>

### TABLE IV

<table>
<thead>
<tr>
<th>Robot mass (30 kg)</th>
<th>Initial Position (1.15, -8.59)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex.</td>
<td>Experiment results</td>
</tr>
<tr>
<td>Modeling</td>
<td>Verification (average)</td>
</tr>
<tr>
<td>Damping ratio $\xi$</td>
<td>0.0120</td>
</tr>
<tr>
<td>Natural frequency $\omega_n$ (rad/s)</td>
<td>2.1517</td>
</tr>
<tr>
<td>Damped frequency $\omega_d$ (rad/s)</td>
<td>2.1515</td>
</tr>
<tr>
<td>Damped period $T_d$(s)</td>
<td>2.9203</td>
</tr>
</tbody>
</table>

### C. Input shaping

After the natural frequency and damping coefficient, which are physical properties of vibration, are determined, input shaping is used as a method to control vibration. This method reduces vibration by applying an impact of $A_2$, considering the period of vibration and the damping ratio when an impact of $A_1$ is applied to the system, as shown in Fig. 9. To control the DAR, it is first accelerated via input-shaping based on the properties of the rope with respect to the position. The verification experiment for modal decomposition was conducted to verify the damping ratio ($\xi$) and damped period ($T_d$) of the x-direction vibration at a height of 1.41 m. The damping ratio ($\xi$) and period ($T_d$) are applied to input shaping in order to reduce robot’s x-direction vibrations. The acceleration for applying the input-shaping method is as follows:

\[
\begin{bmatrix}
A_1 \\
\frac{1}{t_i}
\end{bmatrix} =
\begin{bmatrix}
1 & K \\
1 + K & \frac{T_d}{2}
\end{bmatrix}
\]

(15)

where $K = \frac{-\xi \omega_n}{\left(\frac{T_d}{2}\right)^2}$, $t_i = \frac{\pi}{\omega_d} A_i$ is the ratio of input size, and $t_i$ is the duration of input. According to (15), an acceleration
trajectory was created. The position trajectory was generated based on the acceleration trajectory and used as the input for the motor.

D. Experiment for input shaping

The input shaping experiment was conducted under conditions similar to those of the w/o input shaping experiment; Both w/o input shaping and input shaping experiments were conducted such that the robot was moved and stopped at a speed of 0.5 m/s from point (0.8, 1.41). The w/o input shaping experiment involved a target speed of 0.5 m/s by maintaining an acceleration of 5 m/s² for 0.1 s. In the case of input shaping, the acceleration was divided into two sets according to (15) and maintained for 0.1 s in each set.

The results of the w/o input shaping and w/ input shaping experiments are presented in Fig. 10 (a, b, c). Both experiments measured the x-direction position of the robot moving at a speed of 0.5 m/s and then stopping. Compared with the w/o input shaping experiment, the results of the input shaping experiment exhibited a significant difference in terms of the maximum amplitude and setting time. Compared to the values for the w/o input shaping experiment, the maximum amplitude of input shaping was reduced by 48.3%, whereas the setting time decreased by 34.8% on right; these results are listed in Table V.

<table>
<thead>
<tr>
<th>x-direction vibration</th>
<th>w/o input shaping</th>
<th>w/ input shaping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum amplitude</td>
<td>0.373</td>
<td>0.193 (48.3%)</td>
</tr>
<tr>
<td>Settling time</td>
<td>8.92</td>
<td>5.82 (34.8%)</td>
</tr>
</tbody>
</table>

VI. CONCLUSION

The results obtained via the w/o input shaping and w/ input shaping experiments verified the vibration-reduction effects of input shaping through modal decomposition, as presented in Table V. This enhances the safety of robots using two flexible ropes. However, the vibration was not completely eliminated, which could be explained by three factors. First, the wire encoder on the test bench, which is used to measure position, alters the system characteristics because it applies a certain force to the robot owing to its own characteristics. So, it made the difference between left and right, which can be seen in the Fig 11 (b, c). However, this has a small effect on the system, so there is no big difference between the left and the right. The second factor is the hysteresis of the nylon fiber rope. As mentioned in Section IV. A, there was a difference between the actual rope and the rope model, because the rope experiences hysteresis over time. The third factor is the discrepancies in kinematic characteristics. In this study, the vibration characteristics of a mobile robot was attempted to predict and control using only a flexible rope model. However, even if the vibration characteristics of the DAR are predicted and controlled by the characteristics of the rope, there are disturbances caused by factors such as the weight of the rope, torsional stiffness and joint damping. Therefore, as a follow-up study, external devices such as thruster will be attached to the DAR to attenuate the vibration of the robot.

Fig. 10. Experiment result; (a) Vibration diagram when DAR moves right; (b) Vibration diagram when DAR moves left; (c) DAR's position graph of the w/ input shaping and w/o input shaping experiments.

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