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A Routing Framework for Heterogeneous Multi-Robot Teams in Exploration Tasks

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Abstract— **This paper proposes a routing framework for heterogeneous multi-robot teams in exploration tasks. The proposed framework deals with a combinatorial optimization problem and provides a new solving algorithm, for Generalized Team Orienteering Problem (GTOP). In this paper, a route optimization problem is formulated for a heterogeneous multi-robot system. A novel problem solver is also proposed based on self-organizing map. The proposed framework has a strong advantage in its scalability because the processing time is independent from the number of robots and the heterogeneity of the team. The validity of the proposed framework is evaluated in the exploration and mapping tasks by heterogeneous robot team with overlapping abilities. The simulation results show the effectiveness of the proposed framework and how it outperforms the conventional greedy exploration scheme.**

I. Introduction

Technical barrier to developing small and cheap robots is dramatically lowered recently thanks to the improvement of electronics. As a natural consequence, researches on multi-robot systems (MRS) gains strong attention aiming for practical applications. MRS techniques have a wide range of application fields such as search and rescue [1], security [2], environment monitoring [3], and planetary exploration [4]. From a practical point of view, operating multiple robots at the same time is in many cases much more difficult than operating a single robot. Therefore autonomy is the basis for realising MRS in industry and this is an interesting but challenging part. Such problems have been studied as multi-robot task allocation problem (MRTA), and several frameworks have been proposed [5], [6], [7].

Heterogeneous MRS which is composed of more than one type of robots in a team is studied as a motivating topic among MRS. Heterogeneous MRS is an attractive solution for many practical application scenarios where a severe restriction on resources is imposed such as disaster rescue or space exploration. However, introducing heterogeneity often complicates automation algorithms. In particular, routing is one of the most difficult classes of problems in MRTA. In terms of the definition of the heterogeneity, although the definition and the measurement of heterogeneity is a crucial research topic [8] presenting precise metrics for heterogeneity is out of the scope of this study. In this study, heterogeneity of a team of robots is defined by the number of types of robots which is given by the set of abilities of each robot as is described in later sections. Conventional researches on MRS tend to focus on systems composed of a few different types of robots with independent roles such as UGVs and UAVs collaborating in exploration or delivery tasks [9], [10]. It is unclear whether or not the ad-hoc solutions obtained in such context can be generalized with equal performance to systems with a different constitution in terms of number and types of robots. A framework which can deal with various forms of heterogeneous MRS is the area which needs more investigation.

The focus of this paper is to propose a routing framework that can deal with various forms of heterogeneity in terms of complexity and disparity of types in robot teams for exploration tasks. The proposed framework consists of two main parts:

1) A novel combinatorial optimization problem formulation, called Generalized Team Orienteering Problem (GTOP), for the routing of heterogeneous MRS in distributed tasks.

2) A solving algorithm for GTOP based on Self Organizing Map (SOM) with a constant time complexity in the number and the types of robots in the system. This characteristic alleviates the scalability limitations encounter with centralized planning approaches.

In this paper, the properties of the proposed framework are evaluated in a two-stage manner. Firstly, the solution quality and computational complexity of the solving algorithm are evaluated under 2D static pre-known task distributions. The solution quality of the proposed framework is compared with that of the greedy solution. Secondly, the effectiveness of the proposed framework is examined under an exploration and mapping task in a two-dimensional environment where a team of robots are equipped with different and overlapping abilities. The framework is required to solve problems repetitively in this scenario because of the nature of the exploration tasks in which new information appears as the exploration proceeds. Therefore the constant complexity of the proposed solving algorithm plays a key role. The performance of exploration is compared with conventional greedy exploration scheme.

This paper is organized as follows. Section 2 reviews related works in the field of MRTA for heterogeneous MRS. Section 3 introduces the proposed framework, which is composed of a combinatorial problem formulation and the SOM based solving algorithm. In section 4, the performance of the proposed framework is evaluated and compared to conventional approaches. Finally, section 5 concludes the paper.

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II. Related Works

Multi-Robot Task Assignment (MRTA) is an intensively studied topic [11], [12]. Previous works have proposed terminologies for classifying MRTA according to its complexity by using the terms Single-Task (ST) or Multi-Task (MT) and Single-Robot (SR) or Multi-Robot (MR) and Instantaneous-Assignment (IA) or Time-extended-Assignment (TA). Problems in which each task requires only one robot to be executed, each robot can only perform one task at the same time, and only one task is assigned to one robot, are of the class ST-SR-IA. If a task requires multiple robots and a robot can perform multiple tasks at one time and the sequence of tasks is ordered to be performed in the future, then this problem is class MT-MR-TA. Routing is a TA class of problem because the solution requires the sequence of tasks to be executed. The computational complexity of the problems in the class TA belongs in the most cases to the class NP-hard like the well known Travelling Salesman Problem (TSP), meaning that solving algorithms for these problems are computationally demanding. In exploration scenarios, the task distribution is by nature not static as new points of interest are discovered as the exploration proceeds. Thus a lot of papers which discuss exploration problems propose approaches that iteratively solve IA problem at each exploration step in order to avoid too heavy computational load [13], [14]. However, some researchers reveal that solving the routing problem at each exploration step largely improves the exploration performance [15], [16]. But these methods are restricted to small problems because of the complexity of the problems and only tested for single robot cases.

Several routing frameworks that explicitly deal with heterogeneous MRS have been proposed [17], [18]. Recent research introduces frameworks which can deal with a complicated heterogeneous system with overlapping abilities. The main difference between these approaches and the proposed framework in this paper lies in the computational complexity. The proposed algorithm has a constant complexity in the number of types and robots. This is an important property for exploration tasks for which repetitive planning is required.

The proposed framework in this paper solves ST-MR-TA routing problems for exploration while keeping the computational load small enough. The solution is obtained by solving the combinatorial problem GTOP in a centralised manner. The proposed problem, GTOP is a generalised version of TOP (Team Orienteering Problem). TOP is a route optimization problem for multi-robot in order to maximize the reward which can be collected by following the route while considering the maximum cost budget that each robot can afford [19]. Different from TOP, GTOP focuses on heterogeneous requirements of tasks and heterogeneous abilities of robots in a team. This paper introduces a SOM based algorithm for solving the GTOP. SOM based methods for solving routing problems such as TSP or MTSP can be found in [21], [22]. In these works, the solution quality of the SOM based approaches fall behind to the state of the art approximated algorithms in such well known and studied problems. On the contrary for newly defined problem framework like GTOP which is not well studied yet and effective approximation is not found, introducing a SOM based approach is supposed to be reasonable. Moreover, the scalability of the SOM based algorithm is attractive for applications where large numbers of robots are used.

III. The Proposed Scheme

The proposed framework is composed of a combinatorial route optimization problem which explicitly deals with heterogeneous types, abilities and cost budgets of robots in a team and a SOM based solving algorithm.

A. Generalised Team Orienteering Problem (GTOP)

This paper defines a combinatorial optimization problem, the GTOP which is a routing problem for heterogeneous MRS. Tasks which have heterogeneous requirements are distributed in an environment and the heterogeneous team of robots move in the environment to carry out tasks and gain some reward with considering the restriction of their maximum cost budget. Such cost is defined as the length of the robot's route in this paper. The objective is to find an optimal sequence of tasks to be executed by the team of robots without violating the maximum cost budgets of each individual robot. Fig.1 illustrates the concept of GOTP in an exploration scenario. There are numbers of observation tasks which require different types of sensor or mobility distributed in the field. Each task gives different values of reward for each ability which means that higher reward is obtained when the arrived robot have the suitable ability set for the task. Additionally, an important feature of this problem is that the solution of GTOP does not necessarily include all the tasks in space due to the maximum cost constraints. Due to this constraint, the solution is practical for field robot applications which are often imposed severe energy limitation such as disaster rescue or space exploration.

Suppose that N_R is the number of robots considered in the problem, then a set of robots is represented as $\mathcal{R} : \{R_1, \cdots, R_{N_R}\}$. A robot $R_i \in \mathcal{R}$ is a tuple of a coordinate, a set of abilities and a maximum affordable cost represented by $R_i = (RP_i, A_i, C_i^{Max})$. $\mathcal{A} : \{a_1, \cdots, a_{N_A}\}$ is a set of all abilities where N_A is the total number of abilities considered in the problem. A robot has more than one abilities from this ability set, i.e. $A_i \subset \mathcal{A}$. In this research, tasks are distributed in two dimensional space, thus $RP_i = (X_i, Y_i)$. In this study, heterogeneity of the system is defined by the number of the types of robots N_{Ty}. A group of robots with the identical combination of abilities is counted as a type i.e., if $N_A = 2$, $1 \leq N_{Ty} \leq 3$.

A set of tasks is represented by $\mathcal{T} : \{T_1, \cdots, T_{N_T}\}$ where N_T is the number of tasks considered in the problem. A task $T_i \in \mathcal{T}$ is a tuple which consists of coordinate and reward, thus $T_j = (TP_j, Rwd_j)$ where $TP_j = (X_j, Y_j)$. Reward value of T_j is defined for each individual ability, therefore Rwd_j is a N_A dimensional vector as it is represented by $Rwd_j = (rwd_j^{a_1}, \cdots, rwd_j^{a_{N_A}})$.

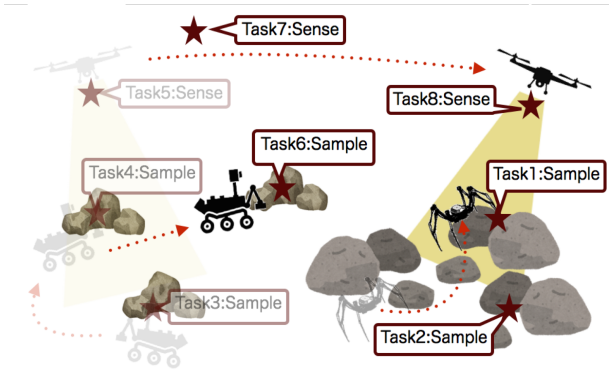

Fig. 1. GTOP is a route optimization problem dealing with the heterogeneity of the team of robots. The objective is to maximize the total reward obtained by the team. Not all tasks are necessarily included in the solution considering the maximum cost budget of robots. Transparent tasks described in the figure represent tasks that are already executed by robots.

The solution $\mathcal{S}$ is a set of routes (sequences of tasks) for the team of robots thus $\mathcal{S} : \{\mathcal{S}_1, \cdots, \mathcal{S}_{N_R}\}$ and $\mathcal{S}_i : \{T_i^1, \cdots, T_i^{N_{S_i}}\}$ where $T_i^k \in \mathcal{T}$. N_{S_i} is the number of tasks assigned to R_i, and N_{S_i} can be 0 if needed. $\mathcal{S}$ does not need to include all the tasks in the problem, i.e. $\sum_i^{N_R} N_{S_i} \leq N_T$. C_i is the cost for executing $\mathcal{S}_i$ and it is given by the Euclidean distance as described by Eq.(1). In Eq.(1), the integral symbols and arrows represents calculation of the Euclidean distance between two points.

$$C_i = \sum_{j=1}^{N_{S_i}-1} \int \{TP_i^j \rightarrow TP_i^{j+1}\} dl + \int \{RP_i \rightarrow TP_i^1\} dl \tag{1}$$

Each robot R_i has the maximum cost budget C_i^{Max}. The obtained solution $\mathcal{S}_i \in \mathcal{S}$ must satisfy $C_i \leq C_i^{Max}$. When R_i arrives at T_j, the obtained reward is calculated by Eq.(2).

$$RWD(Rwd_j, A_i) = \sum_{a \in \mathcal{A}_i} rwd_j^a \tag{2}$$

The objective of the problem is to obtain $\mathcal{S}$ that maximise total reward collected by the team as expressed by Eq.(3).

$$\text{Maximize} \sum_{\mathrm{R_i} \in \mathcal{R}} \sum_{\mathrm{T_j} \in \mathcal{S}_\mathrm{i}} \mathrm{RWD}(\mathrm{Rwd_j}, \mathrm{A_i}) \tag{3}$$

GTOP is an ST-MR-TA task assignment problem. This problem is NP-hard and this is inferred from the fact that this problem structure consists of MTSP (Multi Travelling Salesman Problem) and KP (Knapsack problem), both being NP-hard problems. For practical applications, a way of finding a solution for such a problem in a reasonable amount of time is needed. Therefore the proposed framework adopts a heuristics based on SOM algorithm. This algorithm has a strong advantage in terms of scalability as shown in the following part.

B. The Solving Algorithm

1) The Self Organizing Map: SOM is an unsupervised learning framework formed by a two-layered competitive neural network that presented by Kohonen's in the 1980s [23]. The idea of SOM is to obtain a topological representation of data in high dimensional space using a graph-based structure in low dimensional space. In the SOM algorithm, simulated neuron structure adapts their topologies to the given data similarly to the biological neural networks adapting to given stimulations. For routing problems, the result of the neuron structures become approximated solutions. In this study, the set of high dimensional data given to SOM is the two dimensional coordinate of tasks $TP_j \in \mathcal{T}$. The structure of a SOM is one-dimensional neuron sequence where each neuron has a weight vector $n \in R^k$. Then k is identical to the dimension of the given data ($k = 2$ in this study). Thus a SOM is represented by a sequence of weight vector of neurons, i.e. $\mathcal{SOM} = \{n_1, \cdots, n_{N_n}\}$ where N_n is the number of neurons in the SOM. Adaption process is realised by updating the weight vectors $n \in \mathcal{SOM}$ using the "winner take all" learning principle (Eq.(4),(5)).

$$v = \min_{n_p \in \mathcal{SOM}} ||TP_j - n_p|| \tag{4}$$

$$n_q = n_q + \mu h_G(t, v, n_q)(TP_j - n_q) \tag{5}$$

$$h_G(t, v, n_q) = e^{\frac{-||v - n_q||^2}{G(t)^2}} \tag{6}$$

For a given coordinate $TP_j \in \mathcal{T}$, a winner neuron from $\mathcal{SOM}$ is selected by Eq.(4). Then the winner neuron and its neighbouring neurons in $\mathcal{SOM}$ update their weight vectors by following Eq.(5) to Eq.(6). In the equations, μ is called the "adaptation rate". Then $h_G(t, i, v)$ is called neighbourhood function, which defines the range of neighbour in $\mathcal{SOM}$ from the winner neuron. This function is governed by $G(t)$, which decreases by "decreasing rate" α with iteration. Due to this decreasing rate, the adaptation process converges when sufficient iteration passes. The weight values of the neurons are updated during the adaptation process while the alignment of sequences is preserved. This causes a topology change of the SOM structure and the converged topology can be seen as an approximate solution of a routing problem (Fig.2).

2) The Proposed Algorithm Structure: The proposed algorithm adds modification to pure SOM adaptation mechanism in terms of pseudo adaptation rate of μ' and a constraint checker so that the converged topology of SOM becomes an approximated solution of GTOP. The main process of the proposed algorithm is shown in Algorithm 1. The outer loop from line 4 to 20 is called an epoch. The algorithm takes a set of tasks $\mathcal{T}$, a set of robots $\mathcal{R}$ and the initial positions of robots RP_0 as inputs. The output is the solution of GTOP, $\mathcal{S}$. A set of multiple one dimensional SOMs $\mathcal{SOM}_{all} : \{\mathcal{SOM}_1, \cdots, \mathcal{SOM}_{N_R}\}$ is initialized at line 1. $\mathcal{SOM}_i$ corresponds to the route for $R_i \in \mathcal{R}$ and it is

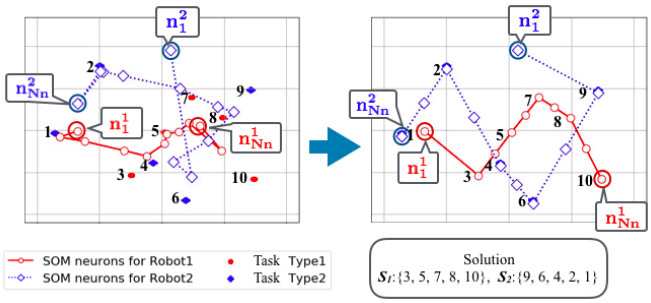

Fig. 2. The adaptation process of one dimensional SOM neuron sequences in two dimensional task space is illustrated. The solution can be obtained by tracing the converged topology of the SOMs. n_0^i is fixed to the initial position of robot R_i throughout the adaptation process.

composed of weight vectors of a sequence of neurons i.e. $\mathcal{SOM}_i : \{n_1^i, \cdots, n_{N_n}^i\}$ as is discussed in the previous subsection. The total number of SOM neurons must be larger than N_T. In our study, the total number of neurons are determined as 3 times N_T ($N_R N_n = 3N_T$). In each adaptation epoch, a task $T_j \in \mathcal{T}$ is given to $\mathcal{SOM}_{all}$ in permutated order. A pseudo adaptation rate of μ' is calculated by Eq.(7).

$$\mu' = \frac{RWD(Rwd_j, A_i)}{\sum_{a \in A_i} rwd_j^a} \tag{7}$$

The idea of the pseudo adaptation rate is that a large adaptation occurs to the route of a robot which has a suitable ability set for the requirement of the task. The subroutine "Adapt" at line 10 performs the SOM adaptation defined from Eq.(4) to Eq.(6). The Violation Check at line 11 is a violation checker testing whether or not the Euclidean length of the temporal solution violates the cost budget or not. The result of the adaptation $\mathcal{SOM}_i'$ is accepted only when the total length of the neuron sequence satisfies the constraint. In one adaptation epoch, only one $\mathcal{SOM}_i \in \mathcal{SOM}_{all}$ is allowed to perform adaptation to a given coordinate TP_j which is selected by the subroutine "BestRouteSelection" (Line 15 in Algorithm 1 and described in Algorithm 2). At the end of the adaptation epoch, the parameter G (Eq.(6)) is decreased by the rate α then finally the topology of $\mathcal{SOM}_{all}$ converges after a certain number of iterations. After the convergence, the solution $\mathcal{S}$ is obtained by tracing the weight vectors of $\mathcal{SOM}_{all}$ (Line 21 in Algorithm 1 and described in Algorithm 3). Although the authors observe that the algorithm stably converges with the finite number of iterations, convergence is not theoretically guaranteed within the algorithm structure. Providing convergence guarantee to the proposed algorithm is the future work of the authors.

The advantage of the proposed algorithm is that the computational complexity is $O(N_T^2)$ and constant in N_R and N_{Ty}. Intuitively, the complexity seems to scale in both N_T and N_R as the double loop of $\mathcal{T}$ and $\mathcal{R}$ is observed in Algorithm 1. However, the computation for SOM adaptation requires the order of N_n, thus the second loop actually scales in the total number of neurons in $\mathcal{SOM}$, i.e. $N_R N_n$. Remember that this value is determined by an integer multiple of N_T ($3N_T$ in this study), the complexity of the algorithm actually scales in N_T^2 and is constant to other numbers. Such complexity is advantageous when routing MRS which has large N_R and N_{Ty}. Moreover, it is even possible to reduce the computational time when the number of robots increases if the system can distribute the calculation process from line 8 to 13 in Algorithm 1 to each robot. The proposed algorithm is only applicable to GTOP where tasks are distributed in the Euclidean space. But many practical applications of multi-robots such as automated delivery using robots, search and rescue and exploration can be modelled in the Euclidean space.

Algorithm 1 MainProcess

INPUT $\mathcal{T}$, $\mathcal{R}$, RP_0
OUTPUT $\mathcal{S}$
PARAMETERS μ_0, G_0, α

1: $\mathcal{SOM} \leftarrow$ **InitSOM**(RP_0)
2: Error $\leftarrow$ Inf
3: $\mathrm{G} \leftarrow \mathrm{G}_0$
4: **while** Error $>$ Threshold **do**
5: **Perm**($\mathcal{T}$)
6: **for** each $T_j \in \mathcal{T}$ **do**
7: $\mathcal{SOM}' \leftarrow$ **Copy**($\mathcal{SOM}$)
8: **for** each $R_i \in \mathcal{R}$ **do**
9: $\mu \leftarrow \mu' \mu_0$
10: $\mathcal{SOM}_{Tmp} \leftarrow$ **Adapt**($\mathcal{SOM}_\mathrm{i}'$, $\mathrm{TP_j}$, μ, G, α)
11: **if** not **ViolationCheck**($\mathcal{SOM}_\mathrm{Tmp}$, C_i^{Max}) **then**
12: $\mathcal{SOM}_r' \leftarrow \mathcal{SOM}_{Tmp}$
13: **end if**
14: **end for**
15: $i \leftarrow$ **BestRouteSelection**($\mathcal{SOM}'$, $\mathrm{TP_j}$, $\mathcal{R}$)
16: $\mathcal{SOM}_i \leftarrow \mathcal{SOM}_i'$
17: Error $\leftarrow$ **max**(Error, $\min_{\mathrm{n} \in \mathcal{SOM}_\mathrm{i}} ||\mathrm{TP_j} - \mathrm{n}||$)
18: **end for**
19: $\mathrm{G} \leftarrow (1-\alpha)\mathrm{G}$
20: **end while**
21: $\mathcal{S} \leftarrow$ **Trace**($\mathcal{SOM}$, $\mathcal{T}$)

Algorithm 2 BestRouteSelection

INPUT TP_j, $\mathcal{SOM}$, $\mathcal{R}$
OUTPUT Index

Score $\leftarrow \phi$
for each $R_i \in \mathcal{R}$ **do**
Distance $\leftarrow 0$
for each $n_l^i \in \mathcal{SOM}_i$ **do**
Distance $\leftarrow$ Distance $+ ||\mathrm{TP_j} - \mathrm{n_l^i}||$
end for
s $\leftarrow$ RWD($\mathrm{Rwd_j}$, $\mathrm{A_i}$)/Distance
Score $\leftarrow$ **Add**(Score, s)
end for
Index $\leftarrow$ $\mathbf{argmax}_{\mathrm{s} \in \mathrm{Score}}$(s)

Algorithm 3 Trace

INPUT $\mathcal{SOM}$,$\mathcal{T}$
OUTPUT $\mathcal{S}$

$\mathcal{S} \leftarrow \phi$
for each $R_i \in \mathcal{R}$ **do**
  $\mathcal{S}_\mathrm{i} \leftarrow \phi$
  for $n_l^i \in \mathcal{SOM}_i$ **do**
    $\mathrm{T_j} \leftarrow \mathrm{GetClosest}(\mathrm{n_l^i}, \mathcal{T})$
    if $||TP_j - n_l^i|| \leq \mathrm{Threshold}$ **then**
      $\mathcal{S}_i \leftarrow \mathbf{Add}(\mathcal{S}_\mathbf{i}, \mathrm{t})$
    end if
  end for
  $\mathcal{S} \leftarrow \mathbf{Add}(\mathcal{S}, \mathcal{S}_\mathbf{i})$
end for

Algorithm 4 GreedyGTOPSolver

INPUT $\mathcal{T}$, $\mathcal{R}$, RP_0
OUTPUT $\mathcal{S}$

$\mathcal{T}_\mathrm{Available} \leftarrow \mathbf{Copy}(\mathcal{T})$
$\mathrm{Pos} \leftarrow \mathrm{Pos_0}$
$\mathrm{Flag} \leftarrow \mathrm{True}$
$\mathcal{S} \leftarrow \phi$
$\mathrm{Perm}(\mathcal{R})$
for $r_i \in \mathbf{R}$ **do**
  $\mathcal{S}_\mathrm{i} \leftarrow \phi$
  while Flag **do**
    $\mathrm{Flag} \leftarrow \mathrm{False}$
    $\mathcal{S}_\mathrm{i}' \leftarrow \mathcal{S}_\mathrm{i}$
    $\mathrm{t} \leftarrow \max_{\mathrm{T_j} \in \mathcal{T}_\mathrm{Avairable}} \mathrm{RWD}(\mathrm{Rwd_j}, \mathrm{A_i})/||\mathrm{TP_j} - \mathrm{RP_i}||$
    $\mathcal{S}_\mathrm{i}' \leftarrow \mathbf{Add}(\mathcal{S}_\mathbf{i}', \mathrm{t})$
    if $\mathrm{not}\mathbf{ViolationCheck}(\mathbf{S_i'}, \mathbf{C_i^{Max}})$ **then**
      $\mathcal{S}_\mathrm{r} \leftarrow \mathcal{S}_\mathrm{i}'$
      $\mathcal{T}_\mathrm{Avairable} \leftarrow \mathbf{Remove}(\mathcal{T}_\mathrm{Available}, \mathrm{t})$
      $\mathrm{Flag} \leftarrow \mathrm{True}$
    end if
    $\mathcal{S} \leftarrow \mathbf{Add}(\mathcal{S}, \mathcal{S}_\mathbf{i})$
  end while
end for

IV. Evaluation

A. SOM Based Algorithm

In this section, the solution quality and the computation complexity of the proposed GTOP solving algorithm (Algorithm 1) are evaluated on static random task distributions. The solution quality is the total of collected reward by the team of robots, as defined in Eq.(3). In the following tests, $\mu = 0.9$, $\alpha = 0.04$ and $G_0 = 100$ are chosen as SOM hyper-parameters. Tasks are distributed in 50[m] square environment with the maximum affordable cost of each robot C_i^{Max} is 50[m] equally to each $R_i \in \mathcal{R}$. The tests are performed for 100 patterns of trials by using a desktop PC equipped with Intel Core i7 CPU and 40GB RAM.

1) Solution quality: The quality of the GTOP solutions that are obtained by the proposed algorithm is compared to the quality of solutions which are obtained by the greedy algorithm (Algorithm 4). For the comparison, problem size of $N_T = 100$ and $N_A, N_{Ty} = 4$ is used. There are 4 types of tasks $T_i^1, T_j^2, T_k^3, T_l^4 \in \mathcal{T}$ with inclined reward distributions, i.e. $Rwd_i^1 = \{1, 2, 3, 4\}, Rwd_j^2 = \{2, 3, 4, 1\}, Rwd_k^3 = \{3, 4, 1, 2\}, Rwd_l^4 = \{4, 1, 2, 3\}$ for any i, j, k, l ($\sum T_i^1 + \sum T_j^2 + \sum T_k^3 + \sum T_l^4 = N_T$) are defined in the test environments. The evaluations are performed under 100 patterns of task distributions and the result is shown in Fig.3. It is confirmed that the quality of the solutions of the proposed algorithm outperforms that of the greedy algorithm for every N_R.

2) Complexity: The computational complexity of the proposed algorithm in N_R, N_{Ty} and N_T is also evaluated. In the same manner as in the previous comparison, GTOP for randomly distributed static tasks is solved by the proposed algorithm for 100 patterns of environments. Fig.4 shows the computational time scalability in the number of robots N_R and the number of types of the team N_{Ty} under the fixed number of tasks $N_T = 50$, and Fig.5 shows the computational time scalability in N_T and N_{Ty} under fixed number of robots $N_R = 8$. It can be seen that the computation time of the proposed algorithm scales only in T_N^2 as is discussed in the previous section. In Fig.5, τn^2 is shown as reference where $\tau = 1.0 \times 10^{-5}$.

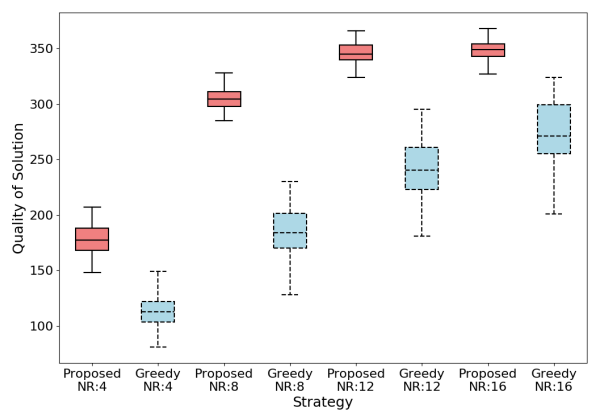

Fig. 3. Quality of the solution obtained by the proposed algorithm in red and by the greedy solver in blue.

B. Application for Exploration and Mapping Tasks

In this section, the proposed framework is applied to an exploration and mapping task which backgrounds planetary cave exploration by heterogeneous MRS. In exploration tasks, repetitive task allocation is required since new points interest are discovered as the exploration proceeds. To reduce the computational requirement, many prior works in this domain proposed approaches that solve IA task assignment problems repetitively at each exploration step as discussed in Section 2. However, the back and forth motion of robots is often observed if the algorithm does not consider tasks in future time steps. By taking advantage of the low computational complexity of the proposed framework, exploration

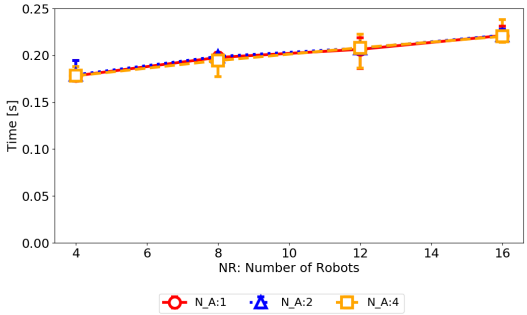

Fig. 4. Scalability of computation time in N_R in an epoch.

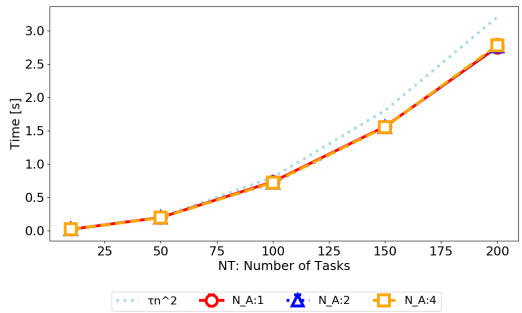

Fig. 5. Scalability of computation time in N_T in an epoch.

performance of a heterogeneous team of robots, where TA task assignment is repetitively solved by the proposed framework, is examined in this section.

An exploration and mapping task which models moon caves exploration scenario is introduced for the simulation setting. Moon caves are attractive targets to investigate in the next decades [24], [25]. Compared with the conventional single robot exploration approaches, MAS have advantages on the capability of establishing a communication network and tolerance to unexpected contingencies for such applications. In this setting, it is assumed that a heterogeneous team of robots are brought to the bottom of the cave by a tethered mother robot [25]. Then the robots explore the bottom of the cave to construct an environment map in terms of the geometry and the spatial distribution of materials and water resources. The team is composed of robots which are equipped with heterogeneous sensors.

In this exploration and mapping task, the region of interest (ROI) is defined in areas that science interests probably exist. These ROIs are assumed to be able to be defined by analyzing images obtained by robots. During exploration, robots are assumed to be communicable to the centralised module which remains at the initial location where robots were deployed. It is also assumed that localization errors of robots are negligible in this simulation. The objective of

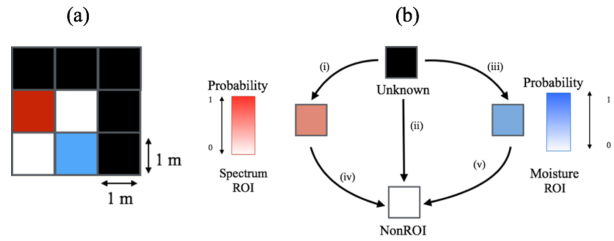

Fig. 6. The environment map representation and the model of state transition of cells.

exploration is to explore the area as broad as possible as well as to find as many ROIs as possible to give them observation by corresponding sensors.

1) Robot Model: A heterogeneous team of robots with a set of abilities (Camera, Spectrometer, Moisture Sensor) is investigated in this paper. Camera represents a stereo camera and light source to obtain geometric information to find ROI in the cave environment. Spectrometer represents a sensor module to obtain material information and Moisture sensor represents a module that can investigate the existence of water resources on the spot. Assuming that the team is consists of 4 types of robots {Hoppers, Small Rover1, Small Rover2, Medium Rover} where each type of has ability set as follows, {(Camera), (Camera, Spectrometer), (Camera, Moisture Sensor), (Camera, Spectrometer, Moisture Sensor)}. Hoppers are hopping robots which are only equipped with cameras. The robots of this type contribute to expanding the map to find ROIs. Small Rover1s and Small Rover2s are small size rovers which are equipped with only one type of science sensors, respectively. Medium Rovers are medium size rovers which are equipped with all types of sensors. The robots of this type play a versatile role in the exploration. Additionally, the following assumptions are introduced in simulations. Sensors can sense every direction and sensing range of different types of sensors are identical, and robots can move in any direction straightly. Uncertainty in sensing and mobility is not considered in this simulation. The battery is consumed only by moving. Energy consumption for sensing and communication are not considered.

2) Environment Map Representation: The environment map is represented by a two-dimensional grid map. The exploration area is gridded by 1[m] square cells as shown in Fig.6(a), and the condition of state transitions of a cell is depicted in Fig.6(b). The state transition (i) is caused when the ”Unknown” cell is a ”Spectrum ROI” and the cell is visited by robots which do not have the ability ”Spectrometer”. If this cell is visited by robots which have the ability ”Spectrometer”, scientific observation is given and the cell turns to ”NonROI” (State transition (ii) and (iv)), and the same way for Moisture ROIs (state transition (ii), (iii), (v)).

3) Task Generation and Reward Definition: To complete this exploration and mapping task, exploration tasks and measurement tasks are generated. Exploration tasks are defined on ”Unknown” cells which face boundary of the

explored region by following the conventional frontier cell exploration scheme [26]. Measurement tasks are generated on ROIs which require scientific measurements by corresponding sensors. Note that the existence and locations of ROIs are not known at the beginning of the exploration. For an exploration task T_e, the reward Rwd_e is defined by $\{\mathrm{R}, 0, 0\}$ for abilities (Camera, Spectrometer, Moisture sensor) using a natural number $R > 0$. For a measuring spectrum task T_s and a measuring moisture task T_m, Rwd_s and Rwd_m are defined by $\{0,\mathrm{R}, 0\}$ and $\{0, 0,\mathrm{R}\}$ respectively.

C. Evaluation of the Exploration Performance

The exploration performance P of the team of robots is evaluated at the end of the exploration by using the Eq.(8). TE_i is a set of tasks that executed by R_i and RWD_{All} is the obtained reward if the area is perfectly explored (The states that all cell turned to "NonROI").

$$P = 100 \times \sum_{R_i \in \mathcal{R}} \sum_{s \in TE_i} RWD(s, A_i)/RWD_{All} \quad (8)$$

D. Simulation Result

1) Parameter Setting: The area of exploration is assumed to be 50[m] square and the number of the spectrum ROIs and the moisture ROIs hidden in the exploration area is set to be 125 each. The maximum travelling distance due to battery limitation of each robot is assumed to be 50[m]. The radius of the sensing range of each robot is assumed to be 2[m]. The reward value $R = 10$ is used in the following simulation. Simulation tests are performed under 20 patterns of the environment.

2) Exploration Performance: The performance is compared to a simple IA exploration scheme (greedy approach). In this greedy approach, the exploration area is divided by each type of robots as shown in Fig.7. In Fig.7 and Fig.8, the exploration performances by the heterogeneous team robots are plotted ($\tau_0 < \tau_1 < \tau_2 < \tau_f$). {Hopper, Small Rover1, Small Rover2, Medium Rover} are represented by {Circle, Triangle, Diamond, Pentagon} respectively. Fig.9 shows the comparison of the performance between two schemes. The performance of the proposed framework outperforms the performance of greedy exploration method at any N_R and the gap is larger for larger N_R.

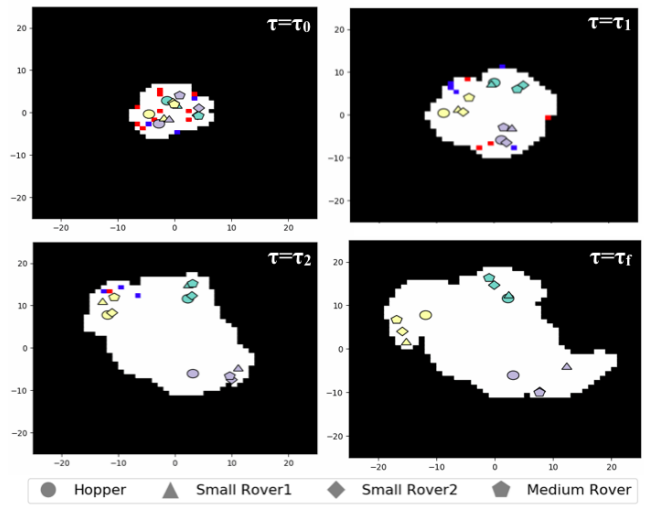

Fig. 7. Exploration performance of the heterogeneous team of robots ($N_R = 12$, $N_{Ty} = 4$) using the greedy scheme.

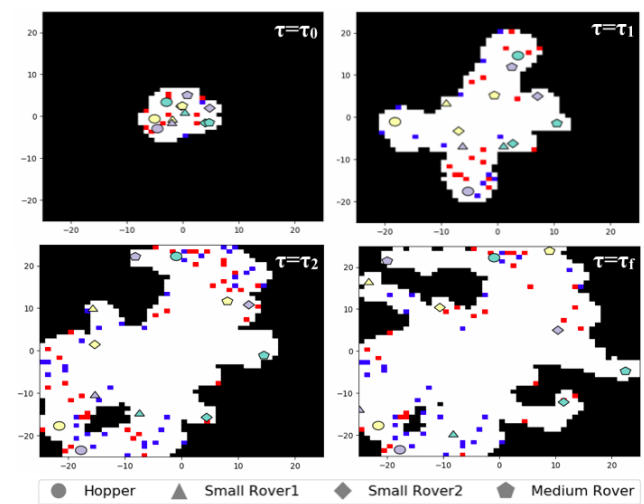

Fig. 8. Exploration performance of the heterogeneous team of robots ($N_R = 12$, $N_{Ty} = 4$) using the proposed scheme.

V. Conclusion

This paper proposes a routing framework for a heterogeneous team of robots by introducing a mathematical problem formulation and a SOM based solving algorithm for exploration and mapping tasks. The performance of the proposed framework is evaluated in simulations which model the moon cave exploration scenario. The proposed approach has a great advantage in terms of scalability to the number of robots and heterogeneity of the team. The results show that the gap in performance between the proposed method and the considered counterpart increases with the number of robots. The contributions of this paper are listed as follows:

- Introducing a mathematical framework, GTOP for multi-robot task execution which explicitly deals with the heterogeneity of the team of robots.
- Proposing a SOM based algorithm to solve the problem formulation GTOP which does not computationally scale to the size of the team of robots.
- An evaluation of the proposed framework in an exploration task which models planetary cave exploration.

The current state of our method neglects several problems that robot systems encounter in real-world applications such as communication distance limits and uncertainties in terms of mobility and sensing. Future work includes modifying the framework to deal with those conditions.

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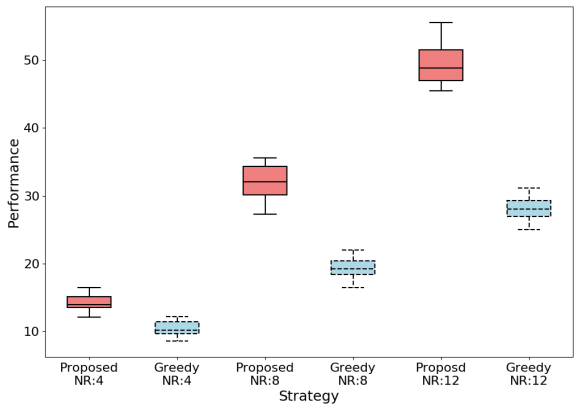

Fig. 9. Comparison of exploration performance between two schemes. Red: the proposed approach. Blue: greedy exploration scheme

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