

Dynamic Median Consensus for Marine Multi-Robot Systems Using Acoustic Communication

Goran Vasiljević¹, Tamara Petrović¹, Barbara Arbanas¹ and Stjepan Bogdan¹

Abstract—In this paper, we present a dynamic median consensus protocol for multi-agent systems using acoustic communication. The motivating target scenario is a multi-agent system consisting of underwater robots acting as intelligent sensors, applied to continuous monitoring of the state of a marine environment. The proposed protocol allows each agent to track the median value of individual measurements of all agents through local communication with neighbouring agents. Median is chosen as a measure robust to outliers, as opposed to average value, which is usually used. In contrast to the existing consensus protocols, the proposed protocol is dynamic, uses a switching communication topology and converges to median of measured signals. Stability and correctness of the protocol are theoretically proven. The protocol is tested in simulation, and accuracy and influence of protocol parameters on the system output are analyzed. The protocol is implemented and validated by a set of experiments on an underwater group of robots comprising of aMussel units. This experimental setup is one of the first deployments of any type of consensus protocol for an underwater setting. Both simulation and experimental results confirm the correctness of the presented approach.

I. INTRODUCTION

One of the envisioned goals of the subCULTron project was to develop a marine multi-robot system for intelligent long-term monitoring of underwater ecosystems [1]. The underwater system is comprised of 3 different types of robots (Figure 1). Artificial mussels (aMussels) are sensor hubs attached to the sea bottom, which monitor natural habitat, including biological agents like algae, bacterial incrustation, and fish. They serve as the collective long-term memory of the system, allowing information to persist beyond the runtime of other agents, enabling the system to continue developing from previously learned states [2], [3]. On the water surface, artificial lily pads (aPads) interface with humans, delivering energy and information influx from ship traffic or satellite data [4]. Between those two layers, artificial fish (aFish) move, monitor and explore the environment and exchange information with aMussels and aPads.

An underwater swarm consisting of a large number of units (100 aMussels, 10 aFish and 5 aPads) was developed within the project. The ability of such a system to compute (in a decentralized manner) common estimates of unknown quantities (such as measurements), and agree on a common

This work was in part supported by the EU-H2020-FETPROACT-2-2014 project subCULTron (grant agreement No. 640967) and EU-H2020 CSA project AeRoTwin (grant agreement No. 810321).

¹Goran Vasiljević, Tamara Petrović, Barbara Arbanas and Stjepan Bogdan are with University of Zagreb Faculty of Electrical Engineering and Computing, Laboratory for Robotics and Intelligent Control Systems (LARICS), Unska 3, Zagreb 10000, Croatia; goran.vasiljevic@fer.hr



Fig. 1: Robots of the subCULTron swarm – aMussels (left), aFish (middle) and aPad (right)

view of the world, is critical. Consensus protocols (algorithms) are particularly compelling for implementation in multi-agent systems due to their simplicity and a wide range of applications. Foundation of consensus protocols in multi-agent systems lies in the field of distributed computing. In networks of agents, *consensus* means to "reach an agreement regarding a certain quantity of interest that depends on the state of all agents" [5]. A consensus protocol is a series of rules that define information exchange between an agent and all of his neighbors on the network, as well as internal processing of the obtained information by each agent.

A good overview of consensus protocols can be found in [5], [6], [7]. Our primary target is subCULTron system or more precisely, a network of underwater robots measuring environment parameters such as oxygen or turbidity, with sensors prone to faults/errors (outliers). Consensus protocols can be exploited to increase the reliability of such a system by i) developing protocols for detection of faulty agents (such as trust consensus protocols [8], [9]), or ii) developing consensus protocols that implicitly account for and nullify the influence of outlier values.

In this paper we apply the later approach - the goal is for each agent to reach a consensus on the real value of the measuring signal. The most common type of consensus problems - average consensus ([10], [11]) - is not appropriate for this application since even one faulty agent with abnormal values might skew the consensus to incorrect values, which do not represent the real value of the measuring signal. On the other hand, authors in [12] and [13] use *median* as the chosen measure, which is inherently robust to outliers, and we follow upon their work in this paper.

Further, due to the time-varying nature of environment parameters and requirement for spatial distribution of sensors, measurements are performed by a multi-robot system applying the *dynamic* median consensus protocol, where agents track the median of locally available *time-varying* signals, executing local computations and communicating

with neighboring agents only [14]. Dynamic protocol for the average case is available, for both static and varying communication topology ([15]).

Work presented in this paper is based upon [12] and [13], where authors present a dynamic median consensus protocol. In this paper we modify their approach so that protocol is functional under scheduled communication protocol, like the one using acoustics for underwater communication. Systems with such a communication scheme are called *switching systems* and their analysis is more complex compared to analysis of static systems. Our previous work [2] and [3] deals with consensus using switching systems, but the work presented in this paper reaches the median value rather than the average. Another difference, compared to our previous work, is addition of the dynamic component to the consensus protocol. To conclude, **the main contribution** of the paper is analysis of novel consensus protocol that is dynamic, works on a switching communication topology and converges to median value, a combination that (to the best of our knowledge) no other papers study.

There have not been many advancements in the area of underwater consensus protocols. As far as we know, the only underwater consensus applications include formation control of tethered and untethered underwater vehicles [16], [17], [18] and tracking of underwater targets using acoustic sensor networks (ASN) [19]. Both of those approaches have been validated only in the simulation environment, using ideal communication channels. Among other things, this paper contributes to the field by providing the first experimental application of a consensus protocol in an underwater multi-agent system, acting as a distributed sensor network.

The paper is organized as follows. In the next section, we define some preliminary definitions and notations regarding the systems we study. We present the implemented method for dynamic median consensus over scheduled acoustic communication in Section III. Simulation results and analysis are presented in Section IV, while the section V describes the robotic platform we used for experiments and shows the achieved results. Finally, we give a conclusion in Section VI.

II. PRELIMINARIES

A. Problem description

We consider a network of n agents communicating over a single communication channel. The underlying communication graph is defined as a directed graph $\mathbf{G} = (\mathbf{V}, \mathbf{E})$, where the set of nodes $\mathbf{V} = \{1, 2, \dots, n\}$ corresponds to agents, and the set of edges $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$ corresponds to communication links between agents. \mathbf{E} is usually described with a corresponding adjacency matrix \mathbf{A} such that $a_{ij} = 1$ if there is a communication link from agent j to agent i , and $a_{ij} = 0$ otherwise.

Each agent i has a local reference signal, denoted $z_i \in \mathbb{R}$, $\mathbf{z} = [z_1 z_2 \dots z_n]$. In this paper this signal is a measurement made by the agent, but in general it could be the value of some other internal or external variable. Internal state of

TABLE I: List of variables used

Variable	Explanation
x_i^k	internal state of i th agent at step k (communicated over the network)
y_i^k	additional internal state of i th agent at step k (not communicated over the network)
z_i^k	measurement of agent i at step k
a_{ij}	1 if agent i is neighbour of agent j , 0 otherwise
$\beta, \alpha, \gamma, \kappa$	algorithm tuning parameters
n	number of agents
r_i	number of neighbours of agent i

agent i in step k is denoted as $x_i^k \in \mathbb{R}$. Let us introduce the following notations.

A *median value* of vector $\mathbf{z} = [z_1 z_2 \dots z_n]$, where elements are listed in ascending order, can be defined as ([13]):

$$m(\mathbf{z}) \in \begin{cases} \{z_{\frac{n+1}{2}}\} & \text{if } n \text{ is odd} \\ [z_{\frac{n}{2}}, z_{\frac{n}{2}+1}] & \text{if } n \text{ is even} \end{cases} \quad (1)$$

Agent dynamics in discrete-time domain can in general be written as:

$$x_i^{k+1} = x_i^k + u_i^k, \quad i \in \{1, 2, \dots, n\} \quad (2)$$

where $u_i^k \in \mathbb{R}$ takes into account both agent's internal states as well as states of neighbouring agents, which are obtained through communication. We say that the system running (2) converges to consensus value c (denoted $x_i^k \rightarrow c$) iff:

$$\exists \delta > 0, \exists k_0 > 0, |x_i^k - c| < \delta, \forall i \in \{1, 2, \dots, n\}, \forall k > k_0 \quad (3)$$

The most common type of such consensus protocols are designed to converge to the *average* value of local reference signals (z_i) of all agents. In this paper we deal with *median* consensus protocol, that is, we propose and analyse a protocol such that all the agents converge to the same value, corresponding to the median of their measurements, $x_i^k \rightarrow m(\mathbf{z})$, which means that internal states of all agents x_i^k will converge to the median of their current measurements \mathbf{z} . Further, since the proposed protocol allows individual agents to track a *time-varying* median of reference signals (measurements), it belongs to a class of *dynamic* consensus protocols.

B. Communication scheme

A communication scheme is such that agents, due to the characteristics of acoustic signals, must take turns transmitting messages. In other words, in order to avoid interference of acoustic signals, one must ensure that agents do not transmit messages in the same time. We opt for the simplest solution, which is a sequential assignment of time-slots to agents, repeated cyclically (*round-robin* [20]). Each time-slot is reserved for sending of messages of one agent. The result of the described scheme is a switching communication topology, as given in Figure 2. The underlying communication graph \mathbf{G} is an union of communication graphs \mathbf{G}_i , $i \in \{1, \dots, n\}$ over one cycle of described round-robin

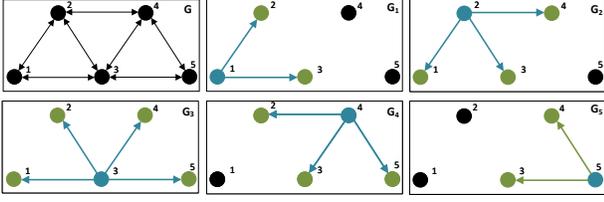


Fig. 2: Scheduled acoustic communication - graph G_i for each time-slot and the underlying graph G . Sending agent (blue), receiving agents (green).

schedule. The same communication scheme was used in [2] and [3], where it was more thoroughly elaborated. Other approaches might be used as well and do not affect the outcome of the proposed protocol.

III. DYNAMIC MEDIAN CONSENSUS

We propose the following decentralized protocol, defined using a local update rule of each agent:

$$\begin{aligned} x_i^{k+1} &= x_i^k + a_{ij}^k (\beta(x_j^k - x_i^k) + \frac{\alpha}{r_i} \text{sign}(z_i^k - x_i^k) + y_i^k) \\ y_i^{k+1} &= y_i^k + a_{ij}^k (\gamma((x_j^k - x_i^k) - \kappa y_i^k)) \end{aligned} \quad (4)$$

where β , α , γ and κ are tuning parameters, y_i^k is an additional internal state of agent i in step k , x_j^k is the value of agent j which is transmitting information over communication channel in step k (when $a_{ij}^k \neq 0$) and r_i is total number of neighbours of agent i , obtained from graph G (including agent i itself). List of the variables used is shown in Table I. Initial value is $x_i^0 = z_i^0$. With respect to other median-reaching algorithms ([12], [13]), the novelty is introduction of the internal state variable y_i , which is needed to ensure convergence for the underlying switching topology. The signum component of the local rule, characteristic for median-reaching consensus, is here specified to account for number of neighbor agents (r_i), which is as well needed to ensure stability due to switching nature of the system.

Theorem 1. For a local rule of each agent defined as (4), and if $y_i \in [-\frac{2\alpha}{r_i}, \frac{2\alpha}{r_i}]$, $\gamma < \beta < \frac{1}{n^2}$, and $\kappa\gamma < 1$, all of the agents converge to the value $m(\mathbf{z})$.

Proof: In order to simplify the equations, let us introduce the following notation $\max_{i \in \mathbf{V}} x_i^k = x_{max}^k$, $\min_{i \in \mathbf{V}} x_i^k = x_{min}^k$. Similar as in [13], we consider a discrete non-smooth Lyapunov candidate:

$$V(\mathbf{x}^k, \mathbf{y}^k) = x_{max}^k - x_{min}^k + y_{max}^k - y_{min}^k \quad (5)$$

In order for a system to be stable, the following conditions need to be met:

$$\begin{aligned} V(0) &= 0 \\ V(\mathbf{x}^k, \mathbf{y}^k) &> 0, \forall \mathbf{x}^k, \mathbf{y}^k \neq \mathbf{0} \quad (6) \\ dV &= V(\mathbf{x}^{k+1}, \mathbf{y}^{k+1}) - V(\mathbf{x}^k, \mathbf{y}^k) \leq 0, \forall \mathbf{x}^k, \mathbf{y}^k \end{aligned}$$

The first two conditions in (6) are always satisfied. The second equation could have value 0 in case when all x_i and all y_i have the same value. However, this case will never occur, because in equation (4), for x_i to be in consensus, all y_i cannot have the same value. The third condition in (6) yields:

$$\begin{aligned} dV &= x_{max}^{k+1} - x_{min}^{k+1} + y_{max}^{k+1} - y_{min}^{k+1} - \\ &\quad - (x_{max}^k - x_{min}^k + y_{max}^k - y_{min}^k) \end{aligned} \quad (7)$$

By taking into account local interaction rule (4), the worst case, from the stability point of view, can be written as:

$$\begin{aligned} x_{max}^{k+1} &= x_{max}^k + \beta(x_j^k - x_{max}^k) + \frac{\alpha}{r_{min}} + y_{max}^k \\ x_{min}^{k+1} &= x_{min}^k + \beta(x_j^k - x_{min}^k) - \frac{\alpha}{r_{min}} + y_{min}^k \quad (8) \\ y_{max}^{k+1} &= y_{max}^k + \gamma((x_j^k - x_{min}^k) - \kappa y_{max}^k) \\ y_{min}^{k+1} &= y_{min}^k + \gamma((x_j^k - x_{max}^k) - \kappa y_{min}^k) \end{aligned}$$

By substituting (8) in (7), we obtain the following:

$$dV = (1 - \gamma\kappa)(y_{max}^k - y_{min}^k) - (\beta - \gamma)(x_{max}^k - x_{min}^k) + 2\frac{\alpha}{r_{min}} \quad (9)$$

For a system to be asymptotically stable $dV < 0$ has to be satisfied. Given $\beta > \gamma$, and, as stated in Theorem 1, y_{max} and y_{min} are limited to $[-\frac{2\alpha}{r_{min}}, \frac{2\alpha}{r_{min}}]$, with $\kappa\gamma < 1$, we get:

$$x_{max}^k - x_{min}^k > \frac{6\alpha - 4\kappa\gamma\alpha}{r_{min}(\beta - \gamma)} \quad (10)$$

The condition (10) is the worst possible case, that is, the right side of the inequality is the lowest theoretical bound on the values of $x_{max}^k - x_{min}^k$.

When (10) is true, the energy in the system will dissipate until the condition in (10) becomes false. In that moment $x_{max} - x_{min}$ will start increasing until equation (10) becomes true. By getting closer to the consensus, y_i^k will converge to either $\frac{\alpha_i}{r_i}$ or $-\frac{\alpha_i}{r_i}$.

Hence, the system is stable under the given condition, however, the system (4) does not reach a steady state in the classical sense. We assume that the steady state is reached when the values over one communication cycle of length n become stationary, that is, when $x_i^{k+n} = x_i^k$, $y_i^{k+n} = y_i^k$, $\forall i$, $\forall k > k_0$. From (4) we get:

$$\begin{aligned} x_i^{k+n} &= x_i^k + \beta \sum_{j=1}^n a_{ij}(x_j^{k+j} - x_i^{k+j}) + \sum_{j=1}^n a_{ij} y_i^{k+j} \\ &\quad + \frac{\alpha}{r_i} \sum_{j=1}^n a_{ij} \text{sign}(z_i - x_i^{k+j}) \\ y_i^{k+n} &= y_i^k + \gamma \sum_{j=1}^n a_{ij}(x_j^{k+j} - x_i^{k+j}) - \gamma\kappa \sum_{j=1}^n a_{ij} y_i^{k+j} \end{aligned} \quad (11)$$

By summing the expressions for y_i^{k+n} in (11) for all agents i in stationary state we get:

$$\begin{aligned} & \sum_{i=1}^n \sum_{j=1}^n a_{ij} (x_j^{k+j} - x_i^{k+j}) = \\ & \kappa \sum_{i=1}^n \frac{-\alpha}{r_i(1+\beta\kappa)} \sum_{j=1}^n a_{ij} \text{sign}(z_i - x_i^{k+j}) \end{aligned} \quad (12)$$

In equation (12) if we assume that during n steps, the value of $\text{sign}(z_i - x_i^{k+j})$ doesn't change for any i , the following must hold:

$$\sum_{i=1}^n \sum_{j=1}^n a_{ij} (x_j^{k+j} - x_i^{k+j}) = \frac{-\alpha\kappa}{1+\beta\kappa} \sum_{i=1}^n \text{sign}(z_i - x_i^k) \quad (13)$$

Under the assumption that the system reaches consensus of all agents, the following holds:

$$\sum_{i=1}^n \sum_{j=1}^n a_{ij} (x_j^{k+j} - x_i^{k+j}) \approx 0 \quad (14)$$

which in turn gives:

$$\sum_{j=1}^n \text{sign}(z_i - x_i^{k+j}) \approx 0 \quad (15)$$

In the case when all agents have exactly the same value, $c = x_i^k \forall i$, it is clear from (15) that c must be the median value of vector \mathbf{z} . However, according to the definition of consensus (3), not all x_i^k need to have exactly the same values. For that reason there is a need for further analysis.

Without the loss of generality we can sort the elements under the sum in the equation (15) in the ascending order of their elements z_i :

$$\sum_{i=1}^n \text{sign}(z_{l(i)} - x_{l(i)}^k) \approx 0 \quad (16)$$

where $l(i)$ rearranges indices from (15) in such a way that $z_{l(i)} < z_{l(i+1)}, \forall i$. It is important to note that each element in the sum in equation (15) has corresponding element in the sum in the equation (16).

After that we can split the sum into two sums:

$$\sum_{i=1}^{n/2} \text{sign}(z_{l(i)} - c + \delta) + \sum_{i=n/2+1}^n \text{sign}(z_{l(i)} - c - \delta) \approx 0 \quad (17)$$

where $x_{l(i)}^k \in [c - \delta, c + \delta]$, as defined in (3).

The exact value of c depends on the values in vector \mathbf{z} , but it is clear that for

$$c \in [m(z) - \delta, m(z) + \delta] \quad (18)$$

the system will converge to the median values of measurements \mathbf{z} .

Finally, since matrix \mathbf{A} is symmetrical, every element $(x_j^{k+j_1} - x_i^{k+j_1})$ in the sum in (12) has its pair element $(x_i^{k+j_2} - x_j^{k+j_2})$, except elements in the diagonal which by

default have value 0. Hence, from equations (11) and (12), one can determine β from the worst case scenario:

$$\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\beta\kappa\alpha - (-\beta\kappa\alpha)}{1 + \beta\kappa} < \frac{\kappa\alpha}{1 + \beta\kappa} \quad (19)$$

which leads to:

$$\beta < \frac{1}{n^2} \quad (20)$$

IV. SIMULATION RESULTS

A. Convergence Analysis

In this section we present simulation results of the proposed dynamic median consensus protocol. We conducted simulations for system with 3, 5 and 31 agents. In the experimental section, we conducted tests with 3 and 5 agents. A more extensive network of 31 agents is chosen to illustrate the algorithm's performance on a larger scale. For each number of agents, two different communication topologies were used, a complete and chain topology, as given in Figure 3. These topologies correspond to the best and worst case, since in order to propagate information from agent i to agent j we need only one step for Figure 3a (best case) and n steps for Figure 3b (worst case). Graphs given in this figure correspond to the overall communication topology, while topology in each step behaves as given in subsection II-B. Adjacency matrices for the above topologies are:

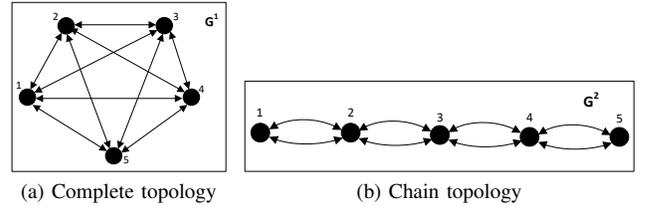


Fig. 3: Communication topologies used in simulation

$$\mathbf{A}^1 = \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 1 & 1 & \dots & 1 \end{bmatrix}$$

$$\mathbf{A}^2 = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & \dots & 0 & 0 \\ \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 1 & 1 \end{bmatrix}$$

Table II shows the number of agents, consensus tuning parameters (as given in Eq. (4)) and results, for six different simulation setups. The numerical results are presented with 3 parameters: t_s settling time (the number of steps needed for all agents to reach value that is within 5% of measurements median value), t_c convergence time (the number of steps needed for maximal distance between two agents to reach value that is within 5% of measurements median value) and ϵ_{ss} error in steady state (value of maximal distance of agents from measurements median value with respect to the median

TABLE II: Tuning parameters and results in each simulation run

Parameter	Sim 1	Sim 2	Sim 3	Sim 4	Sim 5	Sim 6
n	3	3	5	5	31	31
α	9	9	3	3	2	2
β	0.08	0.08	0.04	0.04	0.001	0.001
γ	0.003	0.003	0.0015	0.0015	0.0005	0.0005
κ	0.1	0.1	0.1	0.1	0.1	0.1
\mathbf{A}	\mathbf{A}^1	\mathbf{A}^2	\mathbf{A}^1	\mathbf{A}^2	\mathbf{A}^1	\mathbf{A}^2
t_s	51	238	110	729	26856	342520
t_c	53	230	111	753	11811	440231
ϵ_{ss}	2.25%	2.46%	0.39%	0.63%	0.04%	1.6%

value) Influence of each of the tuning parameters on the system behaviour is analysed later in this section.

We set an initial measured value for each agent, and then make a step change of individual measurements so that the overall median changes stepwise. Results for 3 agents are shown in Figures 4 and 5. Figures show responses of individual agent values x and their additional internal states y for different communication topologies. Results for 5 and 31 agents are given in Figures 6 - 9. Results show that, after a transient period, the system converges to correct values of median, and is able to dynamically track changes in median values. Convergence time is almost seven times shorter for a complete topology, than for a chain topology. Stationary values of y for each agent i are $\pm \frac{\alpha}{r_i}$, as given in Eq. (4).

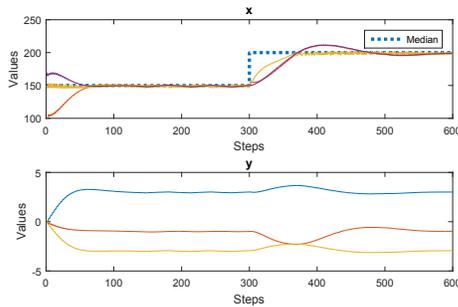


Fig. 4: Simulation results for 3 agents (complete topology)

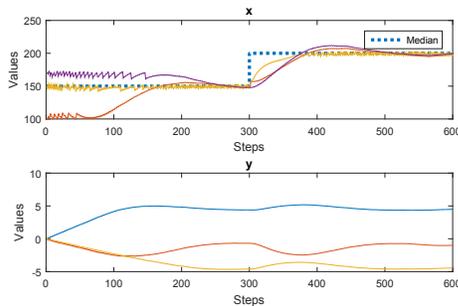


Fig. 5: Simulation results for 3 agents (chain topology)

Results for higher number of agents show that settling (convergence) time increases for higher number of agents. Range of values around the median will also influence the convergence time. In general, larger range leads to slower

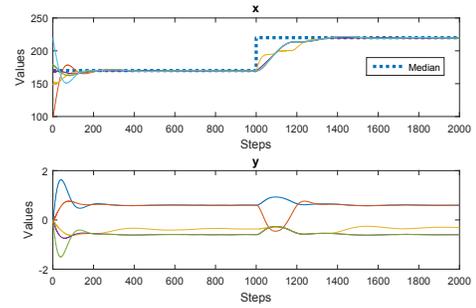


Fig. 6: Simulation results for 5 agents (complete topology)

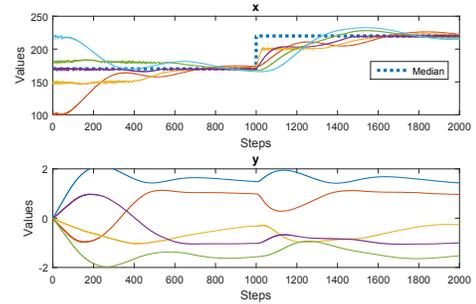


Fig. 7: Simulation results for 5 agents (chain topology)

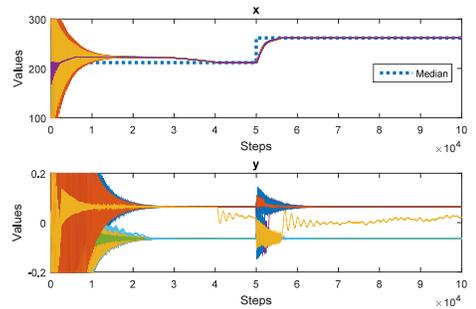


Fig. 8: Simulation results for 31 agents (complete topology)

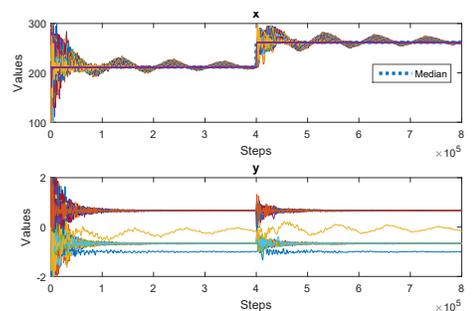


Fig. 9: Simulation results for 31 agents (chain topology)

convergence, which is a consequence of the existence of the sign part in the equation for local interaction rule.

The method was also tested for the case of the lost communication packages. Table III shows influence of random loss of transmitted packages on settling time t_s . Results were obtained by simulation for 5 agents with complete topology. For each percentage of lost packages, 100 simulations were run and the table presents mean settling time t_{savg} and its

TABLE III: Influence of the random loss of transmitted packages on the settling time, tested on 100 simulation for the case of 5 agents with complete topology

Lost packages	t_{savg}	$\sigma(t_s)$	ϵ_{avg}	$\sigma(\epsilon)$
0%	110	0	0.43%	0%
10%	964	1942	0.39%	0.035%
20%	1527	2627	0.38%	0.037%
30%	1982	3303	0.38%	0.034%
40%	2773	3831	0.37%	0.038%
50%	4587	5508	0.38%	0.041%

standard deviation $\sigma(t_s)$, as well as mean steady state error ϵ_{avg} and its standard deviation $\sigma(\epsilon)$. The results show that the system converges to the median of measured values, but it takes longer time with a higher percentage of lost messages. Simulations have shown that even with higher percentage of lost communication packages, the system converges towards the median of the measurements. Simulations have also shown that lost messages have no influence on the steady state error.

System was further tested for sine-shaped measurements of each agent, representing a more dynamical measurement signal. Results (x, y) , together with measurements for each of the 5 agents (z) , are shown in Figure 10. After a transient period the system reaches consensus on the median value and is able to continuously track the median value of all measurements. The second half of the response shows the case of faster changing sinus function. The results show that the system is still able to track the median, but with a delay.

Results are given for an odd number of agents. For an even number, the convergence value is in the interval defined with (3), and exact value depends on the protocol parameters and values of measured signals. The presented method works for both odd and even number of agents. However, results were presented only for an odd number of agents, since in this case the median is a single value, as opposed to system with even number of agents where median is a range of values.

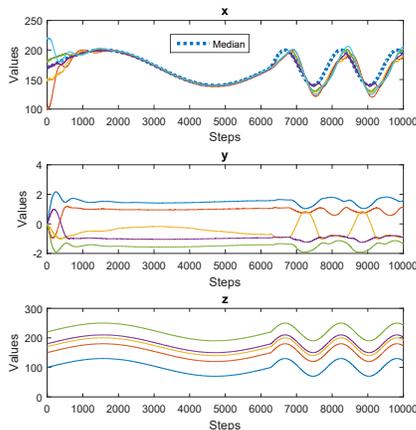


Fig. 10: Simulation results for sine measurements

TABLE IV: Influence of tuning parameters to responses

Parameter	t_c	t_s	ϵ_{ss}
$\alpha = 1$	1082	949	0.36%
$\alpha = 3$	753	729	1.12%
$\beta = 0.01$	2659	1669	1.07%
$\beta = 0.08$	660	463	1.12%
$\gamma = 0.0015$	753	729	1.12%
$\gamma = 0.01$	1415	1044	1.12%
$\kappa = 0.02$	1047	679	0.69%
$\kappa = 0.4$	1639	794	4.12%

B. Analysis of tuning parameters

Each of the algorithm tuning parameters in Table I influences the behaviour of the system:

- Increase of parameter α causes faster convergence, but according to the equation (10) increases the area of instability around the median value.
- Increase of parameter β shortens the time needed for agents to reach a common value (which initially does not have to correspond to the median).
- With parameter γ we influence the rate of change of the state y , which then influences the convergence speed of x . At the same time, its increase towards β increases the instability of the system.
- κ is the forgetting factor of the state y , too large values cause instability of the system, and with too small values, system converges to the wrong value.

The influence of parameters on system response is shown in Figure 11, which shows two graphs for each parameter change: i) initial start of response, which presents the way the agents converge towards the median value and ii) steady state response, which presents the influence of the parameter to the steady state. Numerical results are shown in Table IV. These results were obtained with 5 agents, using tuning parameters Sim 4 from Table II, where only one parameter changed its value.

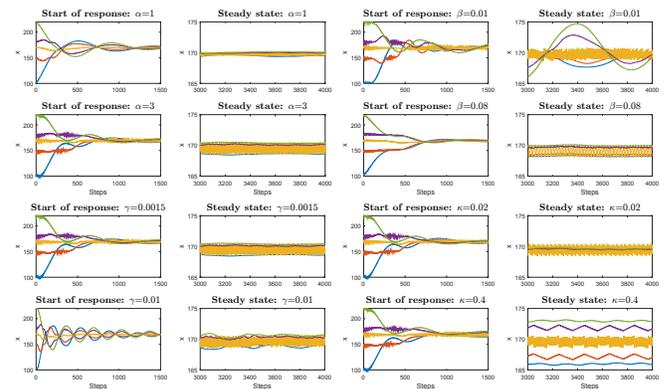


Fig. 11: Influence of tuning parameters to response

V. EXPERIMENTAL RESULTS

In this section we present the results of experiments performed with aMussels (Fig. 12). The aMussel platform (described in [21]) is a robotic unit intended for long-term monitoring of underwater areas. It is designed for low



Fig. 12: Experiment setup using aMussel platforms

power operations on sea-bed, where it takes measurements in regular intervals and goes to low power mode in between. It is equipped with various sensors used to perceive the environment, and two underwater communication devices to share acquired data and control signals with other robots. It is capable only of 1D movement, as it can change its depth using simple yet effective buoyancy system. The aMussel is powered from two single-cell LiPo batteries, where each of them powers different elements of the system. As a communication channel between agents (aMussels) in presented experiments, we have used a nanomodem acoustic unit. For a successful communication, only one of the aMussels in the group is allowed to transmit data using a nanomodem at single moment. For these reasons we have developed a time scheduling scheme, where each of the aMussels has an assigned timeslot in which it is allowed to transmit information using acoustics. Detail about time-scheduling implementation are described in [21] and in section II-B. Each aMussel is equipped with a nanomodem, long-range acoustic communication device.

A. Experiment Setup

We have executed three experiments, one with 3 aMussels and two with 5 aMussels with the corresponding parameters in Table II. Since aMussels were close to each other, it is assumed that the graph is complete, corresponding to the connection matrix \mathbf{A}^1 , but it is possible that some individual messages did not reach all the agents. In all experiments one communication step lasts for 5 seconds.

For the purpose of the experiment with 3 aMussels, measurements are kept constant until moment k_{Step} when aMussel3 changes its measured value from 100 to 180, making a change in system median. This experiment was conducted with aMussels next to each other where for each aMussel, the measured value is a preprogrammed number that does not correspond to any physical value.

Experiments with 5 aMussels were conducted in Jarun lake in Zagreb. Each aMussel was tied to a rope with different length, so each of them was on different depth (Figure 12). Since aMussels were placed close to each other on different depths, a significant loss of communication packages

was expected. During the experiment, aMussels measured pressure in hPa, where one hPa corresponds approximately to one cm of depth. The surface pressure is around 1000 hPa. To enforce a change in measured values, the aMussel with the longest rope was moved towards the surface in the middle of the experiment, and thus became the aMussel with the smallest depth. The experiment goal was for aMussel to agree on the median of their depths (which corresponds to their measured pressure).

Additional experiment with 5 aMussels was conducted to show the influence of communication loss on the results. Same as in the previous experiments, aMussels were tied to a rope on different depths. After they reached consensus, one of them was removed from the water for 5 minutes and then returned to the same depth. After that, two aMussels were removed from the water and left to communicate between each other. After some time, they were returned to the water and placed on different depths.

B. Results

Results of experiments with 3 aMussels (Figure 13) show that values of individual aMussels converge towards the median values. After the measurements of the aMussel3 change, the median value of the group changes, which leads to convergence of individual aMussel values towards the new median value. The results are slightly oscillatory compared to the simulation, because all values transferred over acoustics are quantized. The experiment was run 4 times with similar results.

In the experiment with 5 aMussels, they first reach consensus which corresponds to the measurement of aMussel27 (Figure 14). After the aMussel39 changes its depth, aMussels manage to reach the consensus around the measurement of aMussel18, which represents the new median value. The measured total loss of packages during the experiment was around 15%, with aMussel39 having a loss of around 30%. This experiment was run 2 times with similar results.

The results of communication loss experiment are shown in Figure 15. After 600 steps aMussel27 was removed from the water for 5 minutes, and the results show that this short term communication loss did not influence the system. When aMussel31 and aMussel39 were removed from water and left to communicate only between each other, they started to agree on the consensus between only their measurements. After returning them into the water on different depths, all aMussels in the water started converging to the new median value. This experiment was run once.

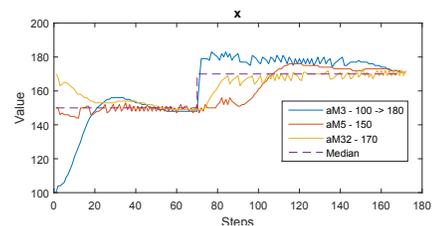


Fig. 13: Experimental results for 3 aMussels

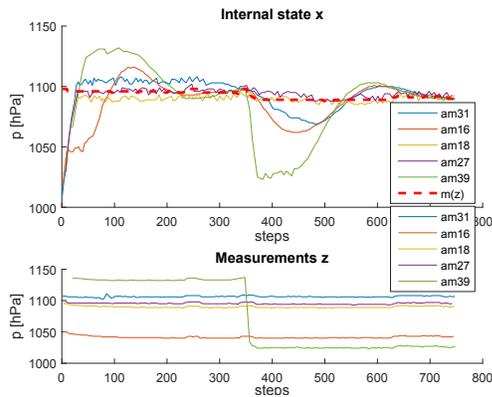


Fig. 14: Experimental results for 5 aMussels

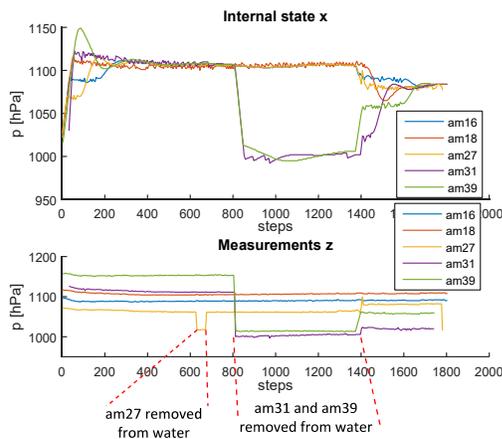


Fig. 15: Experimental results for communication loss experiment

VI. CONCLUSION

In this paper we have presented a method for determining the median value of measurements for a group of agents communicating using scheduled acoustic communication channel. The convergence of the protocol towards the median value is proven theoretically. In order to validate the presented protocol we tested it in a simulation setting, by creating a model of the multi-agent system using scheduled communication. This model was used to gather simulation results of the presented dynamic median method for different number of agents, different connectivity matrices and tuning parameters. We have also tested the presented method on the underwater robotic platforms aMussel, which are equipped with acoustic communication units. Simulation, as well as experimental results, have shown that the presented method converges towards the median of the measurements and that parameters of the consensus protocol can be tuned so that desired speed of convergence and accuracy of the system output obtain desired values. Results also show that the protocol work correctly even for a higher percentage of communication losses.

REFERENCES

- [1] R. Thenius, D. Moser, J. C. Varughese, S. Kernbach, I. Kuksin, O. Kernbach, E. Kuksina, N. Mišković, S. Bogdan, T. Petrović, A. Babić, F. Boyer, V. Lebastard, S. Bazeille, G. W. Ferrari, E. Donati, R. Pelliccia, D. Romano, G. J. V. Vuuren, C. Stefanini, M. Morgantini, A. Campo, and T. Schmickl, "subCULTron - cultural development as a tool in underwater robotics," in *Artificial Life and Intelligent Agents*. Springer International Publishing, 2018, pp. 27–41.
- [2] B. Arbanas, T. Petrovic, and S. Bogdan, "Consensus protocol for underwater multi-robot system using two communication channels," in *2018 26th Mediterranean Conference on Control and Automation (MED)*. IEEE, June 2018.
- [3] —, "Consensus protocol for underwater multi-robot system using scheduled acoustic communication," in *2018 OCEANS - MTS/IEEE Kobe Techno-Oceans (OTO)*. IEEE, May 2018.
- [4] A. Babić, F. Mandić, G. Vasiljević, and N. Mišković, "Autonomous docking and energy sharing between two types of robotic agents," *IFAC-PapersOnLine*, vol. 51, no. 29, pp. 406–411, 2018.
- [5] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, Jan. 2007.
- [6] W. Ren and Y. Cao, *Distributed Coordination of Multi-agent Networks*. Springer London, 2011.
- [7] J. Qin, Q. Ma, Y. Shi, and L. Wang, "Recent advances in consensus of multi-agent systems: A brief survey," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 6, pp. 4972–4983, June 2017. [Online]. Available: <https://doi.org/10.1109/tie.2016.2636810>
- [8] P. Mazdin, B. Arbanas, T. Haus, S. Bogdan, T. Petrovic, and N. Miskovic, "Trust consensus protocol for heterogeneous underwater robotic systems," *IFAC-PapersOnLine*, vol. 49, no. 23, pp. 341–346, 2016.
- [9] T. Haus, I. Palunko, D. Tolić, S. Bogdan, F. L. Lewis, and D. G. Mikulski, "Trust-based self-organising network control," *IET Control Theory & Applications*, vol. 8, no. 18, pp. 2126–2135, 2014.
- [10] A. Jadbabaie, J. Lin, and A. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, June 2003.
- [11] W. Ren and R. Beard, *Distributed Consensus in Multivehicle Cooperative Control: Theory and Applications*. London, UK: Springer, 2008.
- [12] M. Franceschelli, A. Giua, and A. Pisano, "Finite-time consensus on the median value by discontinuous control," in *2014 American Control Conference*. IEEE, June 2014.
- [13] —, "Finite-time consensus on the median value with robustness properties," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 1652–1667, Apr. 2017.
- [14] S. S. Kia, B. V. Scoy, and J. C. et al., "Tutorial on dynamic average consensus: The problem, its applications, and the algorithms," *IEEE Control Systems*, vol. 39, no. 3, pp. 40–72, June 2019.
- [15] M. Zhu and S. Martinez, "Dynamic average consensus on synchronous communication networks," in *2008 American Control Conference*. IEEE, June 2008.
- [16] M. Joordens and B. Champion, "Underwater swarm robotics," in *Handbook of Research on Design, Control, and Modeling of Swarm Robotics*. IGI Global, pp. 718–740.
- [17] V. Putranti, Z. H. Ismail, and T. Namerikawa, "Robust-formation control of multi-autonomous underwater vehicles based on consensus algorithm," in *2016 IEEE Conference on Control Applications*, 2016.
- [18] M. Mirzaei, N. Meskin, and F. Abdollahi, "Robust distributed consensus of autonomous underwater vehicles in 3d space," in *2016 International Conference on Control, Decision and Information Technologies (CoDIT)*. IEEE, Apr. 2016.
- [19] J. Yan, B. Pu, X. Tian, X. Luo, X. Li, and X. Guan, "Distributed target tracking using consensus-based bayesian filtering by underwater acoustic sensor networks," in *2017 36th Chinese Control Conference*. IEEE, July 2017.
- [20] W. Stallings, *Operating Systems: Internals and Design Principles*, 6th ed. Upper Saddle River, NJ, USA: Prentice Hall Press, 2008.
- [21] I. Lončar, A. Babić, B. Arbanas, G. Vasiljević, T. Petrović, S. Bogdan, and N. Mišković, "A heterogeneous robotic swarm for long-term monitoring of marine environments," *Applied Sciences*, vol. 9, no. 7, p. 1388, Apr. 2019.