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# Game Theoretic Formation Design for Probabilistic Barrier Coverage

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***Abstract*— We study strategies to deploy defenders/sensors to detect intruders that approach a targeted region. This scenario is formulated as a *barrier coverage*, which aims to minimize the number of unseen paths. The problem becomes challenging when the number of defenders is insufficient for a full coverage, requiring us to find the most effective location to deploy them. To this end, we use ideas from game theory to account for various paths that the intruders may take. Specifically, we propose an iterative algorithm to refine the set of candidate defender formations, which uses the payoff matrix to directly evaluate the utility of different formations. Given the set of candidate formations, a mixed Nash equilibrium gives a stochastic policy to deploy the defenders. The efficacy of the proposed strategy is demonstrated by a numerical analysis that compares our method with an existing graph-theoretic method.**

## I. Introduction

Surveillance and monitoring of large areas is a fundamental task that is important to many civilian and military defense applications. This paper considers a scenario where moving agents (intruders) must be detected before they reach a target/goal area. Specifically, we are interested in an adversarial scenario where intruders actively avoid detection by selecting paths that are less likely to be covered.

Given a sufficient number of defenders/sensors, one can design a barrier, or a chain of sensors, that completely surrounds the targeted region [1]. If the sensor model is deterministic, then such schemes will guarantee that the intruders will be detected regardless of their paths. The problem becomes more challenging when there is not enough sensors to completely surround the targeted region [2], and also when the detection model is probabilistic [3].

We tackle both of the above challenges: insufficient number of sensors and probabilistic sensing model. As an example, we consider a road network that describes different ways in which the intruders can traverse the space to reach a target (see Fig. 1). From a set of options, the intruders may select the start and goal locations as well as the path to move between the two. The challenge for the defenders/sensors is to ensure detection when these paths selected by the intruders are unknown.

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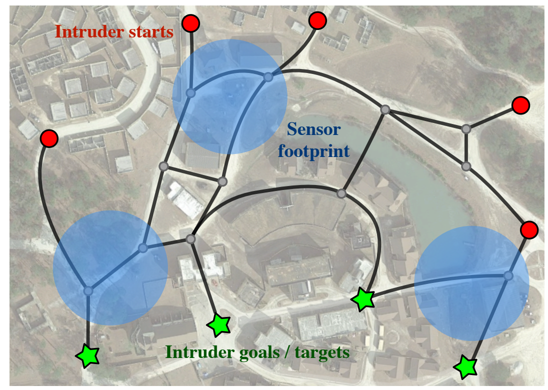


Fig. 1: Example scenario in which each intruder can select an arbitrary path from any of the start locations (red circles) to any of the goal locations (green stars). Three defenders (blue circles) are deployed to detect the intruders.

We use ideas from game theory to address this uncertainty in the opponent's behavior. Specifically, we start by discussing how the intruders and the defenders should select their actions from their strategy sets: candidate paths and formations. Using the detection probability as the payoff function, we pose the problem as a finite matrix game and derive the optimal strategy as a mixed-strategy Nash equilibrium [4].

A prerequisite for the above formulation is the enumeration of candidate formations in order to have a finite strategy set for the defender team. The design of effective formations is challenging for various reasons. First, the sensor footprint of each defender may affect multiple paths in a non-trivial manner, which makes it difficult to solve the problem geometrically. Second, each candidate formation must account for multiple intruder paths to maximize the coverage. Finally, to limit the number of candidate formations for computational tractability, we must have a method to evaluate the effectiveness of different formations and to prune the list as necessary.

To address the above challenges, we propose an iterative method to refine the set of candidate formations. The core idea is to use the payoff matrix to directly evaluate the relative effectiveness as well as the similarity of a given formation with respect to others. The general sample-and-prune procedure that we propose is independent of the choice of sensor models, and is able to handle problems with complex geometries.

The main contributions of the paper are the game theoretic approach to (i) design candidate defender formations, and (ii) find the best action as a stochastic mix of those formations. Due to the game-theoretic formulation and the probabilistic sensor models, the deployment strategy proposed in this paper is more robust to the variations in the intruder's behavior and problem parameters as compared to those strategies based on deterministic approaches.

## II. Related Work

Coverage is a fundamental problem related to Wireless Sensor Networks (WSNs). The majority of theoretical studies consider a full coverage of the area of interest, i.e., every part of the environment must be within the sensing range of at least one sensor [5], [6]. However, some portions of the environment may not be covered in practice when the number of sensors is limited.

For intrusion detection scenarios, the *trap coverage* is among the first to consider partial coverage in WSNs. It was first introduced in [7], studying a 'hole diameter' which is the maximum linear distance that an object can move before being detected. To accommodate a more realistic sensing model, the problem was extended to *probabilistic trap coverage*, which considers a probabilistic sensor model [3]. In this generalization, the maximum displacement is subjected to a minimum detection probability [8], [9].

A closely related problem is called *barrier coverage* [10], [11], for which the placement of the sensors must ensure the detection of any object that passes through the region. For a case with limited number of defenders/sensors, [2] studied a periodic monitoring strategy to achieve barrier coverage. In this context, variants of the problem have considered the use of mobile agents for the creation or adaptation of barrier of nodes [12], [13] or to fix holes and gaps that might exist [14], [15], [16].

Finally, the *partial barrier coverage* problem was considered in [17], where concepts from game theory and graph theory were used to design optimal strategies. The defenders/sensors are deployed along the "bottle neck" of a polygonal environment, which corresponds to a minimum edge cut of a graph-based representation of the environment. Such reduction was effective in [17] because the defender's sensor footprint was modeled as a line segment.

In this work, we consider the case where the intruder's paths and defender's sensor footprint may have more complex geometries, which makes it hard to perform graph-theoretic reduction. To deal with such complex geometry, we propose a method to design defender formations utilizing the payoff matrix in finite static games.

## III. Problem Formulation

Consider a road network on a planar environment $\mathcal{E}$, that contains multiple starting nodes, $\mathcal{N}_\text{S}$, and multiple target/goal nodes, $\mathcal{N}_\text{G}$. The intruders seek to traverse the road network from $\mathcal{N}_\text{S}$ to $\mathcal{N}_\text{G}$ without being detected by the defenders. Each intruder may start from any of the nodes in $\mathcal{N}_\text{S}$ and end at any of the nodes in $\mathcal{N}_\text{G}$. A set of $n$ sensors/defenders $\mathbf{x} = \{x_1, .., x_n\}$ are deployed to detect the intruders, where $x_i \in \mathbb{R}^2$ for $i \in \mathcal{D} \triangleq \{1...n\}$. We refer to $\mathbf{x}$ as a *formation* or a *deployment*.

We assume that both parties know the road network (i.e., the set of possible intruder paths $\mathcal{U}_A$) and the number of defenders $n$. However, the intruders do not know the defender location, and the defenders do not know which paths will actually be taken by the intruders. Therefore, each party must consider possible actions that the opponent may take and decide on its own action.

We also assume that the defenders have probabilistic detection models. A combination of a specific path taken by an intruder, $\boldsymbol{\gamma}$, and a specific robot formation, $\mathbf{x}$, gives the probability, $P(\boldsymbol{\gamma}, \mathbf{x})$, that this intruder is detected somewhere along the path.

Let $\boldsymbol{\Gamma} = \{\boldsymbol{\gamma}_1, ..., \boldsymbol{\gamma}_m\}$ denote the list of all paths taken by $m$ intruders (with possible redundancy). Then the **expected detection probability** across all these intruders is

$$V(\boldsymbol{\Gamma}, \mathbf{x}) = \frac{1}{|\boldsymbol{\Gamma}|} \sum_{\boldsymbol{\gamma} \in \boldsymbol{\Gamma}} P(\boldsymbol{\gamma}, \mathbf{x}), \tag{1}$$

where $|\cdot|$ denotes the cardinality of a set. The quantity $V$ may be interpreted in relation to the expected **fraction of undetected intrusion**:

$$\frac{\mathbb{E}[m_\text{int}]}{m} = \frac{1}{|\boldsymbol{\Gamma}|} \sum_{\boldsymbol{\gamma} \in \boldsymbol{\Gamma}} (1 - P(\boldsymbol{\gamma}, \mathbf{x})) = 1 - V, \tag{2}$$

where $\mathbb{E}[m_\text{int}]$ denotes the expected number of intruders that reach $\mathcal{N}_\text{G}$ without being detected, and we assumed the detection to occur independently across defenders.

The intruder team seeks to minimize $V$ by selecting their paths $\boldsymbol{\Gamma}$, whereas the robot team seeks to maximize $V$ by selecting their formation $\mathbf{x}$.

**Problem 1** (Probabilistic Barrier Coverage)**.** *Given an environment $(\mathcal{E}, \mathcal{N}_S, \mathcal{N}_G)$, what is the best formation $\mathbf{x}$ to maximize the detection probability $V$ against intruders that employ unknown paths $\boldsymbol{\Gamma}$?*

In Sec. V we discuss solutions in a stochastic setting. For example, consider the scenario in which a batch of intruders arrive every day. The intruder team may assign different paths to their members and/or change these assignments every day to "mix" their strategies. The defenders on the other hand can secretly change their formation during night-time to prepare for the next day. In this way, the strategy for each party can be viewed as a probability distribution over the candidate strategies.

Note that we assume that the intruders cannot observe the defender locations and therefore cannot employ any feedback strategy. Similarly, we assume that the defenders do not react to the observed intruders either. Hence the considered strategies are purely offline. The study of more dynamic strategies is a subject of future work.

## IV. Detection Probability

This section discusses an approach to compute the detection probability $P(\boldsymbol{\gamma}, \mathbf{x})$ when $\boldsymbol{\gamma}$ and $\mathbf{x}$ are given. However, note that the game-theoretic framework we present in this paper is generic, and is independent of any specific models or methods used in this section.

## A. Sensor Model

Let $\mathcal{B}_i$ denote the footprint of sensor $i$. We consider a probabilistic sensing model within this sensor footprint. Specifically, we employ Poisson distribution to quantify the detection probability. Consider a sensor/defender located at $x_i$ and an intruder at $x_j \in \mathcal{B}_i$. If the intruder spends $t$ seconds at this location, then the probability of detection is given by

$$p_{ij} = p(x_i, x_j) = 1 - e^{-\lambda_{ij} t}, \tag{3}$$

where $\lambda_{ij}$ (Hz) is a detection frequency parameter that quantifies the average frequency of detection per unit time.

The probability that the intruder is *undetected* decays exponentially over time. This rate of decay is captured by the detection frequency parameter $\lambda_{ij}$, which can be a function of the intruder's relative position with respect to the sensor: $\lambda_{ij} = f(x_i, x_j)$. For example, one can use a decreasing function $f(\cdot)$ where the argument is the distance $d_{ij} = \|x_i - x_j\|$ [18].

When computing the detection probability of a path, we utilize the following quantity:

**Definition 1** (Detection gain)**.** *If $p_{ij}$ is a detection probability, then*

$$\phi_{ij} = -\ln(1 - p_{ij}) \tag{4}$$

*is the corresponding detection gain [3].*

The detection gain is useful in simplifying the probability computation from multiplications to summations. The probability that the intruder is detected by at least one of the $n$ sensors (indexed by $i$) is computed by

$$p_j = 1 - \prod_{i \in \mathcal{D}} (1 - p_{ij}), \tag{5}$$

which is described with detection gain as

$$\phi_j = \sum_{i \in \mathcal{D}} \phi_{ij}. \tag{6}$$

This additive property is convenient in the subsequent analyses.

## B. Detection Gain of a Path

We consider intruder paths generated by a graph traversal. This representation accommodates a variety of situations. For example, paths for ground intruders that are constrained to move on paved roads are represented by road networks (e.g., Fig. 1). When the intruders can move more freely in the environment, their paths maybe generated from a grid-based lattice graph. Such representation is commonly used in robot path planning problems [19].

To focus on the design of defender formations, we assume in this work that candidate intruder paths are given as part of the problem specification. Consider the $j$th path $\boldsymbol{\gamma}_j(s)$ parametrized by the arc length $s \in [0, L_j]$. Recall from (3) and (4) that we have $\phi_{is} = \lambda_{is} t = f(x_i, \boldsymbol{\gamma}_j(s))t$, for any intruder position $\boldsymbol{\gamma}_j(s)$. Also note that an intruder moving at speed $v$ will spend $dt = ds/v$ seconds to traverse through an infinitesimal segment $ds$. Therefore, the detection gain of each path due to a single defender $x_i$ can be computed by the following line integral:

$$\phi(\boldsymbol{\gamma}_j, x_i) = \frac{1}{v} \int_0^{L_j} f(x_i, \boldsymbol{\gamma}_j(s)) ds, \tag{7}$$

where $f$ is a function that models the detection frequency (see the paragraph below (3)), $v$ is the intruder speed, and $L_j$ is the total length of the $j$th path.

The detection gain of a path $\boldsymbol{\gamma}_j$ is given by

$$\phi(\boldsymbol{\gamma}_j, \mathbf{x}) = \sum_{i \in \mathcal{D}} \phi(\boldsymbol{\gamma}_j, x_i) \tag{8}$$

Finally, the probability of detection can be recovered by

$$P(\boldsymbol{\gamma}_j, \mathbf{x}) = 1 - e^{-\phi(\boldsymbol{\gamma}_j, \mathbf{x})}. \tag{9}$$

This expression can be substituted in (1) to give the objective function $V$. The next section discusses how candidate formations and the defenders' deployment strategy can be designed in the context of Problem 1.

# V. Deployment Strategy

This section details our strategy to deploy defenders to maximize the detection probability defined in (1). Based on a game theoretic analysis, we find the optimal deployment strategy as a probability distribution over the candidate formations (mixed-strategy).

## A. Matrix Game and Nash Equilibrium

This section explains how we interpret our problem as a two player non-cooperative game, and discusses the optimal defender strategy when a set of candidate formations are given. The design of those set of formations will be discussed in Sec. VI.

Let the defender team and the intruder team be the two players. We consider a *static game* (simultaneous game) where each player picks a strategy without the knowledge of the decision made by the other [4]. The defender team's strategy set is given by the set of formations $\mathcal{U}_D$. For the intruder team, consider the strategies $\boldsymbol{\Gamma}_i = \{\boldsymbol{\gamma}_i, \boldsymbol{\gamma}_i, .., \boldsymbol{\gamma}_i\}$ for $\boldsymbol{\gamma}_i \in \mathcal{U}_A$, corresponding to all the intruders taking the same path. Note that, later in this section, we will consider a stochastic mix of these single-path strategies to design a general intruder team strategy where agents take different combinations of candidate paths.

Finally, we assume that these sets, $\mathcal{U}_A$ and $\mathcal{U}_D$, are known to both players (i.e., perfect information game). This is a conservative assumption from the defender team's perspective, since their candidate formation may not be known by the intruders, while the candidate paths can be enumerated from the road network.

We consider a zero-sum game where the defender's payoff is $V$ defined in (1) and the intruder's payoff is its negative, $-V$. Note when the defenders and the intruders employ the strategy $\mathbf{x}_i$ and $\boldsymbol{\Gamma}_j$ respectively, the payoff will be $V(\boldsymbol{\Gamma}_j, \mathbf{x}_i) = P(\boldsymbol{\gamma}_j, \mathbf{x}_i)$, since $\boldsymbol{\Gamma}_j$ contains identical paths. The collection of such strategies and their outcome for a zero-sum static game can be conveniently represented using a payoff matrix $\mathbf{V} \in \mathbb{R}^{N\times M}$, where $N \triangleq |\mathcal{U}_D|$ and $M \triangleq |\mathcal{U}_A|$. The elements of the payoff matrix are defined by

$$[\mathbf{V}]_{i,j} \triangleq V(\mathbf{\Gamma}_j, \mathbf{x}_i) = P(\boldsymbol{\gamma}_j, \mathbf{x}_i), \tag{10}$$

which is computed by (9). Note that the first index $i$ (rows) corresponds to the defender formations $\mathbf{x}_i$.

The optimality in static games are given as a Nash equilibrium [4]. Specifically, we say that the selected intruder team strategy $\mathbf{\Gamma}^*$ and the formation $\mathbf{x}^*$ constitute a Nash equilibrium if the following condition holds:

$$V(\mathbf{\Gamma}^*, \mathbf{x}) \leq V(\mathbf{\Gamma}^*, \mathbf{x}^*) \leq V(\mathbf{\Gamma}, \mathbf{x}^*). \tag{11}$$

This relation implies that the intruder team cannot reduce the detection probability by changing the strategy from $\mathbf{\Gamma}^*$, as long as the defenders stick to their formation $\mathbf{x}^*$. Similarly, the defenders cannot achieve a better detection probability by deviating from $\mathbf{x}^*$ if the intruders stick to their equilibrium strategy $\mathbf{\Gamma}^*$.

### B. Mixed Strategies

When the strategies are deterministic, such equilibrium may or may not exist. Even if it exists, its uniqueness is not guaranteed. On the other hand, there always exists an equilibrium when the strategies can be stochastic, which is called the *mixed-strategy Nash equilibrium* [4], [20]. Now the strategies are given as a probability distribution over the paths and formations.

Let $\boldsymbol{\sigma}_D \in \mathbb{R}^N_+$ denote the defender's mixed strategy which satisfies $[\boldsymbol{\sigma}_D]_i \geq 0$ for all $i$ and $\sum_i [\boldsymbol{\sigma}_D]_i = 1$. The defender will use formation $\mathbf{x}_i$ with probability $[\boldsymbol{\sigma}_D]_i$. Similarly, the intruder strategy is denoted by $\boldsymbol{\sigma}_A \in \mathbb{R}^M_+$: i.e., they will use path $\boldsymbol{\gamma}_i$ with probability $[\boldsymbol{\sigma}_A]_i$.

Given a payoff matrix, we can efficiently compute the unique mixed-strategy Nash equilibrium $(\boldsymbol{\sigma}^*_A, \boldsymbol{\sigma}^*_D)$ using various methods [21]. The overall payoff, or the *optimal* expected detection probability is given by

$$V^* = \sum_i^N \sum_j^M [\boldsymbol{\sigma}^*_A]_j [\boldsymbol{\sigma}^*_D]_i V(\mathbf{\Gamma}_j, \mathbf{x}_i) = \boldsymbol{\sigma}^{*\intercal}_D \mathbf{V} \boldsymbol{\sigma}^*_A, \tag{12}$$

where $^\intercal$ denotes matrix transpose. The optimality of these strategies are described by

$$\boldsymbol{\sigma}^\intercal_D \mathbf{V} \boldsymbol{\sigma}^*_A \leq \boldsymbol{\sigma}^{*\intercal}_D \mathbf{V} \boldsymbol{\sigma}^*_A \leq \boldsymbol{\sigma}^{*\intercal}_D \mathbf{V} \boldsymbol{\sigma}_A. \tag{13}$$

As discussed in Sec. III, stochastic formulation is applicable in scenarios where a static game is played repeatedly:

**Definition 2** (Repeated Discrete Scenario)**.** *In a repeated game scenario, intruders arrive in batches where the engagement between one batch of intruders and one formation of the defenders can be treated as an episode. In addition, the time intervals between episodes are sufficiently long so that the defenders can change their formation.*

The mixed strategies may be realized by randomly selecting the paths and formations in each episode of the game. More concretely, the intruder team can realize the mixed strategy by randomly assigning a path from $\mathcal{U}_A$ to each intruder according to the distribution $\boldsymbol{\sigma}_A$. Similarly, the defenders randomly select their formation based on the distribution $\boldsymbol{\sigma}_D$ for each episode.

## VI. Designing Candidate Formations

The deployment strategy in the previous section selects the best mix of formations from a finite set of candidate formations $\mathcal{U}_D = \{\mathbf{x}_1, ..., \mathbf{x}_N\}$. This is natural when $\mathcal{U}_D$ is already given from a logistical or geometric constraints (e.g., buildings or roads). However, the enumeration of the formations may be challenging when the design space is continuous. Here we discuss the design of candidate robot formations, $\mathbf{x}_i$, when the robots may be placed anywhere in the environment $\mathcal{E}$.

### A. Utility of a Formation

We discuss how the effectiveness of a given formation can be evaluated using the payoff matrix, $\mathbf{V}$, defined in the previous section. In this section, we use

$$\mathbf{v}_i \triangleq [\mathbf{V}]_i \in \mathbb{R}^N \tag{14}$$

to denote the $i$th row of the payoff matrix corresponding to the $i$th defender formation.

First, we introduce a concept called *strategic dominance*, which helps us find redundant strategies that will not be used in the mixed-strategy Nash equilibrium.

**Definition 3** (Dominated strategy)**.** *A pure strategy $\mathbf{x}_i$ is dominated by a mixed strategy $\boldsymbol{\sigma}_D$ with $[\boldsymbol{\sigma}_D]_i = 0$ if*

$$\mathbf{v}_i \preccurlyeq \boldsymbol{\sigma}^\intercal_D \mathbf{V}, \tag{15}$$

*where $\mathbf{v}_i$ is defined in* (14)*, and the inequality is element wise. In other words, there exists a superior mixed strategy $\boldsymbol{\sigma}_D$ which produces equal or higher payoff than $\mathbf{x}_i$, regardless of the intruders' action.*

Therefore, there is no rationale to use a dominated strategy. A procedure called the *iterated elimination of dominated strategy by a mixed strategy* (IEDSMS) removes such dominated strategies from each player alternately until there is no such strategy anymore [22]. The following is a strong result which is intuitively straight forward:

**Theorem 1** ([22])**.** *If $\mathbf{V}'$ is an outcome of IEDSMS from $\mathbf{V}$, then $\boldsymbol{\sigma}_D$ is a Nash equilibrium of $\mathbf{V}$ iff it is a Nash equilibrium of $\mathbf{V}'$.*

Essentially, the theorem implies that we can remove dominated strategies from the set $\mathcal{U}_D$, since they should never be used. This result fortifies our iterative algorithm presented in the next section.

Another property we consider is the similarity between different strategies. Even when one formation $\mathbf{x}_i$ is not dominated by another formation $\mathbf{x}_j$, they may be redundant in terms of how they contribute to the "richness" of the defender strategy:

**Definition 4** (Strategic similarity)**.** *We call a pair of formations $\mathbf{x}_i$ and $\mathbf{x}_j$ are* **strategically $\varepsilon$-similar** *if*

$$\|\mathbf{v}^\intercal_i - \mathbf{v}^\intercal_j\|_2 < \varepsilon, \tag{16}$$

*where* $\mathbf{v}$ *is defined in* (14)*, and* $\|\cdot\|_2$ *denotes the 2-norm of a vector.*

If the two formations are $\varepsilon$-similar, then by electing one strategy over the other, the overall payoff defined in (12) will not make any difference greater than $\varepsilon$. Therefore, we may eliminate one of the two if the number of candidate formations are limited to ensure computational tractability.

Importantly, a strategic similarity of formations $\mathbf{x}_i$ and $\mathbf{x}_j$ does not immediately imply that $\|\mathbf{x}_i - \mathbf{x}_j\|_2$ is small. Figure 2 shows an example where we have three intruder paths and three defenders. The two formations are spatially quite different, but their contribution to the overall game are similar in the sense that $\mathbf{v}_1 \approx \mathbf{v}_2$.

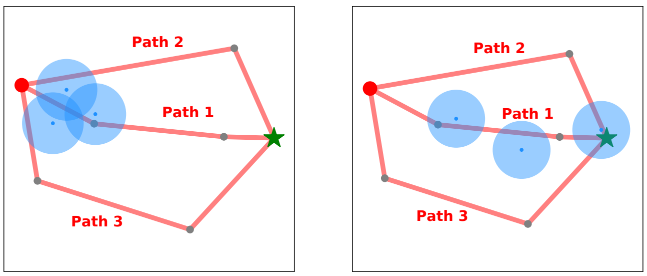


Fig. 2: An example of $\varepsilon$-similar formations with $\varepsilon = 0.06$. The left formation achieves $\mathbf{v}_1 = [0.55, 0.27, 0.12]$, and the right achieves $\mathbf{v}_2 = [0.53, 0.21, 0.14]$.

**Remark 1.** *The dominance and the similarity of the strategies are the key ideas that allow us to evaluate the utilities of the strategies purely based on the payoff matrix. Such an approach is useful in geometrically complex problems. In our case, the effect of each defender on different paths are highly dependent on the proximity and intersections of those paths, and the direct treatment of such coupling will be very difficult. Once they are converted into a payoff matrix, we can immediately evaluate which formation is more effective.*

Building upon the ideas presented in this section, the next section proposes a method to refine the set of defender formations that effectively covers various intruder paths.

## B. Iterative Refinement Algorithm

Given a set of candidate intruder paths, $\mathcal{U}_A$, Algorithm 1 iteratively samples new formations to the set $\mathcal{U}_D$ and eliminates ones that are unnecessary.

**Algorithm 1** Iterative Refinement of Formation Set

```
U_D ← U_{D,0}
while stopping criteria not met do
    x_new ← sample_new_formation(V, U_A)
    U_D ← U_D ∪ x_new
    Recompute V
    U_D, V ← elim_dominated(U_D, V)
    U_D, V ← elim_similar(U_D, V, N_max)
end while
return U_D, V
```

*Eliminating dominated formations:* The subroutine `elim_dominated` goes through each defender strategy (every row of $\mathbf{V}$), identifies and removes the ones that are strategically dominated. The difference with the IEDSMS process (see [22]) is that here we do not modify the intruder team's strategy set. This is to avoid erroneous removal of intruder paths based on suboptimal set of defender formations. Whether a strategy is dominated by a mixed strategy or not can be efficiently found by a linear program.

*Eliminating similar formations:* The parameter $N_{\max}$ is a user specified parameter that determines at most how many defender formations should be carried in $\mathcal{U}_D$. The selection of $N_{\max}$ may depend, for example, on the computational resources. If the number of formations exceeds the limit (i.e., $|\mathcal{U}_D| > N_{\max}$) then the function `elim_similar` prunes the list by eliminating similar strategies. This elimination is done by repeating the following process: find the most similar pair and eliminate one of the two. This elimination based on similarity helps us find a wider range of formations to cover various paths.

*Sampling new formations:* In each iteration the function `sample_new_formation` randomly generates new formations to improve the effective coverage of all paths. First, it uses $\mathbf{V}$ to find paths that are not well covered by the current formation set. For example, a metric to quantify the current coverage of path $j$ may be the average payoff over all formations: i.e., $c_j = \frac{1}{|\mathcal{U}_D|}\sum_i [\mathbf{V}]_{i,j}$.

To generate a formation that emphasizes paths that are less covered, consider the weighting factor $w_j \triangleq 1 - c_j$. We generate a weighted multi-modal Gaussian distribution based on the weights $w_j$ and the paths $\boldsymbol{\gamma}_j$ for every path in $\mathcal{U}_A$. A candidate defender formation is generated by randomly sampling $n$ points from this distribution. Figure 3 shows an example of this probability distribution. Note that this example focuses on the nodes (intersections) since they are effective locations to cover multiple paths, but one can easily modify this method to cover the entire path.

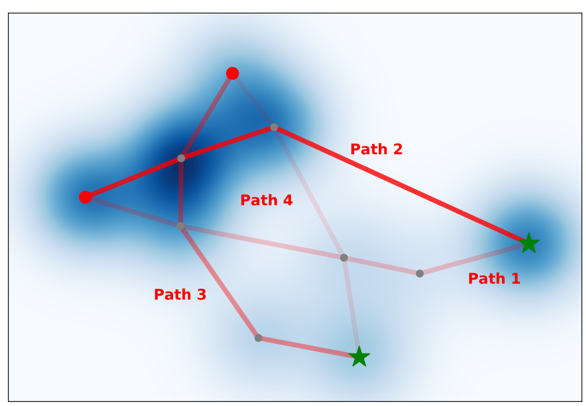


Fig. 3: Example of a density function from which candidate defender positions are sampled. Four paths considered in this example have weights $[w_1, w_2, w_3, w_4] = [0.2, 0.8, 0.6, 0.1]$.

*Termination:* The stopping criteria for the algorithm is application dependent, and it can be a combination of the following: (i) the desired performance is achieved (i.e., probability $V^*$ in (12) exceeds some desired threshold); (ii) the number of iteration exceeds the limit; and/or (iii) the formation set stays unchanged for certain number of iterations. Our numerical analysis in Sec. VII uses (ii) and shows how the resulting performance is dependent on the number of iterations.

### C. Graph-Theoretic Approach

We conclude the section by introducing a benchmark strategy based on [17], which we use in our comparative study presented in Sec. VII. In [17] the authors consider polygonal environments and introduces a *connectivity graph* which models different parts of the environment as nodes, and the edges connecting the neighboring regions. Essentially, this graph is a counterpart of our road network which describes how the intruder can move through the environment.

Each edge has a weight corresponding to the 'width' of the boundary between the two regions, indicating how difficult it is for the defender to form a barrier. Therefore, the defender strategy in [17] was designed based on the *minimum edge cut* of this connectivity graph.

An equivalent strategy in our problem is to consider defender deployment on the minimum cut of the road network. Since it is more efficient to place the defenders on the intersections, we consider minimum node cut. This leads to a candidate formations designed by (i) finding the set of nodes constituting the minimum node cut, and then (ii) finding all combination of $n$ nodes from this minimum cut. We call this the *min-cut strategy*, and use it to evaluate our iterative strategy.

Additionally, we use the formations found by the min-cut strategy as the initial guess $\mathcal{U}_{D,0}$ of our Algorithm 1, which ensures that our final result is at least as good as the min-cut strategy.

## VII. Numerical Analysis

This section demonstrates our theoretical results provided in the previous sections. We first present illustrative examples and then provide statistical results.

### A. Illustrative Examples

We consider a team of 3 defenders deployed in an environment containing a road network with 4 starts and 4 goals. Figure 4 illustrates the min-cut strategy introduced in Sec. VI-C, which is adapted from [17]. A minimum node cut (non-unique for this graph) happens to be the four goal locations in this example, highlighted with dashed circles. Therefore, we have ${}_4C_n = {}_4C_3 = 4$ candidate formations for the min-cut strategy. Figure 4 illustrates one of those candidate formations.

Although there are paths that achieve zero detection probability since the number of defenders is smaller than the cardinality of the min-cut set, intruders cannot confidently select those paths since they are unaware of the actual defender formation in each specific episode. Therefore, the intruders use the mixed Nash equilibrium $\boldsymbol{\sigma}_A^*$ to stochastically allocate the paths to each intruder. This selection/probability is depicted as the brightness of the red intruder paths. Under the optimal/equilibrium strategies on both teams, the final payoff is $V^*_{\text{m.c.}} = 0.291$.

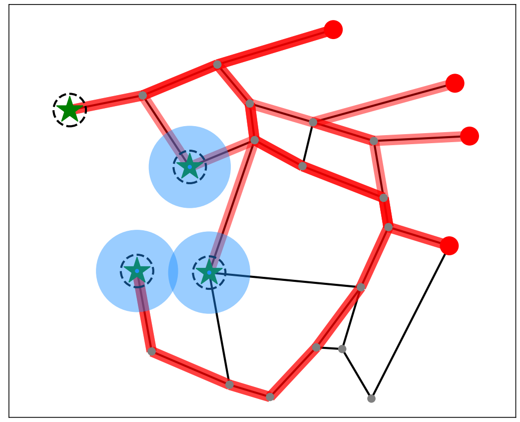

Fig. 4: Example road network with defenders deployed based on the min-cut strategy. The dashed black circles highlight the nodes that belong to the minimum node cut set.

Figure 5 illustrates different formations selected by our iterative method (Algorithm 1), and one formation that will be eliminated by the strategic dominance. Specifically, the mixture of the first three formations ($\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$) dominates the fourth formation $\mathbf{x}_4$, and therefore the fourth formation will not be retained in $\mathcal{U}_D$.

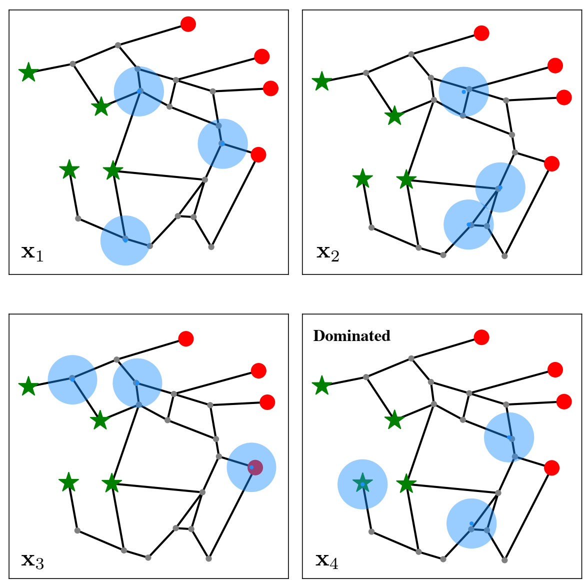


Fig. 5: Examples of the formations generated by Algorithm 1, and one that is dominated by the others (bottom right).

The iterative refinement algorithm identifies formations that cover important locations in terms of the payoff function $V$ (e.g. path intersections). This effective coverage is highly dependent on the geometric properties of the road network and the sensor footprint, which graph-theoretic method (min-cut strategy) fails to identify.

### B. Statistical Results

The performance of our iterative algorithm is also numerically demonstrated by a comparison with the min-cut strategy. Since our algorithm iteratively refines the formation, its performance is dependent on the number of iterations. In addition, the performance is dependent on the maximum number of formations we save in $\mathcal{U}_D$. Figure 6 shows the final payoff $V^*$ as a function of the number of iterations.

We use the same road network as in Fig. 4, and therefore the performance of the min-cut strategy is fixed at $V^*_{\text{m.c.}} = 0.291$. However, since our algorithm randomly searches over the environment, its performance is different in each run. For each data point, we test the algorithm 25 times to obtain the statistics for the box plot.

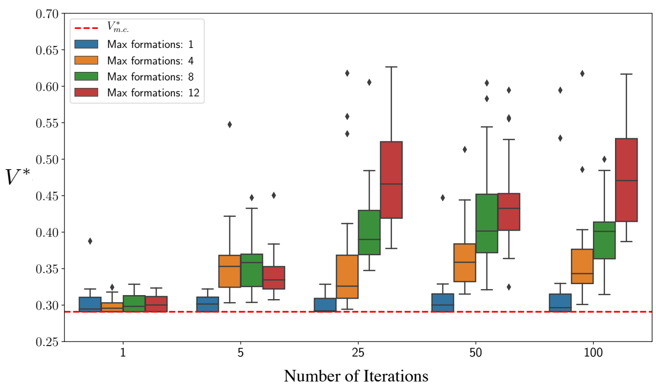


Fig. 6: Improvement over min-cut strategy considering different number of formations kept in $\mathcal{U}_D$ and iterations.

Recall that the performance of our strategy is always at least as good as the min-cut strategy by construction (see Sec. VI-C). The figure shows that the performance increase improves as the number of formations, $N_{\max}$, and the number of iterations increases. However, the figure also shows that the benefit of the number of iterations saturates at 25 for this particular example. We also expect to see a plateau if we increase $N_{\max}$ further.

## VIII. Conclusion and Future Work

This work considers the intrusion detection problem using a team of defenders. The scenario is formulated as a *barrier coverage* which aims to minimize unseen paths. Specifically, this paper addresses two challenges: insufficient number of robots for a full coverage, and probabilistic sensing models. By treating the problem as a two player non-cooperative game, we obtain the optimal defender strategy as a mixed-strategy Nash equilibrium. Our algorithm to design candidate defender formations utilizes the payoff matrix from a finite static game, which bypasses complex geometries of the problem and directly gives us effective strategies. Our stochastic policy accommodates different possibilities for intruders' behaviors, and is a more robust solution when compared to deterministic strategy designs. As an ongoing and future work, we are interested in studying a dynamic version of the game where both teams may observe and adapt to the behavior of the opposing team.

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