2020 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)
October 25-29, 2020, Las Vegas, NV, USA (Virtual)

Energy Autonomy for Resource-Constrained Multi Robot Missions

Hassan Fouad[1] and Giovanni Beltrame[2]

***Abstract*— One of the key factors for extended autonomy and resilience of multi-robot systems, especially when robots operate on batteries, is their ability to maintain energy sufficiency by recharging when needed. In situations with limited access to charging facilities, robots need to be able to share and coordinate recharging activities, with guarantees that no robot will run out of energy. In this work, we present an approach based on Control Barrier Functions (CBFs) to enforce both energy sufficiency (assuring that no robot runs out of battery) and coordination constraints (guaranteeing mutual exclusive use of an available charging station), all in a mission agnostic fashion. Moreover, we investigate the system capacity in terms of the relation between feasible requirements of charging cycles and individual robot properties. We show simulation results, using a physics-based simulator and real robot experiments to demonstrate the effectiveness of the proposed approach.**

I. Introduction

Long term autonomy is considered one of the key ingredients for the practical application of multi-robot system. When performing missions out of the comfort of the lab, limited battery capacity and the recharging ability of robots are one of the most important obstacles to deployment.

Many approaches can be found in literature that deal with this issue. Early efforts took directions as energy aware path planning [1] and node scheduling in wireless sensor networks. Later ideas have been integrated in multi-robot systems, as in [2] in which a mission was split in real time among participating agents depending on their energy level.

Another solution for energy maintenance is through the use of charging stations, whether being static or mobile. In [3], a group of charging robots plans routes to deposit batteries on predefined paths for robots doing surveillance so as to eliminate detours and assure energy sufficiency.

Notomista et al. [4] propose using static charging stations and a control barrier function (CBF) framework to assure energy persistence in a group of robots. This framework provides a constraint based behavioral layer that guarantees the survivability of robots by driving each robot to a dedicated charging station in a minimally invasive way (affecting their original mission as little as possible).

In this paper, we extend [4] by considering a group of robots doing a mission (e.g. coverage or waypoint navigation), but having a single charging station that they need to share. The contribution of this paper is twofold: 1) Augmenting the results in [4] with a CBF-based coordination framework that assures mutually exclusive use of the charging station, and 2) introducing some sufficient conditions that describe the system's capacity and assure the overall feasibility of the coordination.

[1]Hassan Fouad is with Computer and Software Engineering department, Ecole polytechnique de Montreal hassan.fouad@polymtl.ca
[2]Giovanni Beltrame is with Computer and Software Engineering department, Ecole polytechnique de Montreal giovanni.beltrame@polymtl.ca

II. Preliminaries

A. Control Barrier Functions (CBF)

A control barrier function (CBF) [5] is a tool that can be used to assure safety of control systems. In this context, safety means guaranteeing that the states of the system stay in a "safe set" and never wander off to "unsafe" regions.

The safe set is defined to be the superlevel set of a continuously differentiable function $h(x)$ such that [5]:

$$\begin{aligned} \mathcal{C} &= \{x \in \mathbb{R}^n : h(x) \geq 0\} \\ \partial\mathcal{C} &= \{x \in \mathbb{R}^n : h(x) = 0\} \\ Int(\mathcal{C}) &= \{x \in \mathbb{R}^n : h(x) > 0\} \end{aligned} \tag{1}$$

which means that assuring that $h(x) > 0, \forall t > t_0$ implies the safe set $\mathcal{C}$ is positively invariant and the system is safe. For a control affine system of the form

$$\dot{x} = f(x) + g(x)u$$

having a control action u that achieves

$$\underbrace{L_f h(x) + L_g h(x) u}_{\dot{h}(x)} \geq -\alpha(h(x)) \tag{2}$$

where $\alpha(h(x))$ is an extended type $\mathcal{K}$ function, assures that $\mathcal{C}$ is positively invariant. In this paper we use zeroing control barrier functions (ZCBF) [6] owing to their robustness and asymptotic stability properties [7].

Definition 1: [6] For a region $\mathcal{D} \in \mathcal{C}$ a continuously differentiable function $h(x)$ is called a zeroing control barrier function (ZCBF) if there exists an extended class $\mathcal{K}$ function $\alpha(h(x))$ such that

$$\sup_{u \in U} \left(L_f h(x) + L_g h(x) u + \alpha(h(x))\right) \geq 0 \quad \forall x \in \mathcal{D} \tag{3}$$

We can also define the set K_{zcbf} [6] for the ZCBF $h(x)$ as

$$K_{zcbf} = \{u \in U : L_f h(x) + L_g h(x) u + \alpha(h(x)) \geq 0\} \tag{4}$$

which is the set containing all the "safe" control inputs. Choosing a Lipschitz continuous controller u from K_{zcbf} is sufficient to ensure that the safe set $\mathcal{C}$ is forward invariant [6].

Equation (2) represents a basic requirement on the control action to assure safety, but this control action is not necessarily that of an arbitrary mission. Quadratic Programming (QP) can be used to enforce (2) as a constraint

978-1-7281-6211-9/20/$31.00 ©2020 IEEE

that has to be respected by the mission's nominal control action u_{nom} in the following manner [4]:

$$\begin{aligned} u^* = \min_u \quad & ||u - u_{nom}|| \\ \text{s.t.} \quad & L_f h(x) + L_g h(x) u \geq -\alpha(h(x)) \end{aligned} \tag{5}$$

B. Problem statement

Given a group of n robots, we:

- Ensure that the battery voltage of each robot never goes below a certain desired level (safety).
- Ensure that there is no more than one robot at the charging station at any time (coordination).
- Investigate the number of robots that can be accommodated by one charging station (capacity).

For the sake of simplicity of analysis, we use a robot model similar to [4]:

$$\dot{x} = u, \quad \dot{E} = \begin{cases} -k_e, & ||x - x_c|| > \delta \\ k_{ch}, & \text{otherwise} \end{cases} \tag{6}$$

where $x \in \mathbb{R}^2$ is the robot's position, E is the battery voltage, $k_e > 0$ and $k_{ch} > 0$ are discharging and charging coefficients respectively and δ is an effective charging distance away from the charging station. To be conservative, we take k_e as the worst case discharge rate of the battery. The charging model in (6) is a linear approximation of a continuous charging process that can be obtained from wireless pads [8].

C. Overview of the strategy

We build an energy sufficiency layer that ensures that the voltage of each robot stays higher than a given minimum level (to ensure sufficiency) and lower than a certain upper bound (to avoid overcharging). We also implement a coordination layer that ensures mutually exclusive use of the charging station through proper manipulation of the robots' arrival times to the charging station. Moreover, we describe the system's capacity, namely the relationship between the battery discharge characteristics and the number of robots that can be served by a single charging station, to ensure the overall feasibility of the system.

III. Energy sufficiency framework

A. Energy sufficiency CBF

The candidate CBF we use for energy sufficiency is taken from [4] and is defined as

$$h_s = E - E_{min} - k_c \log \frac{||x - x_c||}{\delta} \tag{7}$$

where E_{min} is the desired minimum voltage, δ is the effective radius of the charging station and $k_c > 0$ is a constant such that the last expression approximates the amount of voltage needed to return back to the charging station.

Theorem 1 and 2 from [4] show that the CBF in (7) is a ZCBF if $||x - x_c|| > \delta$ and that $u \in U = \mathbb{R}^2$ (no restrictions on the control action moving the robots). We investigate the relation between the choice of the $\alpha(h)$ in (2) and the voltage level at the arrival at the charging station. In the following, we use a linear $\alpha(h)$ function of the form

$$\alpha(h) = \beta h \tag{8}$$

Lemma 1: Given a robot with dynamics described in (6), and for the the QP in (5) using the ZCBF in (7), the voltage difference $E - E_{min}$ is bounded by zero from below and by a quantity inversely proportional to β from above.

Proof: After some time $T > t_0$, the nominal control input u_{nom} won't be able to satisfy the constraint in (5), in which case the output of the QP problem will be a control input u that satisfies

$$\dot{h}_s = -\beta h_s \Rightarrow h_s(t) = h_s(T) e^{-\beta(t-T)} \tag{9}$$

but from (7)

$$E - E_{min} - k_c \log \frac{||x - x_c||}{\delta} = h_s(T) e^{-\beta(t-T)} \tag{10}$$

and on arrival at the charging station at time $t = t_a$ we have $||x - x_c|| = \delta$ thus

$$E(t_a) - E_{min} = h_s(T) e^{-\beta(t_a - T)} \tag{11}$$

to show that $E(t_a) - E_{min}$ is bounded from below, it suffices to mention that if the robot starts in the safe set (i.e. $h_s(t_0) \geq 0, \forall t \geq t_0$), then respecting the constraint in (5) for h_s assures that $h_s(T) \geq 0$ as well (by virtue of the fact that h_s is a ZCBF [4]), so the right hand side in (11) is not smaller than zero.

$E(t_a) - E_{min}$ is bounded from above by equation(11), which shows that the bound on this difference decays exponentially with β. ■

Remark 1: It is worth noting that Lemma 1 shows that the ZCBF h_s has a tracking property in the sense that the voltage at the time of arrival at the charging station is close to E_{min} with a margin, the tightness of which can be manipulated.

Since the battery discharge is constant with time and that the difference $E - E_{min}$ is bounded (i.e. by proper choice of β the robot arrives at charging station with $E \approx E_{min}$), we can conclude that the arrival time at the charging station is approximately the time it takes the battery to discharge from its current voltage to E_{min}

$$T_L \approx \frac{E - E_{min}}{k_e} \tag{12}$$

with T_L being the arrival time at the charging station.

To recharge the battery, the robot needs to stay for a sufficient time inside the charging region:

Lemma 2: Given a robot with dynamics described in (6), and for the the QP in (5) using the ZCBF in (7), choosing the $\alpha(h_s)$ function

$$\alpha(h_s) = \begin{cases} \beta_h h_s & , ||x - x_c|| > \delta \\ \beta_l h_s & , \text{otherwise} \end{cases} \tag{13}$$

with $0 < \beta_l \ll \beta_h$, leads to having $D \to \infty$ as $t \to \infty$ inside the charging region, where $D = ||x - x_c||$.

Proof: When a robot heads back to the charging station at $t \geq T$, the control action fulfilling (9) is

$$L_f h_s + L_g h_s u = -\beta h_s \Rightarrow -k_e - \frac{k_c}{D^2}(x - x_c)^T u = -\beta h_s$$

This equation can be written as

$$\begin{aligned}(x - x_c)^T u &= \frac{\beta h_s - k_e}{k_c} \overbrace{(x - x_c)^T (x - x_c)}^{D^2} \\ \Rightarrow u &= \frac{\beta h_s - k_e}{k_c}(x - x_c)\end{aligned} \tag{14}$$

plugging this control action into the robot dynamics, and noting that $\frac{d}{dt}D = \frac{(x-x_c)}{||x-x_c||}\dot{x}$, we have

$$\dot{D} = \frac{\beta h_s - k_e}{k_c} D \tag{15}$$

Inside the charging region, $\beta = \beta_l$ and $E - E_{min} = \Delta E_a + k_{ch}\Delta t$, with $\Delta E_a = E(t_a) - E_{min}$ and $\Delta t = t - t_a$, thus

$$\dot{D} = \frac{1}{k_c}\left(\beta_l\left(\Delta E_a + k_{ch}\Delta t - k_c \log\frac{D}{\delta}\right) - k_e\right) D \tag{16}$$

then if we substitute $\beta_l = 0$, which is the lowest value of β, in (16) we get

$$\dot{D} = -\frac{k_e}{k_c} D \Rightarrow D(t) = \delta e^{-\frac{k_e}{k_c}\Delta t} \tag{17}$$

which means that $D \to 0$ as $t \to \infty$. ■

Lemma 3: For the same conditions of Lemma 2, the choice of (13) assures that the robot stays inside the charging region for a sufficiently long period.

Proof: When the robot approaches the charging station, $\dot{D} < 0$. However, there is a point at which $\dot{D} = 0$ as the battery recharges and starts to move out of the charging region. The time until this reversal point can be calculated by setting $\dot{D} = 0$ in the above equation so

$$\Delta t_r = \frac{1}{k_{ch}}\left(\frac{k_e}{\beta_l} + k_c \log\frac{D_r}{\delta} - \Delta E_a\right) \tag{18}$$

where D_r is the distance from x_c at the reversal time. Since $D \to 0$ as $t \to \infty$ this means that that $D_r \neq 0$ in finite time so $\log\frac{D}{\delta} > -\infty$ and thus it can be seen that the reversal time can be set arbitrarily high by setting a low value for β_l, which in turn means that using the proposed switching in the value of β, the robot can stay for an arbitrary amount of time inside the charging region. ■

B. Overcharge protection CBF

The purpose of this CBF is to ensure that the robot "escapes" the charging region before it overcharges (i.e. keeps charging beyond a desired maximum voltage) in a similar way to what was done in [9]. To this end we propose:

$$h_{ov} = E_{max} - E + k_{ov} \log\frac{D}{\delta} \tag{19}$$

The main intuition behind this choice is that when $E_{max} - E$ decreases as the robot recharges, the $k_{ov} \log\frac{D}{\delta}$ tends to become more positive (or rather less negative since the robot is inside the charging region during recharging) by escaping away from the charging region. In the following, we show that this proposed CBF is indeed a ZCBF, in a very similar way to Theorem 1 in [4].

Theorem 1: The function $h_{ov} = E_{max} - E + k_{ov}\log\frac{D}{\delta}$ is a ZCBF if $U = \mathbb{R}^m$

Proof: Since we are considering the case where $U = \mathbb{R}^m$ (i.e. no saturation on the control action), then showing that h_{ov} is a ZCBF follows from showing that $L_g h_{ov} \neq 0$. For the proposed function h_{ov}

$$L_g h_{ov} = \frac{k_{ov}}{D^2}(x - x_c)^T \tag{20}$$

so we need $||x - x_c|| > 0$ to have $L_g h_{ov}$ defined and not equal to zero which was shown in Lemma 2, and since recharging from E_{min} to E_{max} happens in a finite time, then $x \neq x_c$ in included in this period. ■

Remark 2: The CBFs for energy sufficiency and overcharge protection may be in conflict since one slows down the robot for it to recharge (i.e. h_s), while the other tries to push it outside the charging region (i.e. h_{ov}). To avoid infeasibility of the QP, the next two conditions have to be satisfied

$$-k_e - \frac{k_c}{D^2}(x - x_c)^T u \geq -\beta h_c \tag{21a}$$

$$-k_{ch} + \frac{k_{ov}}{D^2}(x - x_c)^T u \geq -\beta_{ov} h_{ov} \tag{21b}$$

and eliminating u we get

$$\frac{\beta h_s - k_e}{k_c} - \frac{k_{ch} - \beta_{ov} h_{ov}}{k_{ov}} \geq 0, \tag{22}$$

so condition (21b) can be relaxed when the left hand side of (22) is equal to zero, leading h_{ov} to take over and push the robot out of the charging region.

IV. Coordination Framework

The purpose of the coordination framework is to ensure mutually exclusive use of the available charging station. The proposed strategy is through the manipulation of E_{min} for each robot to change their arrival times at the charging station (according to (12)). Our main assumptions are:

1) The underlying communication graph between the robots is a complete graph, meaning that each robot can receive information from all other robots.
2) Robots are homogeneous and have the same battery discharge rate.
3) All robots start at the maximum voltage.
4) The recharge rate of the battery is faster than the discharge rate. This can be a reasonable assumption for systems with powerful wireless charging pads along with capable lipo batteries or for battery swapping platforms[1].

[1]Although battery swapping is discrete process in nature, it can fit in our proposed framework by properly choosing k_{ch} so as to ensure all robots spend at least the amount of time needed for a battery swap process.

The mechanism we propose to change the value of E_{min} is to assume that it changes according to the following dynamics

$$\dot{E}_{min} = \eta \tag{23}$$

where $\eta \in \Theta = \mathbb{R}$ with Θ being the set of all admissible values of η, and the nominal value of choice for η is zero (its default value is zero unless changed by other control laws to fulfill other constraints). Our main strategy is twofold: 1) Introduce a constraint to bound the value of E_{min} from below, as well as conditions that assures the feasibility of the scheduling with respect to the system's capacity, and 2) introduce a CBF $h_{c_{ij}}$ that keeps the difference in arrival times above a desired value δ_t through the manipulation of E_{min}.

A. Bounds on E_{min}

Arbitrary manipulation of E_{min} does not necessarily comply with what could be physical bounds on its value. Asking for a too low value may cause permanent damage to the battery and a too high value cause h_s to be negative.

1) Lower bound on E_{min} : Lower bounding E_{min} corresponds to requiring having an acceptable voltage at the beginning of the charging process to avoid permanent damage to the battery. The proposed CBF for this purpose is

$$h_L = k_p(E_{min} - E_{lb}) \tag{24}$$

where $E_{lb} > 0$ is the desired lower bound voltage and $k_p > 0$ is a scaling gain. The constraint for the QP (5) is:

$$k_p \eta \geq -\alpha(h_L) \tag{25}$$

and we choose

$$\alpha(h_L) = \kappa \cdot \text{sign}(h_L) \cdot |h_L|^{\rho} \quad , \rho \in [0, 1) \tag{26}$$

with $\kappa > 0$.

Lemma 4: For a robot with dynamics (6) and E_{min} obeying (23), h_L is a ZCBF.

Proof: It is not hard to see that h_L is a ZCBF following the same argument in Theorem 2, as $\eta \in \Theta = \mathbb{R}$, so it is always the case that a value of η could be found that satisfies (3) for the constraint (25), so if $E_{min}(t_0) \geq 0$ then (25) ensures positive invariance of the safe set $\mathcal{C}_{E_{min}} = \{E_{min} \in \mathbb{R} : E_{min} \geq E_{lb}\}$. ■

2) Upper bound on E_{min}: The upper bound on E_{min} is correlated with the capacity of the system and how many robots can be served by one charging station. Indeed, there is a relation between the feasible number of robots (that can be served with a minimum separation time δ_t) and certain robot parameters, like discharge and recharge rates and maximum and minimum voltage of the battery.

To demonstrate this, suppose we have n robots in the first recharging cycle (when all robots start at E_{max} and discharge with the same rate, as per assumptions 2 and 3) and that each robot has a specific E_{min}. We consider the first robot to recharge (with E_{min} yet to be determined) and the last one (with $E_{min} = E_{lb}$ in the extreme case). We require that once the first robot arrives at time t_1 and recharges at t_2, it won't recharge again until at least a time δ_t after the last robot has recharged. Thus at t_2

$$\frac{E_{max} - \bar{E}_m}{k_e} - \frac{(\bar{E}_m - k_e(t_2 - t_1)) - E_{lb}}{k_e} \geq \delta_t \tag{27}$$

where $t_2 - t_1 = \frac{E_{max} - \bar{E}_m}{k_{ch}}$ is the time needed to recharge the battery of the first robot, and $\bar{E}_m$ is the value of E_{min} of the first agent. The first expression in the above inequality is the time the first robot takes until it reaches the charging station again, and the second is the time the last agent take to reach the charging station for the first time starting from t_2. Substituting $t_2 - t_1$ in the above inequality:

$$\bar{E}_m \leq \frac{(1 + \frac{k_e}{k_{ch}})E_{max} + E_{lb} - \delta_t k_e}{2 + \frac{k_e}{k_{ch}}} \tag{28}$$

Noticing that the difference in arrival times of any two robots is $\Delta T_L = \frac{\Delta E - \Delta E_{min}}{ke}$ and considering the first recharging cycle where E is the same for all robots who have not recharged yet, this means that ΔE_{min} sets the difference in arrival times and uniformly separated arrival times imply uniform separation in values of E_{min}.

We can calculate the uniform step in E_{min} if we convert the last inequality to an equality and using

$$\Delta \bar{E}_m = \frac{\bar{E}_m - E_{lb}}{n - 1} = \frac{(1 + \frac{k_e}{k_{ch}})(E_{max} - E_{lb}) - \delta_t k_e}{(2 + \frac{k_e}{k_{ch}})(n - 1)} \tag{29}$$

What we require in this case is that the arrival times of the last two robots (without loss of generality) with $E_{min} = E_{lb}$ and $E_{min} = E_{lb} + \Delta E_{min}$ to be at least δ_t:

$$\frac{E_{max} - E_{lb}}{k_e} - \frac{E_{max} - (E_{lb} + \Delta E_{min})}{k_e} \geq \delta_t \tag{30}$$

and substituting (29) into the last equation we get

$$\frac{(1 + \frac{k_e}{k_{ch}})(E_{max} - E_{lb}) - \delta_t k_e}{(2 + \frac{k_e}{k_{ch}})(n - 1)} - \delta_t k_e \geq 0, \tag{31}$$

the critical value of δ_t at which equality is achieved, given system parameters $(k_e, k_{ch}, n, E_{max} - E_{min})$ is

$$\delta_{t_{cr}} = \frac{\left(1 + \frac{k_e}{k_{ch}}\right)(E_{max} - E_{lb})}{k_e \left[1 + \left(2 + \frac{k_e}{k_{ch}}\right)(n - 1)\right]}. \tag{32}$$

The last relation describes the feasible separation in arrival times for the robots given different system parameters, and considering that each robot has a distinct value of E_{min}, which are separated by multiples of $\bar{E}_m$.

Finally, we require $\delta_{t_{cr}}$ above to be more than the time taken to fully recharge a battery from E_{lb} to E_{max}

$$\delta_{t_{cr}} \geq \frac{E_{max} - E_{lb}}{k_{ch}}. \tag{33}$$

Definition 2: For a group of n robots, each with a distinct fixed value of E_{min} and all applying the energy sufficiency and overcharge CBFs constraints (h_s and h_{ov}), a *charging cycle* is defined as the time window taken by the robot with the lowest E_{min} value (i.e. $E_{min} = E_{lb}$) to discharge from E_{max} to E_{lb} and then recharges again.

Lemma 5: For a group of n robots each with a distinct value of E_{min} that satisfies (32), (33) and (29), let z_i be the number of recharges that one robot can have in one charging cycle, then the maximum number of recharges for any robot is $\bar{z}_i = 2$.

Proof: The number of recharges of robot $i = n$ (first robot to recharge) in one cycle is

$$z_n = 1 + \left\lfloor \frac{\frac{(E_{max}-E_{lb})\left(1+\frac{k_e}{k_ch}\right)}{k_e}}{\frac{(E_{max}-\bar{E}_m)\left(1+\frac{k_e}{k_ch}\right)}{k_e}} \right\rfloor \tag{34}$$

where the second expression on the right hand side is the floor of the quotient of the two periods. If we take this quotient and substitute (28) and (32) we get

$$\frac{\left(2 + \frac{k_e}{k_{ch}}\right) A}{A + \left(2 + \frac{k_e}{k_{ch}}\right)} = Q \tag{35}$$

where $A = \left(1 + \left(2 + \frac{k_e}{k_{ch}}\right)(n-1)\right)$. What we want to verify is that $1 < Q < 2$. Checking the difference between numerator and denominator

$$\left(2 + \frac{k_e}{k_{ch}}\right) A - A - \left(1 + \frac{k_e}{k_{ch}}\right) = (A-1)\left(1 + \frac{k_e}{k_{ch}}\right) > 0, \tag{36}$$

satisfying the first inequality. To check the second we need to make sure the numerator is less than twice the denominator

$$\begin{aligned} &2\left(A + \left(1 + \frac{k_e}{k_{ch}}\right)\right) - 2A - \frac{k_e}{k_{ch}} A \\ &= 2\left(1 + \frac{k_e}{k_{ch}}\right) - \frac{k_e}{k_{ch}}\left(1 + \left(2 + \frac{k_e}{k_{ch}}\right)(n-1)\right) \end{aligned} \tag{37}$$

but if we substitute (32) in (33) we have

$$\left(1 + \frac{k_e}{k_{ch}}\right) \geq \frac{k_e}{k_{ch}}\left(1 + \left(2 + \frac{k_e}{k_{ch}}\right)(n-1)\right) \tag{38}$$

which renders (37) positive, meaning that the ratio is upper bounded by 2, which in turn means that the maximum number of recharges of the most needy agent is two per charging cycle. Since the last robot recharges only once in a cycle, this means that any other robot in between can recharge no more than twice per cycle, which completes the proof. ∎

Lemma 6: For a group of n robots, if δ_t satisfies

$$\frac{E_{max} - E_{lb}}{k_{ch}} \leq \delta_t \leq \delta_{t_{cr}} \tag{39}$$

as well as equation (29), then there exists $\mathbf{E}_m = \{E_{min_1}, \ldots, E_{min_n}\}$ such that the difference in arrival times between any two robots is at least δ_t (i.e. the scheduling problem is feasible).

Proof: The idea of the proof is to use the upper bound $\bar{z}_i$ and show that the possible difference between any two landing times is at least $\delta_{t_{cr}}$ even with this worst case scenario.

Based on Lemma 5, the possible number of recharges is $2(n-1)$ and thus the required number of spaces between these recharging events (taking the start and end recharging events of last agent) is $M = 2(n-1) + 1 = 2n - 1$. If we divide the length of a whole charging cycle by this quantity it gives the available time δ_{av} between any two recharging events in the worst case, which should be at least equal to $\delta_{t_{cr}}$. To check this

$$\delta_{av} - \delta_{t_{cr}} = \frac{(E_{max} - E_{lb})\left(1 + \frac{k_e}{k_{ch}}\right)}{k_e}\left[\frac{1}{2n-1} - \frac{1}{A}\right] \tag{40}$$

and the difference of the numerator is $\frac{k_e}{k_{ch}}(n-1) > 0$, which in turn means that for the $2n-1$ intervals, each can be at least $\delta_{t_{cr}}$. This means that there exist values of E_{min_i} for each of these landings that are properly temporally separated.

To complete the proof, we consider two corner cases for E_i at the beginning of each cycle, to ensure the robots are able to adopt new E_{min} values leading to separate landings. The worst case is $E_i = E_{min_i}$ at the beginning of each cycle. In this case, setting $E_{min_i} = E_{lb}$ gives

$$\begin{aligned} T_{L_2} &= \frac{E_{lb} + \Delta\bar{E}_m - E_{lb}}{k_e} = \delta_{t_{cr}} \\ T_{L_3} &= \frac{E_{lb} + 2\Delta\bar{E}_m - E_{lb}}{k_e} = 2\delta_{t_{cr}} \\ &\vdots \end{aligned} \tag{41}$$

the second corner case is if $E_i = E_{max}$ at the beginning of the cycle, which has a solution by design, as value of E_{min_i} separated apart by at least $\Delta\bar{E}_m$ assures having $\delta_{t_{cr}}$ between arrival times by design. ∎

B. Coordination CBF

This CBF aims at separating the arrival times of two robots with at least δ_t, and the core idea is very similar to the collision avoidance strategy proposed in [10], albeit it is collision of arrival times. For that we define a pairwise safe set $\mathcal{C}_{ij}$ as

$$\mathcal{C}_{ij} = \{(E_{min_i}, E_{min_j}) \in \mathbb{R}^2 | h_{c_{ij}} \geq 0\} \tag{42}$$

and we propose the following CBF $h_{c_{ij}}$ between two agents (i, j)

$$h_{c_{ij}} = \log \frac{|T_{L_i} - T_{L_j}|}{\delta_t} \tag{43}$$

where T_{L_i} is the arrival time of robot i as described in (11), and δ_t is the desired separation in arrival times between robots. The resulting constraint for the QP problem is

$$\frac{T_{L_i} - T_{L_j}}{|T_{L_i} - T_{L_j}|} \frac{\Gamma_i - \Gamma_j}{ke} \geq -\alpha(h_{c_{ij}}) \tag{44}$$

where $\Gamma_i = \frac{d}{dt} T_{L_i} = -k_{e_i} - \eta_i$.

It would be more practical to consider a decentralized version of equation (44) and to show that (43) is a ZCBF. The desired decentralization can be done by dropping the term η_j from Γ_j in (44) so we end up with (noticing that robots are assumed to have the same discharge rate)

$$-\frac{T_{L_i} - T_{L_j}}{k_e|T_{L_i} - T_{L_j}|}\eta_i \geq -\alpha(h_{c_{ij}}). \tag{45}$$

For the $\alpha(h_{c_{ij}})$ function in (45) we propose the form

$$\alpha(h_{c_{ij}}) = \gamma_{ij} \cdot \text{sign}(h_{c_{ij}}) \cdot |h_{c_{ij}}|^{\rho} \quad , \rho \in [0, 1)\,. \tag{46}$$

We define the value of γ_i for robot i as

$$\gamma_{ij} = \begin{cases} \gamma_h & \text{, if } ||x_i - x_c|| > \delta \text{ and } ||x_j - x_c|| > \delta \\ 0 & \text{, otherwise} \end{cases} \tag{47}$$

Theorem 2: For a multi robot system with (6) as the dynamics of each robot, then for a pair of robots (i, j) satisfying $||x_i - x_c|| > \delta$ and $||x_j - x_c|| > \delta$, $h_{c_{ij}}$ is a ZCBF for $\eta \in \Theta = \mathbb{R}$, with (45) rendering the set $\mathcal{C}_{ij}$ forward invariant. Moreover, if $(E_{min_i}(t_0), E_{min_j}(t_0)) \notin \mathcal{C}_{ij}$, (45) leads $(E_{min_i}(t), E_{min_j}(t))$ to converge to $\partial\mathcal{C}_{ij}$ in finite time.

Proof: Since $\eta \in \Theta = \mathbb{R}$ there exists a control action η that satisfies (3) (and keeps $\mathcal{C}_{ij}$ invariant), then to show that $h_{c_{ij}}$ is a ZCBF, we need to make sure that $|T_{L_i} - T_{L_j}| \neq 0$.

The only chance that this difference can be equal to zero is when one of the robots enters to the charging region. To show this, consider having two robots (i, j) applying (45) and without loss of generality suppose that robot j arrives at the charging station, so the difference in arrival times is

$$\Delta T_{L_{ij}} = \frac{1}{k_e}\left[E_i - E_{min_i} - (E_j(t_a) + k_{ch}(t - t_a) - E_{min_j})\right] \tag{48}$$

Due to the choice of γ_{ij} in (47) the right hand side of (45) is equal to zero, so the choice of $\eta_{nom_i} = \eta_{nom_j} = 0$ as nominal values actually satisfies (45) for both robots and consequently E_{min_i} and E_{min_j} does not change. This means that (48) is

$$\Delta T_{L_{ij}} = \frac{1}{k_e}\left[E_i - E_{min_i} - (\delta_E + k_{ch}(t - t_a))\right] \tag{49}$$

where δ_E is the difference between the voltage and minimum voltage at the time of arrival to the charging station, expressed in (11). Therefore, it can be seen from (49) that at some point this difference in arrival times is equal to zero (E_i decreases while $k_{ch}(t - t_a)$ increases), rendering (44) undefined. We can conclude that (43) is a ZCBF when both robots are out of the charging region.

The proof of the second part is the same as that of proposition III.1 in [11] and is omitted here for space limits. ∎

It is worth mentioning that the main motivation in the choice of (46) is the idea that each robot can start with a random estimate of its E_{min_i} and generating a control action η_i that satisfies (45) ensures safety in finite time.

In addition, the above theorem is applicable even if one robot i applies (45) with respect to another robot j while robot j is not doing the same. This follows the same argument that since $\eta_i \in \Theta = \mathbb{R}$ then there is always a value of η_i that satisfies (3), as long as both robots are away from the charging station.

In the coordination strategy we propose, we decouple the energy sufficiency behaviour and the coordination behaviour. This should not affect the arguments stated earlier about the ability of the energy sufficiency ZCBF to track E_{min}. This is because (10) and (11) do not put a constraint on the change of E_{min} as long as u is generated in such a way that causes $\dot{h}_s = -\beta h$.

To show this, the constraint in (5) for h_s, assuming that E_{min} changes, is

$$-k_e - \eta - \frac{k_c}{D^2}(x - x_c)^T u \geq -\alpha(h_s) \tag{50}$$

and if we consider E_{min} not to change in this constraint the second term of the LHS of the previous inequality drops out

$$-k_e - \frac{k_c}{D^2}(x - x_c)^T u \geq -\alpha(h_s) \tag{51}$$

If we consider that $u \in U = \mathbb{R}^2$ (which was the case for Theorem 1 in [4] to show that h_s is a ZCBF), then it can be argued that there will be always u' to be substituted in (51) such that

$$\frac{k_c}{D^2}(x - x_c)^T u' = \frac{k_c}{D^2}(x - x_c)^T u + \eta \tag{52}$$

and this indicates the possibility to decouple the coordination from the energy sufficiency CBFs.

C. Feasibility of QP

Our coordination strategy introduces two barrier functions: $h_{c_{ij}}$ which tends to keep two agents' arrival times separate through changing E_{min}, and h_L which bounds E_{min} from below. We need to assure that these constraints are admissible and lead to a feasible QP.

The main problem lies in the fact that the coordination effort is being done by changing only E_{min} which is only 1-D. This way potentially conflicting constraints in the QP (5) may render it infeasible. This issue has been dealt with in the context of control barrier function composition in [12] and [10] and we adapt the basic idea from the latter.

The main idea is that for a local agent, instead of applying the coordination constraint with all other agents, it only has to apply it with the agent that has the closest arrival time among all other agents (hence the need for assumption 1). If during the process the value of E_{min} is about to go below the lower bound, the agent stops the coordination and focuses only on keeping the lower bound of E_{min}.

This way, we make sure that by construction each agent only changes E_{min} in one direction, leading to a feasible QP. This idea is summarized in Algorithm (1).

The final QP is

$$\begin{aligned} \mathbf{u}^* = \min_{\mathbf{u} \in \mathbb{R}^3} \quad & ||\mathbf{u} - \mathbf{u}_{nom}|| \\ \text{s.t.} \quad & A\mathbf{u} \geq B \end{aligned} \tag{53}$$

Algorithm 1: Coordination algorithm

Input: $T_{L_k} \quad, \forall k \in \mathcal{N}_i$
Result: A_c and B_c for QP constraints
$h_{c_{min}} = h_0$
$h_{L_i} = E_{min_i} - E_{lb}$
while j *in* $\mathcal{N}_i$ **do**
  $h_{c_{ij}} = \log \frac{T_{L_i} - T_{L_j}}{\delta}$
  if $h_{c_{ij}} < h_{c_{min}}$ **then**
    $h_{c_{min}} = h_{c_{ij}}$
  end
end
if $h_{c_{min}} < h_{L_i}$ **then**
  $A_c = L_g h_{c_{min}}$
  $B_c = -L_f h_{c_{min}} - \alpha(h_{c_{min}})$ ...(eqn. 45)
else
  $A_c = L_g h_{L_i}$
  $B_c = -L_f h_{L_i} - \alpha(h_{L_i})$ ...(eqn. 25)
end

where

$$A = \begin{bmatrix} A_s^T \\ A_{ov}^T \\ A_c^T \end{bmatrix} = \begin{bmatrix} -\frac{k_c}{D^2}(x - x_c)^T & 0 \\ \frac{k_{ov}}{D^2}(x - x_c)^T & 0 \\ \begin{bmatrix} 0 & 0 \end{bmatrix} & A_c^T \end{bmatrix},$$

$$B = \begin{bmatrix} B_s \\ B_{ov} \\ B_c \end{bmatrix} = \begin{bmatrix} k_e - \alpha_s(h_s) \\ k_{ch} - \alpha_{ov}(h_{ov}) \\ B_c \end{bmatrix}$$

while A_c and B_c are determined from Algorithm (1). In the following, we show that Algorithm (1) indeed achieves the desired coordination task.

Theorem 3: For a multi robot system with dynamics defined in (6) and with the coordination and lower bound constraints defined in (43) and (24), and provided that the inequalities (28) and (39) are satisfied, then Algorithm (1) assures that the difference in arrival time between any two robots is at least δ_t (mutual exclusive use is assured).

Proof: From Algorithm (1), each robot is either applying the coordination CBF $h_{c_{ij}}$ or the lower bound CBF h_L. For the robots which do not apply h_L, from Theorem (2), for a robot i the control action that respects the constraint (44) leads E_{min_i} into safe set $\mathcal{C}_{ij}$ with respect to its neighbor with the closest landing time. Each robot can apply this to its neighbor with the closest landing time $\{(i,j) | j \in \mathcal{N}_i \text{ and } h_{c_{ij}} = \min_{k \in \mathcal{N}_i} h_{c_{ik}}\}$, eventually leading to $E_{min_i} \in \mathcal{C} = \bigcap_{\forall i \neq j} \mathcal{C}_{ij} \quad, \forall i$.

Moreover, since we have established the feasibility of the scheduling problem in Lemma (6), then we know that the sets $\mathcal{C}_{ij}$ are nonempty and that a solution exists.

If a robot i is applying the lower bound h_L, then it can't push its arrival time any further. In this case The nearest robot j that applies the coordination CBF will have a control action η_j that will lead E_{min_j} to $\mathcal{C}_{ij}$ (noticing that $\mathcal{C}_{ij}$ is non empty), and then all other robots applying coordination CBF will coordinate in a pairwise fashion based on the neighbour of closest landing time as discussed in the previous point. ■

V. Results

A. Simulation results

We carried out simulations using ARGoS [13], a physics-based simulator designed to handle multi-robot and swarm systems. The code was written using the Buzz programming language [14].

The mission considered for this simulation is a coverage mission, as in [4], in which 7 robots spread over a given area.

The main requirement is to cover a square of dimensions $6m \times 6m$ with a charging region of radius $0.2m$ around the origin. The robots are required to arrive at the charging station with a separation of $\delta_t = 20$ sec (knowing that it takes $\frac{E_{max} - E_{lb}}{k_{ch}} = 6$ sec to recharge). Table I contains the parameters of the system.

TABLE I: Values of parameters used in simulation

Parameter	k_e	k_{ch}	n	E_{max}	E_{lb}	$\delta_{t_{cr}}$
Value	0.012	0.5	7	13.2	10	$20.77 > \delta_t$

Figure 1a shows the separation in arrival times is as desired. We note that the voltage does not go below E_{lb}. It can be seen as well that the maximum voltage is exceeded in a small number of conditions because of the kinematic model in the simulation, which was for a differential drive robot, so the rotation the robot experiences to pursue the point mass velocity introduces some delay that can cause such peaks.

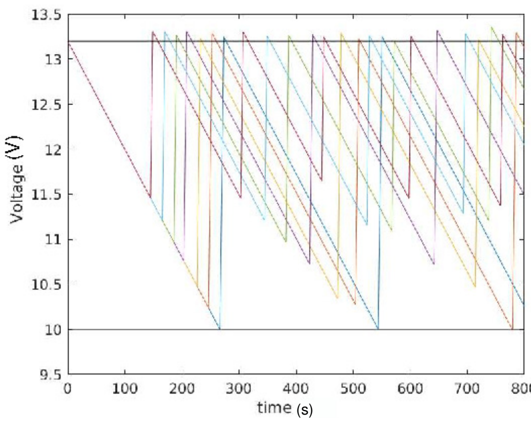
B. Experimental results

We performed a simple waypoint navigation mission with three Khepera IV robots (where $u_{nom} = -k_p(x - x_{target})$), where $k_p > 0$ is a proportional gain). In our experiment, a virtual battery simulated in code was used instead of a real battery for the sake of proving the concept and probing the effects of more realistic operating conditions on the proposed algorithm. The code had a running frequency of 10 Hz, and an optical tracking system was used for position feedback. Figures 1c and 1d show the evolution of E and E_{min} with time.

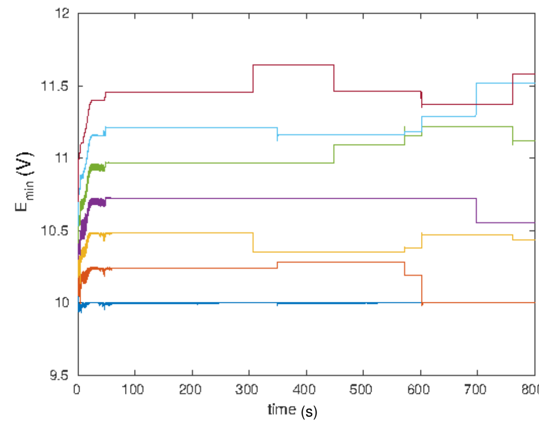
VI. Conclusions

In this paper we present a control barrier function (CBF) based framework for long term autonomy of multi robot systems with limited charging resources. We started by highlighting some tracking properties of the energy persistence CBF in [4] and then we introduced a CBF based framework to achieve the necessary coordination for sharing the charging station.

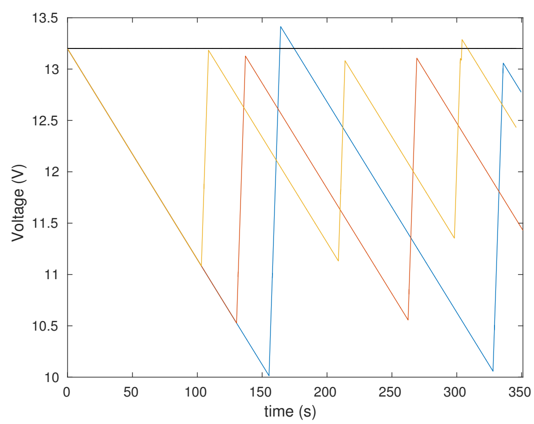
As a future work we consider extending the current results by investigating double integrator robot models with disturbances and examine the effect of such disturbances on the coordination behavior. Moreover, we would like to accommodate our approach to the case of having multiple charging stations and possibly relaxing the assumption of having a complete communication graph.



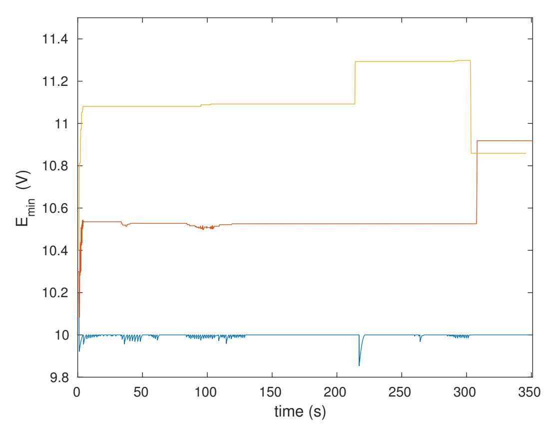
(a)



(b)



(c)



(d)

Fig. 1: Evolution of voltage and E_{min} values for the coverage task in V-A (1a and 1b) and the waypoint navigation task in V-B (1c and 1d). The occasional overshoots of voltage can be mostly attributed to the difference between the single integrator kinematics and that of an actual robot. There is also some jitter in E_{min} due to the switching nature of Algorithm 1.

REFERENCES

[1] Z. Sun and J. Reif, "On energy-minimizing paths on terrains for a mobile robot," in *2003 IEEE International Conference on Robotics and Automation (Cat. No. 03CH37422)*, vol. 3. IEEE, 2003, pp. 3782–3788.

[2] A. Kwok and S. Martinez, "Energy-balancing cooperative strategies for sensor deployment," in *2007 46th IEEE Conference on Decision and Control*. IEEE, 2007, pp. 6136–6141.

[3] Y. Ding, W. Luo, and K. Sycara, "Decentralized multiple mobile depots route planning for replenishing persistent surveillance robots," in *2019 International Symposium on Multi-Robot and Multi-Agent Systems (MRS)*. IEEE, 2019, pp. 23–29.

[4] G. Notomista, S. F. Ruf, and M. Egerstedt, "Persistification of robotic tasks using control barrier functions," *IEEE Robotics and Automation Letters*, vol. 3, no. 2, pp. 758–763, 2018.

[5] A. D. Ames, S. Coogan, M. Egerstedt, G. Notomista, K. Sreenath, and P. Tabuada, "Control barrier functions: Theory and applications," in *2019 18th European Control Conference (ECC)*. IEEE, 2019, pp. 3420–3431.

[6] A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada, "Control barrier function based quadratic programs for safety critical systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 3861–3876, 2016.

[7] X. Xu, P. Tabuada, J. W. Grizzle, and A. D. Ames, "Robustness of control barrier functions for safety critical control," *IFAC-PapersOnLine*, vol. 48, no. 27, pp. 54–61, 2015.

[8] J. Wang, M. Hu, C. Cai, Z. Lin, L. Li, and Z. Fang, "Optimization design of wireless charging system for autonomous robots based on magnetic resonance coupling," *Aip Advances*, vol. 8, no. 5, p. 055004, 2018.

[9] G. Notomista and M. Egerstedt, "Persistification of robotic tasks," *arXiv preprint arXiv:1903.05810*, 2019.

[10] M. Egerstedt, J. N. Pauli, G. Notomista, and S. Hutchinson, "Robot ecology: Constraint-based control design for long duration autonomy," *Annual Reviews in Control*, vol. 46, pp. 1–7, 2018.

[11] A. Li, L. Wang, P. Pierpaoli, and M. Egerstedt, "Formally correct composition of coordinated behaviors using control barrier certificates," in *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2018, pp. 3723–3729.

[12] L. Wang, A. D. Ames, and M. Egerstedt, "Multi-objective compositions for collision-free connectivity maintenance in teams of mobile robots," in *2016 IEEE 55th Conference on Decision and Control (CDC)*. IEEE, 2016, pp. 2659–2664.

[13] C. Pinciroli, V. Trianni, R. O'Grady, G. Pini, A. Brutschy, M. Brambilla, N. Mathews, E. Ferrante, G. Di Caro, F. Ducatelle, *et al.*, "Argos: a modular, parallel, multi-engine simulator for multi-robot systems," *Swarm intelligence*, vol. 6, no. 4, pp. 271–295, 2012.

[14] C. Pinciroli, A. Lee-Brown, and G. Beltrame, "Buzz: An extensible programming language for self-organizing heterogeneous robot swarms," *arXiv preprint arXiv:1507.05946*, 2015.