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Robust Dynamic State Estimation for Lateral Control of an Industrial Tractor Towing Multiple Passive Trailers

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***Abstract*— In this paper, we propose a dynamic state estimation framework for lateral control of a heavy tractor-trailers system using only mass-produced low-cost sensors. This issue is challenging since the lateral velocity of the lead tractor is difficult to measure directly. The performance of existing dynamic model-based estimation methods will also be degraded, as different trailers and payloads cause the tractor model parameters to change. We address this issue by incorporating a kinematic estimator into a dynamic model-based estimation scheme. Accurate and reliable tire cornering stiffness and dynamics-informed lateral velocity of the lead tractor can be output in real-time by using our method. The stability and robustness of the proposed method are theoretically proved. The feasibility of our method is verified by full-scale experiments. It is also verified that the estimated model parameters and lateral states do improve the control performance by integrating the estimator into a lateral control system.**

I. INTRODUCTION

The tractor-trailer system consists of a tractor towing multiple passive trailers. Such systems are widely used in logistics warehouses, airports, terminals, etc., due to their high-efficient and low-cost cargo transportation capabilities. As pointed out in [1], the forward motion of the tractor-trailers system is naturally stable. Therefore, as long as the lead tractor can be controlled accurately and automatically, autonomous control of the entire system can be achieved. However, there are two main challenges faced in this case. First, due to complex industrial environments and sensing errors, kinematic methods are usually difficult to provide long-term reliable dynamic feedback [2]. Second, the different number of trailers and payloads will affect the dynamic behavior of the lead tractor, thereby decreasing the performance of existing model-based estimation and control methods [3], [4]. This paper proposes a robust and easy-to-use estimation scheme suitable for the tractor-trailers system. The output of the estimator can then be used for high-performance path control of the heavy tractor-trailers system.

The dynamic state of a vehicle usually includes the longitudinal and lateral velocity and the yaw rate. In general, the yaw rate can be measured by a gyroscope or an attitude and heading reference system (AHRS) sensor. The longitudinal velocity can be measured by wheel encoders. Existing methods such as [5] can also be adopted for closed-loop estimation of the longitudinal velocity. However, the lateral velocity, sometimes in the form of vehicle sideslip angle, cannot be measured directly unless costly optical or global sensors are used [2]. As an important state of vehicle lateral control, the estimation of the vehicle's lateral velocity (or in the form of sideslip angle), has been widely investigated in the literature. The existing work falls into two main categories: kinematic methods and dynamic model-based methods.

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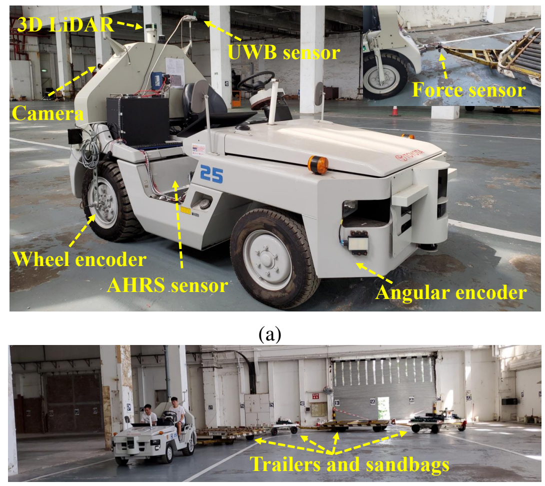

(a)

(b)

Fig. 1. A full-size self-driving tractor and its sensing system.

Kinematic methods for lateral velocity estimation are typically performed based on measurements from inertial sensors and kinematic differential equations that describe vehicle motion [6]. In order to overcome the cumulative error introduced by direct integration, the EKF method and the nonlinear observer method are adopted in [7] and [2], respectively, to achieve closed-loop estimation. Besides, as the most widely used additional sensor, Global Positioning System (GPS) measurements can further improve the performance of kinematic methods for lateral vehicle velocity estimation [7]. The fusion of visual information has also promoted the kinematic estimation performance in GPS-denied environments [2], [8]. Kinematic methods are robust to modeling uncertainties since it is model-free, but it is particularly sensitive to sensor errors [9]. Furthermore, [10] indicated that the kinematic method could support satisfactory lateral velocity estimation when the vehicle is laterally excited.

On the other hand, the dynamic model-based method is relatively robust to sensor errors, as the introduction of the vehicle model helps the convergence of the estimation. Therefore, such methods have been intensively studied [11], [12]. On the downside, dynamic model-based methods rely heavily on model parameters. Especially, tire modeling and parameter identification, in general, are complicated and vary under different driving conditions [13]. In [14], a nonlinear observer was proposed for vehicle state estimation, but highly complex tire models were used. A complicated tire parameter identification process is required before the algorithm runs, which increases the difficulty of practical application. For the normal driving of light vehicles, a linear lateral tire force model based on cornering stiffness and sideslip angle is simple and effective, and is therefore widely used in existing path control studies [3]. However, this is not the case for the lateral control of a tractor-trailers system. Different numbers of trailers and different weights of cargo will result in different hitch forces, and eventually, lead to lateral tire force variations of the lead tractor. As indicated in [13], the increase of the lateral force variation range makes it easier for the tire to enter the nonlinear region, thereby impairing the accuracy of the linear lateral force model.

Several approaches have been devoted to handling lateral tire force model uncertainties. In [9], a scale parameter was introduced to adjust the lateral tire force, and this parameter was then added to the state vector and updated with the estimation system. These approaches rely on empirical tire models. They have been verified for specific tire types to guarantee better velocity estimation performance, even if the tire enters a nonlinear region. But the versatility of the algorithm is limited. In [15], a comprehensive and robust vehicle dynamics state estimation strategy was proposed. Also, [16] handles the uncertainty of lateral tire force modeling by designing an online cornering stiffness update method. However, [15] and [16] require the measurement of driving torques and longitudinal tire forces, respectively, which are difficult to obtain for mass-produced industrial tractors.

In this paper, by integrating the advantages of the kinematic and the dynamic model-based approach, we propose a robust and easy-to-use lateral dynamic state estimator that suitable for a heavy tractor-trailers vehicle. The originality of the paper is summarized as follows: 1) We propose a complete lateral dynamics estimation framework for the tractor-trailers system, which includes model parameter adaptation and lateral state estimation. 2) The stability and robustness of the proposed method are theoretically proved. 3) The proposed method is robust and can be easily implemented using only mass-produced and low-cost sensors, and is suitable for large-scale industrial applications. The validity and performance of the proposed estimator are verified by experiments on a full-scale autonomous tractor-trailers system.

II. System Modeling and Description

The objective of this study is to develop an accurate and reliable lateral dynamics estimation framework, using only cost-optimized sensors, for an industrial tractor-trailers system (Fig. 1). Note that only state estimation in uniform forward motion is considered in this work. In practice, the uniform forward path following of the tractor-trailers system is sufficient to achieve fully autonomous cargo transportation tasks in most scenarios [2]. In addition, for safety reasons, the reversing of such heavy and long systems is not allowed at most industrial sites due to the jack-knife phenomenon.

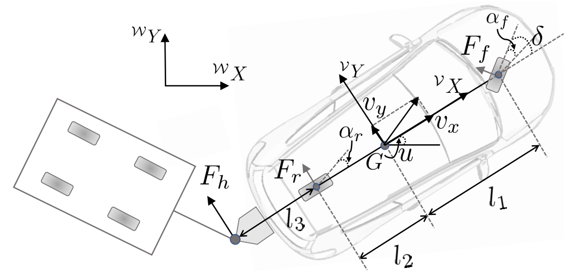

Fig. 2. The diagram of a single-track tractor model.

A. System Modeling

Fig. 2 shows a two degrees-of-freedom model for the lateral motion of the lead tractor. The world frame and the tractor frame are denoted as $\{\mathcal{W}\}$ and $\{\mathcal{V}\}$, respectively, and $\{\mathcal{V}\}$ is attached at the center of gravity G of the tractor. v_x and v_y represent the longitudinal and lateral velocities of the tractor in $\{\mathcal{V}\}$, and u is the yaw rate of the tractor. Be noticed that v_x is a positive constant in our case, then a kinematic differential equation is given below,

$$\dot{v}_y = -uv_x + a_y, \tag{1}$$

which correlates v_y, u, and a_y, and a_y is the lateral accelerations of the tractor at G. To further characterize the heavy and low-speed tractor dynamics, a nonlinear single-track model is adopted. In this context, the tractor is assumed to drive on a flat plane, and the vehicle roll and pitch motion, the air drag, rolling, and internal resistances, as well as suspension dynamics, are neglected [17]. The dynamic equations describe tractor motion are obtained by applying the Newton-Euler method

$$\begin{aligned} ma_y &= F_f\cos(\delta) + F_r + F_h, \\ J_z\dot{u} &= l_1F_f\cos(\delta) - l_2F_r - (l_2+l_3)F_h, \end{aligned} \tag{2}$$

where m is the mass, J_z is the yaw moment of inertia, δ is the front steering angle. l_1 and l_2 are the distances from G to front/rear axle, and l_3 is the distance between the force sensor and the rear wheel. F_h is the measurable lateral force that is exerted on the tractor at the off-axle hitch point by towing trailers and payloads. F_f, F_r represent the lateral forces of the front and rear tires, respectively. For low-speed plane motion, the lateral tire force can be represented by a linear tire model of the respective tire sideslip angles [17]

$$F_f = \bar{C}_f\alpha_f, \quad F_r = \bar{C}_r\alpha_r, \tag{3}$$

$$\alpha_f = \delta - \frac{(v_y + l_1u)}{v_x}, \quad \alpha_r = -\frac{(v_y - l_2u)}{v_x}. \tag{4}$$

where $\bar{C}_f$ and $\bar{C}_r$ denote the constant nominal cornering stiffness of the front and rear tire, respectively. C_f and C_r

are the cornering stiffness to be identified for the current towing configuration. α_f and α_r denote sideslip angles of the front and rear wheels, respectively.

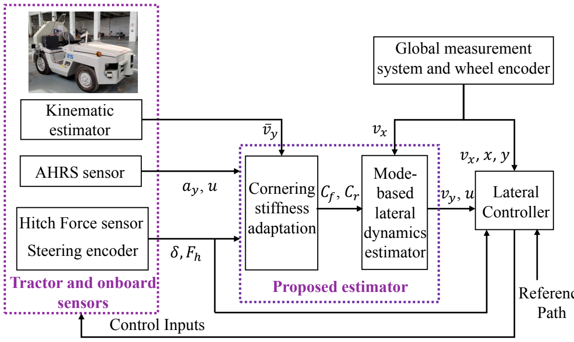

Fig. 3. The proposed estimation framework.

B. System Description

Fig. 3 shows the framework of the proposed estimation system. With the fact that the forward driving of the tractor-trailers vehicle is naturally stable, the control performance of the entire system can be improved by improving the dynamic estimation and control of the lead tractor. Using professional simulation software, we know that the lateral tire force of the tractor keeps at a low level under small lateral excitation, hence the linear tire model with nominal cornering stiffness is accurate. However, different hitch loads cause the tractor tire to easily enter the nonlinear region when turning. From the tire characteristics, the nominal cornering stiffness decreases rapidly outside the linear region [17].

Based on these observations, the proposed estimator is designed by fusing the kinematic and dynamic information. Specifically, when the vehicle is laterally excited, a kinematic method will be employed to provide lateral velocity for tire parameter adaptation of the tractor. Similar to [18], we make the observation that the kinematic estimator (see for example [7] and [2]) can be considered as a sensor that outputs kinematic lateral velocity measurements. Benefiting from the updated model, a dynamic model-based method will be proposed, which can support more reliable lateral velocity estimates. As will be demonstrated in Section IV, both the updated model parameters and the model-aided lateral velocity estimates can help improve the path control performance of the heavy tractor-trailers system.

III. Lateral Dynamics Estimation Approach

A. Cornering Stiffness Adaptation

The linear tire model (3) is widely used in existing research [2], [3], [19]. However, for a heavy tractor-trailers case where the lateral tire force varies greatly, model accuracy will decrease if only nominal cornering stiffness is used. Investigations have shown that introducing a mechanism to update the cornering stiffness in different towing configurations leads to improved lateral velocity estimation and control performance.

In view of the fact that the cornering stiffness varies slowly during normal driving, also, the kinematic method can provide appropriate lateral velocity measurement (denoted as $\bar{v}_y$) when the tractor has lateral excitation. Therefore, when the tractor has lateral motion, $\bar{v}_y$ can be regarded as a reference for updating the cornering stiffness. Eligible lateral motion patterns will be quantified and discussed in the subsequent observability analysis part.

For cornering stiffness adaptation, we define $\boldsymbol{x}_1 = [u, v_y]^T$ as a state vector. The measurement of $\boldsymbol{x}_1$, denoted as $\bar{\boldsymbol{x}}_1 = [\bar{u}, \bar{v}_y]^T$, is available by using the AHRS sensor and the kinematic estimator. By summarizing (1)-(4), we have the following differential equations

$$\dot{\boldsymbol{x}}_1 = \boldsymbol{P}(\boldsymbol{x}_1) + \boldsymbol{Q}(\delta)[C_f, C_r]^T, \tag{5}$$

where

$$\boldsymbol{P}(\boldsymbol{x}_1) = \begin{bmatrix} -\frac{l_2+l_3}{J_z}F_h \\ -v_x u + \frac{F_h}{m} \end{bmatrix}, \ \boldsymbol{Q}(\delta) = \begin{bmatrix} \frac{l_1 cos(\delta)}{J_z}\bar{\alpha}_f & -\frac{l_2}{J_z}\bar{\alpha}_r \\ \frac{cos(\delta)}{m}\bar{\alpha}_f & \frac{1}{m}\bar{\alpha}_r \end{bmatrix},$$

and $\bar{\alpha}_f$, $\bar{\alpha}_r$ are known from (4) and sensor measurements.

Define $\hat{\boldsymbol{x}}_1 = [\hat{u}, \hat{v}_y]^T$ as the estimated state. According to the duality of observation and control, we consider C_f and C_r as control variables. Then, by designing the control input to impose the estimated state $\hat{\boldsymbol{x}}_1$ to converge to the measured one $\bar{\boldsymbol{x}}_1$, the purpose of adapting the cornering stiffness can be achieved. To this end, we define the estimation error $\tilde{\boldsymbol{x}}_1 = \hat{\boldsymbol{x}}_1 - \bar{\boldsymbol{x}}_1$, then, the control input can be designed as

$$[\hat{C}_f, \hat{C}_r]^T = \boldsymbol{Q}^{-1}(\delta)(-\boldsymbol{K}_1\tilde{\boldsymbol{x}}_1 - \boldsymbol{P}(\hat{\boldsymbol{x}}_1)), \tag{6}$$

where $\boldsymbol{K}_1$ is a positive definite gain matrix. Be noticed that $\dot{\bar{\boldsymbol{x}}}_1 = 0$ is used, since the quasi-static evolution for $\bar{\boldsymbol{x}}_1$ is well preserved for our low-speed heavy systems. Then, using (6), the state error dynamics can be derived as

$$\dot{\tilde{\boldsymbol{x}}}_1 = -\boldsymbol{K}_1\tilde{\boldsymbol{x}}_1, \tag{7}$$

which implies the exponential convergence of $\tilde{\boldsymbol{x}}$. This indicates that (6) is the relevant result of adjusting the cornering stiffness online for the current tractor-trailers configuration.

The only troublesome is that the matrix $\boldsymbol{Q}(\delta)$ inversion is needed. We can see that the matrix $\boldsymbol{Q}(\delta)$ is respectively not invertible or ill-conditioned for sideslip angles that are null or close to zero. This situation is often encountered in actual driving scenarios, such as the tractor is traveling along a straight line. Therefore, the cornering stiffness adaptation is only activated when the condition number test of $\boldsymbol{Q}(\delta)$ is satisfactory. This usually happens when the tractor turns, that is, when there is lateral excitation. If $\boldsymbol{Q}(\delta)$ is not invertible or ill-conditioned, which occurs when the tractor is not turning, the previous valid cornering stiffness will be retained for the subsequent estimator. As will be demonstrated in Section IV, for constant road conditions, the cornering stiffness for a specific tractor-trailers configuration will converge to a constant and relevant value during turning.

B. Model-Aided State Estimation

When developing the model-based estimator, the cornering stiffness provided by Section III-A is assumed to be accurate first. The robustness analysis of the estimator in Section III-C will allow this assumption to be removed.

1) Estimator Design: Benefiting from the updated cornering stiffness for the current towing configuration, a model-based lateral dynamic state estimator is developed in this subsection. Compared with the kinematic method, the new estimator can provide reactive lateral velocity estimation in any motion patterns due to the introduction of dynamic information, and it is robust to sensor errors.

From (2), the force and torque equation of the tractor can be lumped as

$$\begin{aligned} ma_y &= F_y(\delta, v_x, v_y, u), \\ J_z\dot{u} &= T_z(\delta, v_x, v_y, u), \end{aligned} \tag{8}$$

where $F_y(\delta, v_x, v_y, u)$ and $T_z(\delta, v_x, v_y, u)$ can be further denoted by F_y and T_z, respectively, for simplicity of notation. By injecting (3) and (4), (8) can be formulated as

$$\begin{aligned} F_y =&(\delta - \frac{v_y + l_1 u}{v_x})C_f cos(\delta) + \frac{l_2 u - v_y}{v_x}C_r + F_h, \\ T_z =&(\delta - \frac{v_y + l_1 u}{v_x})l_1 C_f cos(\delta) - \frac{l_2 u - v_y}{v_x}l_2 C_r \\ &- (l_2 + l_3)F_h. \end{aligned} \tag{9}$$

For lateral state estimation, we define $\boldsymbol{x}_2 = [v_y, u]^T$ as the state vector. Note that u is added to the state vector and estimated with v_y, so that the yaw rate measurements from the AHRS sensor can be fully utilized. The estimated state is defined as $\hat{\boldsymbol{x}}_2 = [\hat{v}_y, \hat{u}]^T$. Then, similar to (8), we have

$$\begin{aligned} m\hat{a}_y &= F_y(\delta, v_x, \hat{v}_y, \hat{u}) = \hat{F}_y, \\ J_z\dot{\hat{u}} &= T_z(\delta, v_x, \hat{v}_y, \hat{u}) = \hat{T}_z, \end{aligned} \tag{10}$$

where $\hat{F}_y$ and $\hat{T}_z$ are calculated from (9), except that v_x and u are replaced by the estimated $\hat{v}_y$ and $\hat{u}$.

Based on the above tractor model and inertial measurements, the lateral state estimator is designed as below

$$\begin{aligned} \dot{\hat{v}}_y &= -\hat{u}v_x + a_y - \rho_1(ma_y - \hat{F}_y), \\ \dot{\hat{u}} &= \frac{\hat{T}_z}{J_z} + \rho_2(u - \hat{u}), \end{aligned} \tag{11}$$

where ρ_1 and ρ_2 are estimator gains. By introducing the tractor dynamics model, a feedback term can be formed between the estimated lateral state and the corresponding measurement one to close the estimation loop. Indeed, this method relies on the accuracy of inertial measurements. Therefore, on the one hand, we adopted a robust intrinsic calibration algorithm for inertial sensors [20] to improve the measurement of lateral acceleration and yaw rate. On the other hand, considering that the unknown vibration excitation from the road leads quite noisy acceleration measurements, a simple median filter-based denoising algorithm is adopted [12]. This method enables the onboard IMU to provide smooth vehicle acceleration measurements without the cost of much phase delay.

***Remark** 1:* The proposed model-aided estimator (11) is inspired by [14], but [14] needs additional tire parameter identification for each towing configuration before implementation, which impairs the practicality of the algorithm. With the help of linear tire models and online adaptation mechanisms, our estimator can be directly applied to industrial tractor-trailers vehicles. The effectiveness of the proposed estimator and its performance improvement on automatic control will be verified in Section IV.

2) Stability Analysis:

***Theorem** 1:* The proposed estimator (11) can uniformly exponentially stabilize the estimation system, i.e. $\lim_{t\to\infty} \hat{\boldsymbol{x}}_2 = \boldsymbol{x}_2$, under the conditions that the gains constraints (19) are satisfied.

Proof: The estimation error is first defined as

$$\tilde{\boldsymbol{x}}_2 = \boldsymbol{x}_2 - \hat{\boldsymbol{x}}_2. \tag{12}$$

Then, from (1), (2) and (11), the error dynamics can be derived as

$$\begin{aligned} \dot{\tilde{v}}_y &= -\tilde{u}v_x + \rho_1(F_y - \hat{F}_y), \\ \dot{\tilde{u}} &= \frac{T_z}{J_z} - \frac{\hat{T}_z}{J_z} - \rho_2\tilde{u}. \end{aligned} \tag{13}$$

Consider the following candidate Lyapunov function

$$L = \frac{1}{2}\tilde{v}_y^2 + \frac{1}{2}\tilde{u}^2. \tag{14}$$

By differentiating the Lyapunov function (14) leads to

$$\dot{L} = \tilde{v}_y\dot{\tilde{v}}_y + \tilde{u}\dot{\tilde{u}}. \tag{15}$$

From the mean value theorem, we have

$$\begin{aligned} F_y - \hat{F}_y =&(\partial F'_y/\partial v_y)\tilde{v}_y + (\partial F'_y/\partial u)\tilde{u}, \\ T_z - \hat{T}_z =&(\partial T'_z/\partial v_y)\tilde{v}_y + (\partial T'_z/\partial u)\tilde{u}, \end{aligned} \tag{16}$$

where $F'_y = F_y(\delta, v_x, v'_y, u')$ and $T'_z = T_z(\delta, v_x, v'_y, u')$, v'_y is any points located between the line segments v_y and $\hat{v}_y$, u' is any point located between the line segments u and $\hat{u}$. The partial derivatives involved in (16) can be calculated using (9). Since we consider lateral estimation and control for conventional heavy vehicles, therefore, v_x is a positive constant, the steering angle is mechanically limited to $|\delta| < 21°$, and $C_f > 0$, $C_r > 0$ always exist, we have

$$\begin{aligned} \partial F_y/\partial v_y <& -\beta_1, \quad \partial F_y/\partial u \le \beta_2, \\ \partial T_z/\partial v_y \le& \beta_3, \quad \partial T_z/\partial u \le \beta_4, \end{aligned} \tag{17}$$

where $\beta_1, \beta_2, \beta_3, \beta_4$ are positive constants. (17) means that the partial derivatives are all bounded. From (16)-(17) and using the Lipschitz condition [21], we have

$$\begin{aligned} (F_y - \hat{F}_y)\tilde{v}_y \le& -\beta_1\tilde{v}_y^2 + \beta_2\left|\tilde{u}\right|\left|\tilde{v}_y\right|, \\ (\frac{T_z}{J_z} - \frac{\hat{T}_z}{J_z})\tilde{u} \le& \beta_3\left|\tilde{u}\right|\left|\tilde{v}_y\right| + \beta_4\tilde{u}^2. \end{aligned} \tag{18}$$

We are now in a position to give the following gain conditions. For some $\epsilon > 0$, we choose

$$\begin{aligned} &\rho_1 > 0, \\ &\rho_2 > \epsilon + \beta_4 + \frac{(\beta_2\rho_1 + v_x + \beta_3)^2}{2\beta_1\rho_1}. \end{aligned} \tag{19}$$

Then, under (19) and by substituting (13), (18) into (15), we have

$$\begin{aligned}\dot{L} &= -\tilde{v}_y v_x \tilde{u} + \rho_1(F_y - \hat{F}_y)\tilde{v}_y + (\frac{T_z}{J_z} - \frac{\hat{T}_z}{J_z})\tilde{u} - \rho_2\tilde{u}^2 \\ &\leq (\rho_1\beta_2 + v_x + \beta_3)\,|\tilde{u}|\,|\tilde{v}_y| - \rho_1\beta_1\tilde{v}_y^2 + \beta_4\tilde{u}^2 - \rho_2\tilde{u}^2 \\ &\leq -(\sqrt{\frac{\rho_1\beta_1}{2}}|\tilde{v}_y| \frac{\beta_2\rho_1 + v_x + \beta_3}{\sqrt{2\rho_1\beta_1}}|\tilde{u}|)^2 + (\beta_4 - \rho_2)\tilde{u}^2 \\ &\quad + \frac{(\beta_2\rho_1 + v_x + \beta_3)^2}{2\beta_1\rho_1}\tilde{u}^2 - \frac{1}{2}\rho_1\beta_1\tilde{v}_y^2 \\ &\leq -\epsilon\tilde{u}^2 - \frac{1}{2}\rho_1\beta_1\tilde{v}_y^2 \\ &\leq -min\{\epsilon, \frac{\rho_1\beta_1}{2}\} \cdot 2L.\end{aligned} \tag{20}$$

Therefore, the estimation error $\tilde{\boldsymbol{x}}_2 = \mathbf{0}_{2\times1}$ is a uniformly exponentially stable equilibrium point of (13) by using (19), (20) and Lyapunov theory [21]. This completes the proof. ■

C. Robustness Analysis

Due to inherent difficulties in modeling the exact tractor and the dynamic effects imposed by different trailers and payloads, robustness analysis of the proposed model-aided estimator (11) is necessary. As a physically appropriate and natural way, additive perturbations are widely used for robustness analysis against vehicle modeling uncertainties and external disturbances. Similar to [2] and [19], we use additive perturbations to obtain a more practical model

$$\begin{aligned} m a_y &= F_f\cos(\delta) + F_r + F_h + Z_y, \\ J_z\dot{u} &= l_1F_f\cos(\delta) - l_2F_r - (l_2 + l_3)F_h + Z_u, \end{aligned} \tag{21}$$

where Z_y and Z_u are uncertain terms caused by the model-tractor mismatch and external disturbances. Then, by substituting (21) into (13), results in a new estimation error dynamics

$$\begin{aligned} \dot{\tilde{v}}_y &= -\tilde{u}v_x + \rho_1(F_y - \hat{F}_y) + \rho_1 Z_y, \\ \dot{\tilde{u}} &= \frac{T_z}{J_z} - \frac{\hat{T}_z}{J_z} - \rho_2\tilde{u} + \frac{Z_u}{J_z}. \end{aligned} \tag{22}$$

Use the same quadratic function (14) and follow the same derivation procedure from (15)-(20), we have

$$\dot{L} \leq -\epsilon\tilde{u}^2 - \frac{1}{2}\rho_1\beta_1\tilde{v}_y^2 + \rho_1\tilde{v}_y Z_y + \frac{Z_u}{J_z}\tilde{u}. \tag{23}$$

Define $\boldsymbol{z} = [\rho_1 Z_y, Z_u/J_z]^T$, then, for $\xi \in (0, 1)$, using (23) we have

$$\begin{aligned} &\dot{L} \leq -min\{\epsilon, \frac{\rho_1\beta_1}{2}\}(1-\xi)L, \\ &\forall\, \|\tilde{x}_2\| \geq \frac{\|\boldsymbol{z}\|}{min\{\epsilon, \frac{\rho_1\beta_1}{2}\}\xi}, \end{aligned} \tag{24}$$

which concludes the input-to-state stability (ISS) of the system (22) [21], namely, estimation errors $\tilde{\boldsymbol{x}}_2$ will always be bounded, as long as Z_y and Z_u are bounded.

***Remark** 2:* The ISS works well for most driving scenarios [2], [19]. As will be presented in Section IV, even for a heavy system, the boundedness of Z_y and Z_u is well maintained during normal driving and has been shown to have a limited impact on estimation and control performance (much less than the impact of cornering stiffness).

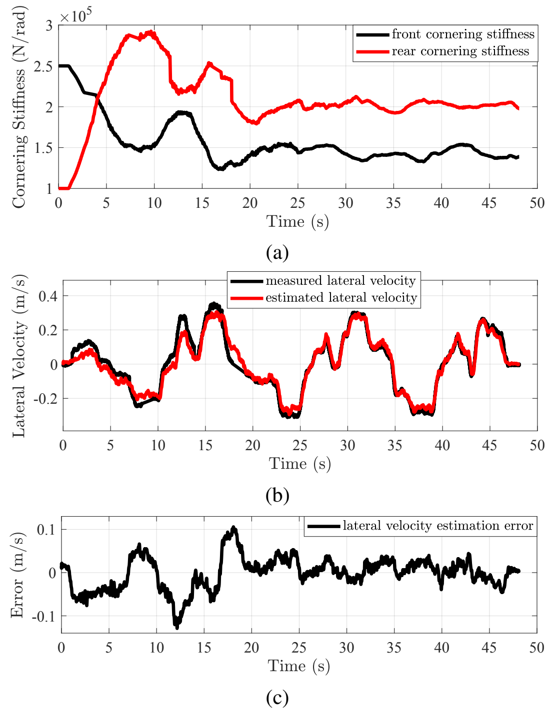

Fig. 4. Estimation results for the single tractor case. (a) Cornering stiffness adaptation results. (b) Lateral velocity estimation results. (c) Lateral velocity estimation errors.

IV. Experimental Validation

A. Experimental Setup

We have carried out the proposed method on a full-size tractor-trailers system in an industrial warehouse. As shown in Fig. 1, a *TOYOTA* diesel tractor is employed for experiments. It is a standard industrial heavy tractor with a front-steer and rear-drive mechanism. With the drive-by-wire retrofit, the tractor is compatible with both automatic and manual driving. The mass of the tractor is 4280kg and the yaw moment of inertia is 2356kg·m^2. The three wheelbases of the tractor are l_1=1.28m, l_2=0.43m, and l_3=0.87m, respectively. Besides, four standard full trailers and some sandbags (total weight of 6000 kg) are also adopted in the experiments. All estimation algorithms run in real-time with *Intel Core i7* CPU and 16 GB RAM. For performance evaluation, an integrated ground truth system [22], which fuses 3D LiDAR, camera, AHRS, and Ultra-Wideband (UWB) sensor information, is used in experiments.

The tractor is equipped with a *ME* K3D160 three-axis force sensor mounted on its hitch point to measure hitch forces at 100Hz. An *Xsens MTi* AHRS sensor is mounted to measure yaw rate and acceleration at 200Hz. Also, the steering angle of the tractor is measured by a *HEIN LANZ* encoder at 100Hz. Kinematic estimates $\bar{v}_y$ can be provided by GPS-based [7] or vision-based [2] method using conventional vehicle sensors. Therefore, the sensing systems needed to implement our algorithm are affordable. Additionally, the introduction of the force sensor enables the effects of different trailers and payloads on tractor dynamics to be captured directly, and then the algorithm proposed in this paper can be adopted to handle these effects properly. This novel and inexpensive method is compatible with any towing configuration and does not require complex identification of the dynamic parameters of the trailer and payload, making it suitable for large-scale industrial applications.

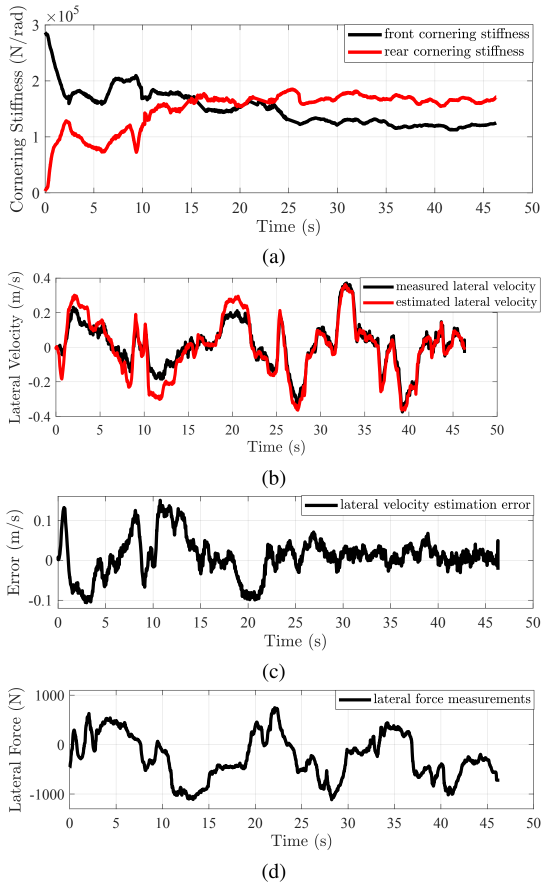

Fig. 5. Estimation results for the case of the tractor towing 4 full trailers. (a) Cornering stiffness adaptation results. (b) Lateral velocity estimation results. (c) Lateral velocity estimation errors. (d) Lateral hitch force measurements.

B. Results of the Estimation Algorithm

In the first experiment, we validate the effectiveness of the proposed estimation algorithm. Due to site regulations, automatic driving is allowed only on designated routes. Therefore, in order to thoroughly verify our online parameter adaptation and state estimation methods, we manually drive the tractor along an arbitrary path at a velocity of approximately 3m/s to support sufficient lateral excitation. In addition, we tested the single tractor and tractor-4-trailers (with sandbags) cases separately to show that our method works well in different towing configurations. In these experiments, kinematic-based $\bar{v}_y$ can be easily obtained following the method in [2].

Fig. 4a and Fig. 5a show the cornering stiffness adaptation results of the front and rear tires for the single tractor and the tractor-4-trailers case, respectively. We assign different initial values to the cornering stiffness of the front and rear tires in both cases. It can be observed that the cornering stiffness of the front tires converges to 144 KN/rad and 125 KN/rad, and the cornering stiffness of the rear tires converges to 205KN/rad and 166 KN/rad, respectively. The difference between the cornering stiffness estimation results in the two cases shows that our algorithm well captures the impact of the trailers on the tractor. This claim will be further supported by automated control comparison experiments later.

The corresponding lateral state estimation results in the two cases are shown in Fig. 4b-4c and Fig. 5b-5c, respectively. Note that in all experiments, the estimated and measured yaw rate are highly matched, hence only the lateral velocity estimation results are presented for brevity. Since the lateral velocity is close to zero when driving straight, and it starts to increase in the presence of lateral excitation, we initialize $\hat{v}_y$ to zero throughout the experiments. It can be seen that the lateral velocity estimation error of our model-aided estimator decreases significantly with the online update of cornering stiffness. After the cornering stiffness convergence, the root-mean-square errors (RMSE) of the velocity estimates in the two towing configurations are only 0.027 m/s and 0.035 m/s, respectively, which validates the effectiveness of our proposed estimation framework. Parameter adaptation and model-aided state estimation can work well in an online form. This enables the potential for the heavy tractor-trailer system to move towards fully autonomous navigation.

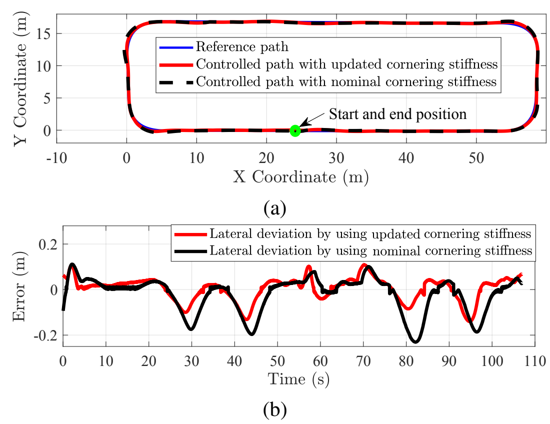

Fig. 6. Automatic control results using different cornering stiffness. (a) Comparison of lateral control paths. (b) Comparison of lateral deviation.

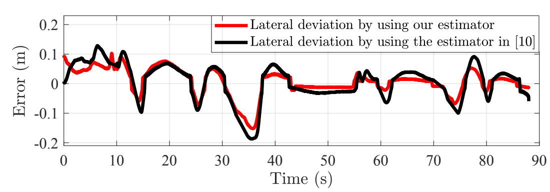

Fig. 7. Automatic control results using different estimator. Only the comparison of lateral deviation is given since lateral control paths are similar to that of Fig. 6.

C. Results of Estimator-Based Lateral Control

In the second experiment, we applied the proposed estimator (11) to an existing lateral controller [2] to explore the effect of the identified cornering stiffness and dynamics-informed lateral velocity estimates on the navigation control performance of a heavy system. The controller in [2] is easy-to-implement for the tractor-trailers system, but its performance is limited by the fact that only kinematic estimators and nominal tire parameters are used.

First, we analyze the impact of cornering stiffness adaptation on automatic control. In order to minimize the interference of other factors, the ground truth system is adopted to provide state feedback. The nominal cornering stiffness in [2] is used to perform lateral control for the tractor-4-trailers case. As a comparison trial, the updated cornering stiffness in Section IV-B is employed when turning, and the nominal cornering stiffness is used for straight driving. The control results are given in Fig. 6. As we can see, for the 4-trailers case, the updated cornering stiffness can support better control performance than using the tractor's nominal value in the controller. Even with the same lateral controller and state feedback, the new cornering stiffness limits the maximum control error from 24 cm to 15 cm.

We then analyze the impact of the proposed model-aided estimator (11) on automatic control of a tractor-4-trailers system. The vision-based kinematic estimator in [2] will also be implemented as a comparison. At this time, the previously updated cornering stiffness is used when turning. The control results are shown in Fig. 7. As can be seen, compared to the cornering stiffness, the difference in control results caused by different estimators is relatively small. But the new model-aided estimator still shows slightly superior control performance, which verifies that the proposed estimator is feasible for dynamic control of heavy systems.

It is worth noting that the current state-of-the-art dynamic model-based estimators, whether EKF-based [9] or nonlinear observer-based [14], require sophisticated tire modeling and known parameters. This makes it difficult to implement these methods on our tractor platform and conduct fair comparison tests. In contrast, our method has better practicality under the premise of ensuring performance. Moreover, the implementation of the kinematic estimator in [2], including most existing vision- or LiDAR-based motion estimation methods, depends on the quality of features in the environment. Instead, the model-aided method is less dependent on the environment and therefore has an advantage in dynamic industrial environments.

V. Conclusion

In this paper, a robust and practical method for lateral dynamics estimation of a tractor-trailers system was proposed. To deal with the effects of varying hitch force on the tractor model, kinematic information was introduced and tire parameters were updated when the tractor has lateral excitation. A dynamic model-based estimator was then designed to achieve more reliable lateral velocity estimation. The proposed estimator enables high-performance model-based tractor control under any towing configuration. The validity of the proposed method is verified theoretically and experimentally. Although the presented work shows good performance in low-speed driving, the robustness of the cornering stiffness adaptation needs to be further studied in the future. The valid range of the tire model we use also needs to be investigated for different road surfaces and combined-slip effects. More rigorous comparative experiments together with the relevant statistics will also be presented.

References

[1] A. González-Cantos and A. Ollero, "Backing-Up Maneuvers of Autonomous Tractor-Trailer Vehicles using the Qualitative Theory of Nonlinear Dynamical Systems," *Int. J. Robot. Res.*, vol. 28, no. 1, pp. 49-65, 2009.

[2] S. Zhou, H. Zhao, W. Chen, M. Miao, Z. Liu, H. Wang, and Y.-H. Liu, "Robust Path Following of the Tractor-Trailers System in GPS-Denied Environments,"*IEEE Robot.Autom.Lett.*, vol.5, no.2, pp.500-507, 2020.

[3] J.-J. Jiang and A. Astolfi, "Lateral Control of an Autonomous Vehicle," *IEEE Trans. Intell. Veh.*, vol. 3, no. 2, pp. 228-237, 2018.

[4] H. Li, J. Zhao, J. Bazin, and Y.-H. Liu, "Robust Estimation of Absolute Camera Pose via Intersection Constraint and Flow Consensus," *IEEE Trans. Image Process.*, vol. 29, pp. 6615-6629, 2020.

[5] E. Hashemi, M. Pirani, A. Khajepour, B. Fidan, S. Chen, and B. Litkouhi, "Fault Tolerant Consensus for Vehicle State Estimation: A Cyber-Physical Approach," *IEEE Trans. Ind. Inform.*, vol. 15, no. 9, pp. 5129-5138, 2019.

[6] D. Selmanaj, M. Corno, G. Panzani, and S. Savaresi, "Vehicle Sideslip Estimation: A Kinematic Based Approach," *Control Eng. Practice*, vol. 67, pp. 1-12, 2017.

[7] J. Yoon and H. Peng, "Robust Vehicle Sideslip Angle Estimation Through a Disturbance Rejection Filter That Integrates a Magnetometer With GPS," *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 1, pp. 191-204, 2013.

[8] S. Zhou, Z. Liu, S. Suo, H. Wang, H. Zhao, and Y.-H. Liu, "Vision-based Dynamic Control of Car-Like Mobile Robots," in *Proc. IEEE Int. Conf. Robot. Autom.*, Montreal, Canada, pp. 6631-6636, 2019.

[9] A. Katriniok and D. Abel, "Adaptive EKF-Based Vehicle State Estimation With Online Assessment of Local Observability," *IEEE Trans. Control Syst. Technol.*, vol. 24, no. 4, pp. 1368-1381, 2016.

[10] A. Rezaeian, A. Khajepour, W. Melek, S.-Ken Chen, and N. Moshchuk, "Simultaneous Vehicle Real-Time Longitudinal and Lateral Velocity Estimation," *IEEE Trans. Veh. Techno.*, vol. 66, no. 3, pp. 1950-1962, 2017.

[11] L. Imsland, T. A. Johansen, T. I. Fossen, H. F. Grip, J. C. Kalkkuhl, and A. Suissa, "Vehicle Velocity Estimation Using Nonlinear Observers," *Automatica*, vol. 42, no. 12, pp. 2091-2103, 2006.

[12] D. Piyabongkarn, R. Rajamani, J. A. Grogg, and J. Y. Lew, "Development and Experimental Evaluation of a Slip Angle Estimator for Vehicle Stability Control," *IEEE Trans. Control Syst. Technol.*, vol. 17, no. 1, pp. 78-88, 2009.

[13] G. Baffet, A. Charara, and G. Dherbomez, "An Observer of Tire-Road Forces and Friction for Active Security Vehicle Systems," in *IEEE/ASME Trans. Mechatronics*, vol. 12, no. 6, pp. 651-661, 2007.

[14] L. Zhao, Z. Liu, and H. Chen, "Design of a Nonlinear Observer for Vehicle Velocity Estimation and Experiments," *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 3, pp. 664-672, 2011.

[15] E. Hashemi, M. Pirani, A. Khajepour, A. Kasaiezadeh, S. Chen, and B. Litkouhi, "Corner-Based Estimation of Tire Forces and Vehicle Velocities Robust to Road Conditions," *Control Eng. Practice*, vol. 61, pp. 28-40, 2017.

[16] S. You, J. Hahn, and H. Lee, "New Adaptive Approaches to Real-Time Estimation of Vehicle Sideslip Angle," *Control Eng. Practice*, vol. 17, no. 12, pp. 1367-1379, 2009.

[17] R. Rajamani, *Vehicle Dynamics and Control*, New York, NY, USA: Springer, 2012.

[18] D. Abeywardena, Z. Wang, S. Kodagoda, and G. Dissanayake, "Visual-Inertial Fusion for Quadrotor Micro Air Vehicles with Improved Scale Observability," in *Proc. IEEE Int. Conf. Robot. Autom.*, Karlsruhe, Germany, pp. 3148-3153, 2013.

[19] J. Guo, P. Hu, and R. Wang, "Nonlinear Coordinated Steering and Braking Control of Vision-Based Autonomous Vehicles in Emergency Obstacle Avoidance," *IEEE Trans. Intell. Transp. Syst.*, vol. 17, no. 11, pp. 3230-3240, 2016.

[20] D. Tedaldi, A. Pretto, and E. Menegatti, "A Robust and Easy to Implement Method for IMU Calibration," in *Proc. IEEE Int. Conf. Robot. Autom.*, Hong Kong, pp. 3042-3049, 2014.

[21] H. K. Khalil, *Nonlinear Systems*, 3rd ed., Upper Saddle River, NJ: Prentice-Hall, 2002.

[22] H. Zhao, Z. Liu, Z. Li, S. Zhou, W. Chen, C. Suo, and Y.-H. Liu, "Modelling and Dynamic Tracking Control of Industrial Vehicles with Tractor-trailer Structure," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, Macau, China, pp. 2905-2910, 2019.