

Collision Reaction Through Internal Stress Loading in Cooperative Manipulation

Victor Aladele¹ and Seth Hutchinson²

Abstract—Cooperative manipulation offers many advantages over single-arm manipulation. However, this comes at a cost of added complexity, both in modeling and control of multi-arm systems. Much research has been focused on determining optimal load distribution strategies based on several objective functions, some of which include manipulability, energy consumption and joint torque minimization. This paper presents an internal loading strategy that is subject to the estimate of the external disturbances along the body of one or more of the arms involved in the manipulation process. The authors of this paper propose a reaction strategy to external disturbances by transforming the disturbance forces into internal forces on the object through appropriate load distribution on the cooperative arms. The goal is to have a set-point on the object, track a given trajectory while compensating for external disturbances along the links of some of the robot arms involved in the cooperative manipulation.

I. INTRODUCTION

Cooperative manipulation has gained significant attention in the last couple decades. With increasing interest in making robots perform more complex tasks, it has become obvious that certain tasks cannot be performed by single-arm robots, either due to payload limitations or the geometry of the object to be manipulated. Therefore, there has been an increased research effort in developing the theoretical background for multi-arm systems [1], [2], [3]. Several areas have benefited and could continue to benefit from the applications of cooperative manipulation, including: Space missions, medical surgery, elderly care and to a great extent, industrial manufacturing [4], [5].

In [6], manipulability was set as the optimization criterion for load distribution while [7] used the total energy consumption of the multi-arm system as an optimization measure. Load sharing between a human and a robot is also one of the recently explored research areas. [8] used the geometric and dynamic properties of the task to design the load sharing strategy of the cooperative task. Some other research work focused on a decentralized control of multi-arm systems, where there is no communication between robots involved in the manipulation task. In [9], control is based on a decentralized estimation of the load parameters and twist. These parameters are estimated based only on local measurements of the kinematics at the contact points. Other approaches to cooperative manipulation involve a *leader-follower* strategy

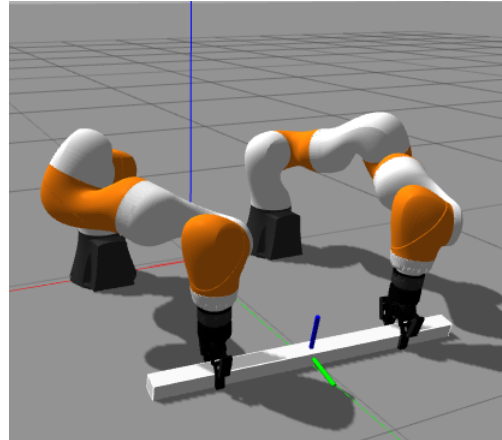


Fig. 1. Two fixed-base robot arms picking up an object

[10], where one of the manipulators is selected to control the main objective, and the other robot is controlled to maintain a relative pose with respect to the leader. This approach has some drawbacks. Should there be a drop in communication between both robots, the manipulation task will fail. In recent years, work has also been done with learning of cooperative motion tasks involving human-robot collaboration [11]. A gesture recognition system is used to predict the motion of the human, and generate motion plans for the robot.

Often in robotic manipulation, collision is inevitable, especially in unstructured environments. Hence, [12] proposed a method for safe collision reaction to external forces along the links of a seven degree-of-freedom (7DOF) robot arm. The method proposed in [12] projects the reaction torques in the nullspace of the primary task, which in this case, involves tracking of an end effector trajectory. This method exploits the redundancy in the task definition.

Internal stress is often avoided or controlled while performing cooperative manipulation. In our paper, we take the reverse approach of using the internal strength of a manipulated object to an advantage. Here, we present a novel method of reacting to collisions along the links of a robot arm by transmitting the disturbance torques through the manipulated object and thereby sharing the effect of this disturbance with the other robots in the cooperative manipulation system. One advantage of this collision reaction strategy over the proposed method in [12] is that in [12], beyond certain thresholds, the task can no longer be preserved. Hence, part of the primary task, such as orientation control of the end effector is sacrificed to provide more redundancy to compensate for the disturbance. However, instead of compromising the task, our method proposes *transferring* the external disturbance to

¹Victor Aladele is with the Department of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30318, USA valad@gatech.edu

²Seth Hutchinson is with the Department of Interactive Computing, Georgia Institute of Technology, Atlanta, GA 30318, USA seth@gatech.edu

the rest of the cooperative manipulation system in such a way that the primary task is not affected. Moreover, our proposed strategy is expected to work with manipulators with less than seven degrees-of-freedom by virtue of the extra degrees-of-freedom provided by multiple manipulators.

Similar work has been done by [13], where external forces on the object are compensated by modifying the contact forces. These contact forces lie in the nullspace of the grasp matrix. However, their approach was not designed to handle external disturbances on the robot itself. Whereas, our approach is designed to compensate for collisions along the links of the robot.

In this paper, we focus on dual-arm systems, with the theories developed here being extensible to multi-arm systems. We evaluate our proposed load distribution strategy by simulating two fixed-base 7DOF kuka robot arms in Gazebo. The two arms are engaged in cooperative manipulation with the manipulated object rigidly grasped by both end effectors of the dual-arms.

The rest of the paper will be organized as follows: in section II, we review the technical background of cooperative manipulation and briefly discuss the estimation of external torques that has been proposed in [14], [15]. In section III, we present our formulation for collision reaction through *internal stress loading*. Section IV gives an overview of our setup in simulation while section V shows the result of our experiment in simulation. In section VI, we review some of the key ideas discussed in the previous sections. Finally, we conclude in section VII and discuss possible future research directions.

II. PRELIMINARIES

A. Manipulator Dynamic Model

The mathematical model of a single arm with N joints is given by

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau \quad (1)$$

where $M \in \mathbb{R}^{N \times N}$ is the symmetric and positive-definite inertia matrix, $C \in \mathbb{R}^{N \times N}$ is the coriolis/centripetal matrix, $g \in \mathbb{R}^N$ is the gravity vector, $q \in \mathbb{R}^N$ represents the joint positions and $\tau \in \mathbb{R}^N$ is the applied joint torques [16]. In the presence of external disturbances, (1) is modified to incorporate the external torques

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau + \tau^{ext} := \tau_m \quad (2)$$

B. External Torque estimation

In this section, we introduce the sensorless collision torque estimation that has been derived in [14], [15]. This method uses a momentum observer to estimate the external torques due to collision along the body of a 7DOF arm. Using observers, [17] designed a pipeline, from the detection of the collision, through the isolation of the collision region, to the estimation of the external torques τ^{ext} due to the collision. The output of the observer $r(t)$, known as the residual vector, gives the estimate of the external torques.

$$r(t) = K_I \left[p(t) - \int_0^t (\tau + C^T(q, \dot{q}) - g(q) + r) ds \right] \quad (3)$$

where $r \in \mathbb{R}^N$, with $r(0) = 0$ and $p(t)$ is the momentum of the system. $K_I > 0$ is an appropriately chosen observer gain matrix which is diagonal. From (3), the dynamics of r can be defined as

$$\dot{r} = -K_I(r + \tau^{ext}) \quad (4)$$

The details of the derivation of (3) and (4) can be found in [12], [17]. This method for estimating the external joint torques is quite attractive as it only requires proprioceptive sensors. Moreover, this approach circumvents the need to derive the jacobian at the collision point, which in certain cases may not be a simple point contact.

C. Cooperative Manipulation

The forces acting on a grasped object by two manipulators are combinations of the motion inducing wrenches, internal wrenches and possibly, external disturbances. As expected, the motion inducing forces on the manipulated object cause the movement of the object along a given trajectory. A point of interest on the object, which is often the object's center of Mass (COM), is selected as the point to track the desired trajectory.

In a cooperative manipulative task involving K manipulators grasping a rigid object, with the assumption of a rigid grasp by the end effectors, the relationship between the end effector wrenches F^d and the desired wrench F_o^d experienced at the COM of the object is given by:

$$F_o^d = G F^d \quad (5)$$

where $F_o^d = (f_o^{dT}, m_o^{dT})^T \in \mathbb{R}^6$ is the target wrench [18] that needs to be applied to the object's COM to track the desired trajectory $x_o^d, \dot{x}_o^d, \ddot{x}_o^d$. $F^d = (F_1^{dT}, \dots, F_i^{dT}, \dots, F_K^{dT})^T \in \mathbb{R}^{6K}$ is the vector of end effector wrenches, with $F_i^d = (f_i^{dT}, m_i^{dT})^T$ and $f_i^d, m_i^d \in \mathbb{R}^3$ for $i \in 1, \dots, K$. f and m represent the end effector force and end effector moment, respectively. $G \in \mathbb{R}^{6 \times 6K}$ is the grasp matrix defined by the contact points $l_i \in \mathbb{R}^3$ of each end effector and the manipulated object [18]. The grasp matrix G can be computed as:

$$G = \begin{bmatrix} I_3 & 0_3 & \dots & I_3 & 0_3 & \dots & I_3 & 0_3 \\ S(l_1) & I_3 & \dots & S(l_i) & I_3 & \dots & S(l_K) & I_3 \end{bmatrix} \quad (6)$$

where $S(*)$ is a skew-symmetric matrix [18]. It is important to note that this grasp matrix is applied in the object frame. Given the desired wrench of the object F_o^d , to calculate the required wrench at the grasp locations, (5) is inverted to obtain the generalized inverse G^+ of the grasp matrix G . This leads to the inverted equation

$$F^d = G^+ F_o^d \quad (7)$$

The key question of cooperative manipulation is solving (7) such that (5) is satisfied. Since G is not a square matrix, the solution to (7) is not unique. Hence a redundancy resolution scheme is employed through optimization of one or more criteria. In the following section, we introduce a resolution that uses a collision reaction strategy to determine the load distribution. Although it is not exactly a resolution, since there is not a unique solution that generates internal stress

on the object. However, our compensation strategy chooses from a subset of the solution set of (7), with the condition that the subset only contains solutions that induce no motion on the object. Hence, the sum of the collision forces and the compensation term, defined in the next section, equals zero. Therefore, any motion that would have been induced by the collision is cancelled.

III. COLLISION REACTION VIA INTERNAL STRESS LOADING

In this section, we discuss the proposed internal stress loading strategy to compensate for the collision torques received along the links of the robot.

A. Multi-arm Cooperative Manipulation Model

We begin by discussing the dynamics of the object being manipulated. When describing the dynamics of a rigid object, we select a point of interest on the object. The point of interest is typically chosen as the center of mass of the object. Therefore, the operational space dynamics is given by [19]

$$M_o(x_o)\ddot{x}_o + C_o(x_o, \dot{x}_o)\dot{x}_o + g_o = F_o \quad (8)$$

where M_o is the inertia mass matrix of the object, C_o is the coriolis matrix and g_o the gravity vector.

For K manipulators in contact with a grasped object, the following equation describes the relationship between the joint torques and the forces at the end effector of the i -th manipulator [19].

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i + J_i^T F_i \quad (9)$$

where $J_i \in \mathbb{R}^{6 \times N}$ is the manipulator jacobian, $F_i \in \mathbb{R}^6$ is the end effector wrench and $\tau_i \in \mathbb{R}^N$ are the joint torques, of the i -th manipulator. With collision on the links of the i -th manipulator, (9) can be modified to yield

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i + J_i^T F_i + \tau_i^{ext} \quad (10)$$

where $\tau_i^{ext} = J_i^T F_i^{ext}$ is the estimated torque due to the collision on the links of the i -th manipulator. This torque can be estimated using the method discussed in section II-B. F_i^{ext} is the resulting wrench at the i -th manipulator end effector due to the collision induced joint torques τ_i^{ext} . Expanding equation (10) further, we obtain

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i + J_i^T F_i + J_i^T F_i^{ext}. \quad (11)$$

B. Internal Stress Loading

The mathematical derivation here focuses on the case where the cooperative system is comprised of two manipulators (i.e. $K = 2$).

Equation (7) does not have a unique solution. Hence we take advantage of the infinitely many solutions by selecting the appropriate end effector wrenches that compensate for the external torque on either arm without affecting the performed task at the object's COM. The reaction forces, if not causing motion, will generate internal stress on the object. Thus, for this formulation to work, the following assumptions need to hold:

- 1) Internal force loading of the object is permissible (i.e. the load can withstand squeezing/bending forces).
- 2) The manipulated object is rigidly grasped by both end effectors.
- 3) Both arms share information about the estimated collision torques on either arm.

For the purpose of brevity with notation, we define the arm involved in collision as *arm-1*, while the arm not involved in any collision, as *arm-2*. Our proposed method selects the appropriate end effector wrench for arm-2 to counter the effect of the external disturbance caused by the collision on arm-1.

When a collision occurs on arm-1, a wrench $(f_1^{ext}, m_1^{ext})^T$ expressed in the object frame, results at the end effector of arm-1. We can compensate for the torque component of this disturbance by simply setting

$$m_2^c = -m_1^{ext} \quad (12)$$

where, $m_2^c \in \mathbb{R}^3$ is the compensation torque that needs to be applied by arm-2. However, to compensate for the force f_1^{ext} , special care must be taken. The compensating force from arm-2 is given by

$$f_2^c = -f_1^{ext} \quad (13)$$

where, $f_2^c \in \mathbb{R}^3$ is the compensation force to be generated by arm-2. However, we still need to compensate for an additional effect of f_1^{ext} on the manipulation system, which is an induced torque about the COM [18]. Hence, we cannot compensate for the force component of the disturbance by simply negating the force f_1^{ext} alone. The induced torque due to f_1^{ext} can be calculated as

$$m_1^{ind} = S(l_1)f_1^{ext} \quad (14)$$

where $l_1 \in \mathbb{R}^3$ is the distance between the arm-1 and the object's COM [18]. We should also not forget to take into account the torque induced due to the compensation force f_2^c in (13). This induced torque can be calculated similarly to (14)

$$m_2^{ind} = S(l_2)f_2^c := -S(l_2)f_1^{ext} \quad (15)$$

Therefore, a total torque compensation strategy is given by

$$m_2^c = -(m_1^{ext} + m_1^{ind} + m_2^{ind}) \quad (16)$$

This procedure works exactly the same way, if the collision is on arm-2 instead of arm-1, as defined here. Similarly, in a situation where both arms experience collision, the compensation would be performed by both arms. This could result in more or less stress on the object, depending on the direction of the resulting end effector wrenches due to the collisions.

C. Control Derivation

In this section we discuss the control structure used to drive the cooperative manipulation system. Adopting the idea of the *Computed-Torque* method [20], we can derive the control law for the manipulated object, as well as, the

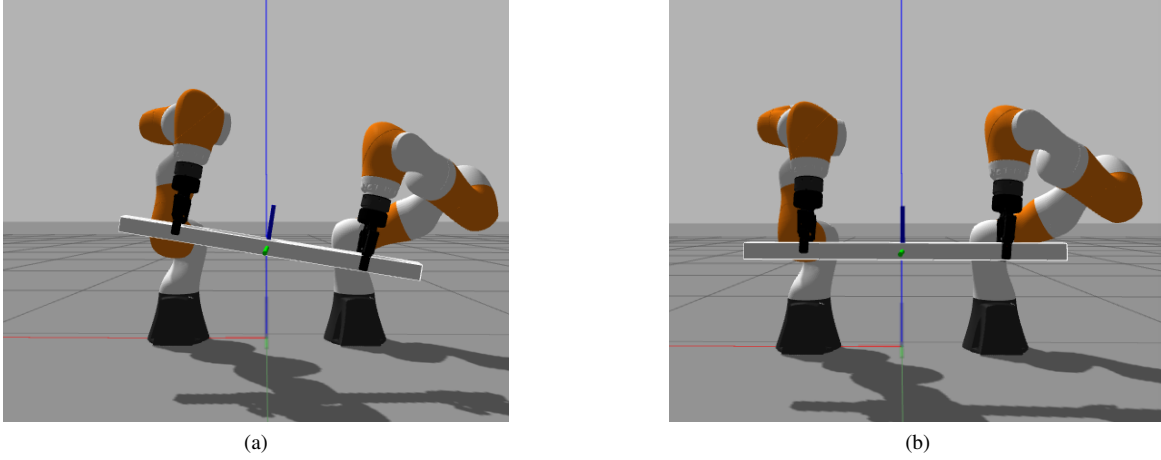


Fig. 2. (a), External force applied on the left arm, in the +z direction without internal loading compensation. (b), External force applied on the left arm, in the +z direction with internal loading compensation

robots performing the manipulation. We begin by modifying equation (7) to incorporate the compensation term \mathcal{N} .

$$F^d = G^+ F_o^d + \alpha \mathcal{N} \quad (17)$$

where $\mathcal{N} \in \mathfrak{R}^{12}$ is a vector of compensation wrenches for both end effectors.

$$\mathcal{N} = \begin{bmatrix} f_1^c \\ m_1^c \\ f_2^c \\ m_2^c \end{bmatrix} \quad (18)$$

$\alpha \in [0, 1]$ is the tuning factor that defines how much compensation is performed. For example, if $\alpha = 0$, no compensation is performed. Whereas, making $\alpha = 1$ would yield a full compensation. There are many reasons why this tuning factor might be useful. A joint limitation on the compensating arm, in this case arm-2, would mean that the required compensation torques cannot be generated by arm-2. This means that although arm-2 does not have the capacity to generate the joint torques needed to fully compensate for the disturbance, it can still perform partial compensation. Hence, we can use the tuning factor to control how much compensation joint torques is demanded from arm-2.

The tuning factor also provides a smooth way to transition into the framework of load distribution, as α can be replaced with a diagonal matrix that selects how much compensation should be performed by each manipulator. Lastly, the assumption made in this section alluding that internal load stressing of the object is permissible, is not always true. Therefore, the tuning factor can act as a switch to turn off ($\alpha = 0$) and on ($\alpha \in (0, 1]$) the compensation scheme. In most cases, the object might be able to withstand some internal stress, but not the total internal stress required to fully compensate for the external disturbance. For example, a glass tray that is transported by two manipulators might break when internal stress is applied, but a plastic tray would withstand the same level of internal stress without being significantly deformed. Therefore, the tuning factor can be used as a constraint variable to regulate the amount

of internal stress acting on the object. Hence, if we knew the stress limit on the object, we could reduce the impact of the external disturbance on the desired trajectory, while not exceeding the stress limit of the object.

$$F_o^d = M_o \ddot{x}_o^r + C_o + g_o \quad (19)$$

where,

$$\ddot{x}_o^r = \ddot{x}_o^d - K_v(\dot{x}_o - \dot{x}_o^d) - K_p(x_o - x_o^d) \quad (20)$$

with the assumption that the desired trajectory $x_o^d, \dot{x}_o^d, \ddot{x}_o^d$ is given. Substituting (19) into (17), we obtain the control law

$$F^d = G^+ (M_o \ddot{x}_o^r + C_o + g_o) + \alpha \mathcal{N} \quad (21)$$

$F^d \in \mathfrak{R}^{12}$, where $F^d = (F_1^{dT}, F_2^{dT})^T$. F_1^d is passed on to the low-level controller for arm-1 and F_2^d is passed on to the low-level controller for arm-2. Each low-level controller then uses the force-based control scheme [16], [21] in the joint space to calculate the joint torques required to obtain the corresponding end effector wrench. The desired trajectories of the robot require low frequency accelerations. Therefore, the resulting control law is shown in equation (22)

$$\tau_i = J^T F_i^d + C(q, \dot{q}) + g(q) \quad (22)$$

where $\tau_i \in \mathfrak{R}^N$ represents the joint torques for the i -th manipulator and $i = 1, 2$.

The manipulators described in this work have seven degrees-of-freedom, each. By definition of the task, these manipulators are redundant. Therefore, it is important to note that equation (22) is incomplete for redundant manipulators [22]. According to [22], there is a nullspace associated with the inverse of the Jacobian transpose and joint torques projected into this nullspace do not affect the forces at the end effector. Thus, we have focused on the operational space where the nullspace projection is irrelevant. However, in future work, we will leverage the property of this nullspace projection to optimize for certain performance objectives, such as, joint torque minimization.

IV. SIMULATION

To evaluate the performance of our compensation approach, we simulated the cooperative system in Gazebo [23]. Two Kuka 7DOF manipulators and a rectangular rod with dimensions $0.8m \times 0.04m \times 0.04m$ were simulated as shown in Figure 3. The robots are controlled at the joint level by a low-level controller. A high-level controller computes the desired wrench required to track a given COM trajectory via a computed-torque control method. The high-level controller also uses the pseudoinverse of the grasp matrix to compute the appropriate end effector wrenches to attain the desired COM wrench. External disturbance compensation is also handled by the high-level controller, as the low-level controller informs the high-level controller of the measured external joint torques/end effector wrench. Some manipulators already provide external torque measurements (e.g. kuka iiwa7), and in many cases, force-torque sensors are installed on the wrist of the arms to measure end effector forces and torques. Hence, in our experiment, we assume the external forces on the end effectors have been provided.

We also obtain the position of the object’s COM from Gazebo’s physics engine. However, we acknowledge that implementing this outside of simulation will require a different source for the COM value. In such scenario, especially where the shape of the object is not trivial, we propose using a vision-based system to estimate the COM of the object.

Our experiment was designed as a set-point trajectory tracking problem. Both end effectors, fitted with a robotiq-85 gripper, pick up the rod, whose COM is initially located at $[0, 0.7, 0]$ and orientation relative to the world is $[0, 0, 0]$, as shown in Figure 3. The end effectors grasp the object at similar positions, but at different ends of the COM along the x-axis: -0.25 and 0.25 , as shown in Figure 3. A goal point for the COM is set to $x_o^d = [0, 0.4, 0.3, 0, 0, 0]$ and the final position of the object is shown to be accurately placed at this location, as shown in Figure 2b.

V. RESULTS

A force of $25N$ was applied on the left arm in the $+z$ direction i.e. $F_1^{ext} = [0, 0, 25, 0, 0, 0]$. In Figure 2a, a significant deviation in the pose of the rod is observed. There is an error in the y-orientation of the COM when no compensation is performed. However, when we introduce our compensation scheme, the system is able to maintain the object’s COM at the desired set-point, as seen in Figure 2b. The plots shown in Figure 3 demonstrate the performance of the compensation strategy we have proposed in this paper.

The plots show that with our compensation scheme, the pose error is significantly reduced as opposed to the case where there is no compensation. More evident is the difference in the error in the z-position and y-orientation. It is clear that the external force of $25N$ in the z - direction induced a torque about the y-axis, causing the object to tilt about this axis. However, our compensation scheme was able to counter the effect of such torque, ensuring that the main task of maintaining the COM at the desired pose, x_o^d , is achieved.

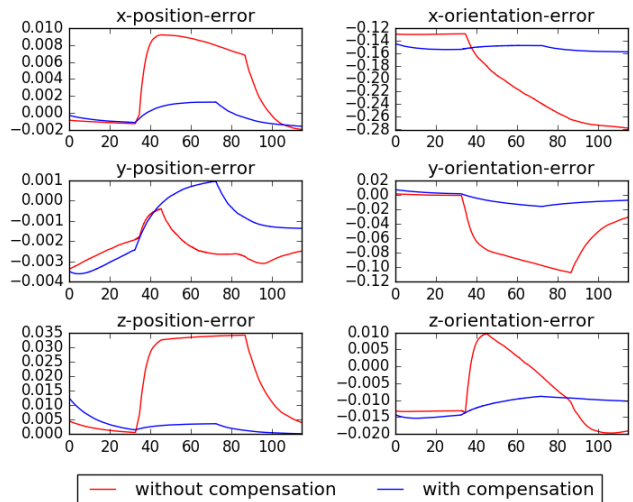


Fig. 3. Plot of objects COM pose error (in meters/radians) against time (in seconds) when a force of $25N$ is applied in the $+z$ direction on arm-1, at about the 35 second mark and removed at about the 90 second mark.

VI. DISCUSSION

In this section, we review some of the points that we have discussed and how they can be viewed from multiple perspectives. Our framework can be adopted to $K > 2$ manipulators, and fits nicely into other frameworks on load distribution. For instance, instead of just one manipulator compensating for all the external torques on the robot, multiple manipulators could share the burden of compensation. Future work will explore methods to determine the value of the tuning factor α , including a constrained optimization approach. The tuning factor could also be derived via an impedance model to control the internal forces that are applied on the object [24].

The authors understand that in the real world, model uncertainty would affect the performance of this cooperative scheme. Moreover, the presence of internal forces, both on the object and on the robots, impels the need to address this model uncertainty. Without directly estimating the model uncertainty, an impedance control approach could be used to regulate the forces of interaction, thereby minimizing the effects of modeling errors [25]. In addition, the manipulators could be retrofitted with force-torque sensors. This will provide force-feedback to compensate for errors in the end effector force/wrench estimates [26].

Another important point to consider is that the nullspace of the grasp matrix can be used to generate end effector wrenches that cause internal stress on the load [1], [27]. However, for purpose of our approach, we did not use the nullspace projection to generate internal loading.

In simulation we were able to use a plugin known as *Grasp-Fix* plugin. This plugin allows us to simulate a rigid grasp between the grippers and the object. However, again we acknowledge that outside simulation environments, ensuring rigid grasps become significantly difficult, and extra steps need to be taken to ensure that the formulation presented in

this paper still holds.

VII. CONCLUSIONS

In this paper, we have discussed how using an internal loading strategy could mitigate the impact of external disturbances along the links of one of the arms involved in the manipulation of a grasped object. Possible future directions will introduce more arms to the system. We hope to further expand this compensation to a more robust framework embedded in load distribution allocation for multi-arm systems. There is also the possibility of adding a mobile base to each of the arms, thereby increasing the workspace and redundancy of the system. Lastly we consider exploring the idea of *human-in-the-loop*, where one of the arms is replaced with a human to perform cooperative manipulation tasks.

REFERENCES

- [1] I. D. Walker, R. A. Freeman, and S. I. Marcus, "Analysis of motion and internal loading of objects grasped by multiple cooperating manipulators," *The International journal of robotics research*, vol. 10, no. 4, pp. 396–409, 1991.
- [2] S. Erhart and S. Hirche, "Internal force analysis and load distribution for cooperative multi-robot manipulation," *IEEE Transactions on Robotics*, vol. 31, no. 5, pp. 1238–1243, 2015.
- [3] N. Michael, J. Fink, and V. Kumar, "Cooperative manipulation and transportation with aerial robots," *Autonomous Robots*, vol. 30, no. 1, pp. 73–86, 2011.
- [4] N. Inaba and M. Oda, "Autonomous satellite capture by a space robot: world first on-orbit experiment on a japanese robot satellite ets-vii," in *Proceedings 2000 ICRA. Millennium Conference. IEEE International Conference on Robotics and Automation. Symposia Proceedings*, vol. 2. IEEE, 2000, pp. 1169–1174.
- [5] H. Lee, H. Kim, and H. J. Kim, "Planning and control for collision-free cooperative aerial transportation," *IEEE Transactions on Automation Science and Engineering*, vol. 15, no. 1, pp. 189–201, 2016.
- [6] H. Shen, Y.-J. Pan, and G. Bauer, "Manipulability-based load allocation and kinematic decoupling in cooperative manipulations," in *2019 IEEE 28th International Symposium on Industrial Electronics (ISIE)*. IEEE, 2019, pp. 1168–1173.
- [7] Y. F. Zheng and J. Luh, "Optimal load distribution for two industrial robots handling a single object," in *Proceedings. 1988 IEEE International Conference on Robotics and Automation*. IEEE, 1988, pp. 344–349.
- [8] M. Lawitzky, A. Mörtl, and S. Hirche, "Load sharing in human-robot cooperative manipulation," in *19th International Symposium in Robot and Human Interactive Communication*. IEEE, 2010, pp. 185–191.
- [9] A. Petitti, A. Franchi, D. Di Paola, and A. Rizzo, "Decentralized motion control for cooperative manipulation with a team of networked mobile manipulators," in *2016 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2016, pp. 441–446.
- [10] A. Tsiamis, C. K. Verginis, C. P. Bechlioulis, and K. J. Kyriakopoulos, "Cooperative manipulation exploiting only implicit communication," in *2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2015, pp. 864–869.
- [11] J. Mainprice and D. Berenson, "Human-robot collaborative manipulation planning using early prediction of human motion," in *2013 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2013, pp. 299–306.
- [12] A. De Luca and L. Ferrajoli, "Exploiting robot redundancy in collision detection and reaction," in *2008 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2008, pp. 3299–3305.
- [13] H.-C. Lin, J. Smith, K. K. Babarhamati, N. Dehio, and M. Mistry, "A projected inverse dynamics approach for multi-arm cartesian impedance control," in *2018 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2018, pp. 1–5.
- [14] A. De Luca and R. Mattone, "Sensorless robot collision detection and hybrid force/motion control," in *Proceedings of the 2005 IEEE international conference on robotics and automation*. IEEE, 2005, pp. 999–1004.
- [15] A. De Luca, A. Albu-Schaffer, S. Haddadin, and G. Hirzinger, "Collision detection and safe reaction with the dlr-iii lightweight manipulator arm," in *2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2006, pp. 1623–1630.
- [16] J. Nakanishi, R. Cory, M. Mistry, J. Peters, and S. Schaal, "Operational space control: A theoretical and empirical comparison," *The International Journal of Robotics Research*, vol. 27, no. 6, pp. 737–757, 2008.
- [17] S. Haddadin, A. De Luca, and A. Albu-Schäffer, "Robot collisions: A survey on detection, isolation, and identification," *IEEE Transactions on Robotics*, vol. 33, no. 6, pp. 1292–1312, 2017.
- [18] A. Z. Bais, S. Erhart, L. Zaccarian, and S. Hirche, "Dynamic load distribution in cooperative manipulation tasks," in *2015 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*. IEEE, 2015, pp. 2380–2385.
- [19] P. Hsu, "Coordinated control of multiple manipulator systems," *IEEE Transactions on Robotics and Automation*, vol. 9, no. 4, pp. 400–410, 1993.
- [20] R. Murray, Z. Li, and S. Sastry, *A mathematical introduction to robotic manipulation*. Boca Raton, FL: CRC press, 1994.
- [21] K. M. Lynch and F. C. Park, *Modern Robotics*. Cambridge University Press, 2017.
- [22] O. Khatib, "Inertial properties in robotic manipulation: An object-level framework," *The international journal of robotics research*, vol. 14, no. 1, pp. 19–36, 1995.
- [23] N. Koenig and A. Howard, "Design and use paradigms for gazebo, an open-source multi-robot simulator," in *2004 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, vol. 3. IEEE, 2004, pp. 2149–2154.
- [24] D. Heck, D. Kostić, A. Denasi, and H. Nijmeijer, "Internal and external force-based impedance control for cooperative manipulation," in *2013 European Control Conference (ECC)*. IEEE, 2013, pp. 2299–2304.
- [25] S. Erhart, D. Sieber, and S. Hirche, "An impedance-based control architecture for multi-robot cooperative dual-arm mobile manipulation," in *2013 IEEE/RSJ International Conference on Intelligent Robots and Systems*. IEEE, 2013, pp. 315–322.
- [26] O. Khatib, K. Yokoi, K. Chang, D. Ruspini, R. Holmberg, A. Casal, and A. Baader, "Force strategies for cooperative tasks in multiple mobile manipulation systems," in *Robotics Research*. Springer, 1996, pp. 333–342.
- [27] R. G. Bonitz and T. C. Hsia, "Force decomposition in cooperating manipulators using the theory of metric spaces and generalized inverses," in *Proceedings of the 1994 IEEE International Conference on Robotics and Automation*. IEEE, 1994, pp. 1521–1527.