Ospheel: Design of an Omnidirectional Spherical-sectioned Wheel

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Abstract—The holonomic and omnidirectional capabilities imparted to the mobile base platform depends mainly on two factors, i.e., the wheel design and its various arrangements in the platform chassis. This paper reports on the development of a novel omnidirectional spherical sectioned wheel named Ospheel. It is modular, and the spherical sectioned geometry of the wheel is driven using two actuators placed inside the housing above the wheel that rotates it independently about two perpendicular axes. The mechanical drive system for Ospheel consists of two gear trains, namely, internal spur gear and crown gear spatially assembled in orthogonal planes and are driven by two driving pinions. The kinematics of a single Ospheel is described, followed by the kinematic equation of a robot equipped with two Ospheels. Forward and inverse kinematic equations are derived explicitly. Experiments were carried out with the two Ospheels at a fixed inclination assembled with the base to illustrate the holonomic motion. The robustness of the wheel design is experimented with different trajectories and on different terrains.

I. INTRODUCTION

The invention of the wheel finds its application traced back as the potter's wheel around 3000 BC to its modernday usage as an integrated driving system wherein driving source and the wheel are coupled with gear trains to move a vehicle, robots, etc. There is a continuous effort in the robotics community to achieve omnidirectional and holonomic locomotion in wheeled platforms by improvisation in the wheel design. The mobility of rigid vehicles on flat ground is characterized by three distinct movements in the form of two translations, i.e., longitudinal and sideways, along with self-rotation of the vehicle. The omnidirectional system requires prior steering action to reorient the wheel in the moving direction, such as independent driven and steering vehicles. It is due to the use of a standard wheel, which is disk-shaped, constraints the side translation along the length of the disk contact with the ground. Whereas, a holonomic vehicle can simultaneously orient itself while translating in an arbitrary direction at a given time. It has to be noted that each novel wheel design has its advantages and limitations.

Conventional wheels integrated with mobile platforms prohibits it to perform side translation. Since the geometry of the wheel, which is disk-shaped, satisfies the rolling

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constraints, and the components orthogonal to it are equal to zero [1]. Modification in conventional wheels by providing the free steering action about the vertical axis (with and without offset) from the point of contact of the wheel with the ground are known as Caster wheels. Later the powered caster wheels were designed to give omnidirectional behavior in mobile vehicles [2]–[4]. To allow sideways rolling, the conventional wheels were modified by providing a hub with the passive rollers arranged across the periphery are known as Omniwheels or Swedish wheels [5] and with passive rollers at an angle in Mecanum wheels [6]. The kinematic analysis of these wheels is discussed in [7]. Recently conventional wheels were modified with active rollers and assembled in the car like vehicle [8].

The objective for the new wheel designs (Fig. 1) was to eliminate the sliding constraints and hence achieve the sideways locomotion. The spherical shape has the symmetry to roll in arbitrary directions on a 2D plane. The spherical wheels developed can be classified into two classes based on the placement of actuators. The internally actuated full spherical shell type of wheels rolls because of the barycenter offset principal as proposed in [9], [10]. The complete spherical geometry actuated from the external actuators in contact with the surface of the sphere was reported in [11]-[13]. A novel design on this principle was presented in [14] where a pair of sliced orthogonal wheels were placed perpendicularly on the axle. Similar design modification was proposed in "Omniball," which is formed by two passive hemispherical sections rotation and one active rotational axis [15]. Steering action achieved using two wheels driven with spatial mechanism, making it a pivoted differential drive, is reported and implemented in [16]-[20]. Some of the cited wheel designs in this paper are depicted in Fig. 1. Some of the features associated with these wheels are:

- Presence of passive rolling sections in the wheel [5], [6], [14], [21] results in slippage and odometric errors.
- The assembly of multiple small parts for passive rollers on the periphery results in a complicated design.
- Limited torque transmission due to the presence the passive components [5], [14], [21] and also with the spherical shelled robots [9]–[11].
- Ability to overcome irregularities in the form of step is minimal during sideways motion [2], [5], [6].
- Inadequacy to be used in outdoor environment say, with partially filled water surface [5], [6], [21].

In this paper, we present a novel design of the omnidirectional spherical-sectioned wheel named *Ospheel*, which combines the advantages of the normal and spherical wheels.

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Fig. 1: Different wheel designs (a) potters wheel (b) conventional wheel (c) Caster wheel without offset (d) Caster wheel with offset (e) Omniwheel [5] (f) Mecanum wheel [6] (g) Active caster wheel with horizontally placed motors [3] (h) Active caster wheel with vertical placement of actuators [4] (i) Spherical wheel with internal actuation [10] (j) Spherical wheel with external actuation [11]–[13] (k) Sliced spherical orthogonal wheels [14] (l) Omniball [15], [21]

The design principle is based on the observations made with the evolution and objectives of the designed wheels in literatures. The design of the wheel is modular and can be assembled at the desired inclination with the ground to the vehicle and is discussed in Section IV. Moreover, with two units of *Ospheel* attached to the chassis helps in achieving the holonomic locomotions, whereas in [5], [6], [21] requires at least three units to be attached with the vehicle chassis. *Ospheel* design can be useful for selfreconfigurable architecture of the robots and will assist in improving its taxonomy level as reported in [22].

The rest of this paper is organized as follows. Section II explains on the design features and mechanical layout of the modular wheel *Ospheel* in detail. Section III describes the kinematic formulation of a single *Ospheel*. Section IV explains the mobility of the vehicle assembled with two *Ospheels*, and the trajectory traced by the vehicle is presented and discussed. Finally Section V concludes the paper.

II. WHEEL DESIGN

In this section, the design principles considered for the modular wheel architecture is discussed. The detailed description of the mechanical design is also presented.

A. Design Principles

From the study of existing wheels design with some of them discussed in the introduction and summarized in Fig. 1, we used generic design principles [23] that are required for the wheels to be omnidirectional. The observation concluded from the literature are: a) The wheel should be able to roll in both the directions, i.e., traverse and sideways or lateral direction b) Both the rolling action of the end geometry in contact with the ground should be actuated which can provide higher torque while locomotion, c) Spherical symmetry has the advantage of rolling point of contact in two directions d) External actuation of the spherical geometry with gearreduction helps to transmit higher torque. These factors were taken into account to design the *Ospheel* as discussed next.

B. Mechanical layout

The exploded view of the assembly is shown in Fig. 2. The spherical-sectioned wheel has two active axes for rotation labelled as steering and wheel axis that are orthogonal to each other. The two identical motors are used to drive the gear trains, which transmit the necessary torque to rotate the spherical sectioned wheel in two rotational directions. The two motors were attached opposite to each other with the motor-holder, which then passes through the holder, which allows the motor-shaft to pass. For the steering action to take place, four of the collar-bearings are held with the four collar-arresters, which allow the relative motion for the rotation and are supported with the groves present on the exterior boundaries of cylindrical-cage which are held all the way using the fork acting as module arrester.

1) Steering Axis: The gear trains to transmit the rotational motion from the motors to the two orthogonal directions are depicted in Fig. 3. For the steering action, the internal spur gear placed inside the cylindrical-cage is driven by the pinion which is connected through the shaft of the motor. The motor-holder let the motor rest on it and the motor shaft to pass through. Hence, the driving unit for the use of steering consists of a) Motor 1, b) Driving pinion 1, and c) Internal spur gear. The gear ratio between the internal spur gear and the pinion (N_s) governs the torque and speed transmission from the motor to the steering unit as per the law of gearing. Here $N_s = 12/62$, and since the number of teeth on the internal spur gear is greater than the pinion, it gives the ability to transmit high torque. It is one of the advantages of this design.



Fig. 2: Exploded view with the labelled components of Ospheel.



Fig. 3: Driving mechanism with the gear trains of Ospheel

2) Wheel Axis: The top view of the gear train for steering is shown in Fig. 3a. The driving pinion is connected with the shaft which is powered by motor-2. The crown gear is embedded with the annular section disk. This annular section is mounted on the wheel-shaft, which is supported on ballbearings. The ball bearing holds the disk, and the shaft has square cross-section to rigidly attach the two spherical cups that rotate along with the rotation of the crown gear. The internal spur gear and the crown gear are spatially located in the two perpendicular planes Π_1 and Π_2 , as shown in Fig. 3b.

III. KINEMATIC MODELING

In this section, the kinematic formulations of single wheel and vehicle with two *Ospheels* are presented. The discontinuity of the surface geometry is discussed. However, during the modeling of the single wheel system and the double wheel system, the formulation assumes no discontinuity on the surface geometry.

A. Geometry of Ospheel

The geometric design of the wheel is a sectioned sphere that consists of the spherical cap sections when $\theta_s \in [\pi/6, 5\pi/6] \cup [7\pi/6, 11\pi/6]$ as shown in Fig. 4a. Assuming



Fig. 4: Geometry of Ospheel and the radius of the sphere r_s . Note that the effective radius for rotation varies with the change in tilt angle α and steering position θ_S

the spherical section of wheels as continuous, the wheel radius r_w that changes due to the change in steering angle is estimated assuming the spherical sectioned wheel as a continuous sphere. The wheel axis rotation occurs with the change in inclined angle α and the steering angle θ_s . However, during this rotation, the equivalent wheel section always pass through point P, which is the contact point between Ospheel and the ground, and normal to the wheel axis. Assigning a moving frame originated from the center of the sphere that rotates together with Ospheel as shown in Fig. 4b, the equation of plane of the equivalent wheel section w.r.t the moving frame using the spherical coordinates is defined as:

$$y + r_s \sin \alpha \sin \theta = 0 \tag{1}$$

In Fig. 4b, point Q is the center of the section, the distance $\overline{OQ} \equiv r_s \sin \alpha \sin \theta$. In addition, the position of point P w.r.t., the moving frame is $[-r_s \sin \alpha \cos \theta - r_s \sin \alpha \sin \theta - r_s \cos \alpha]^T$. The position of the intersection can be obtained by substituting y = 0 and z = 0 in 1 and is equal to $[r_s \sin(\alpha) \cos(\theta_s) \quad 0 \quad 0]^T$. The distance between contact point P and the center is the effective wheel radius at an inclination is given as:

$$r_w(\alpha, \theta_s) = ||QP|| = r_s \sqrt{\cos^2 \alpha + \sin^2 \alpha \cos^2 \theta_s}$$
 (2)

where r_w is the effective radius of rolling due to wheel axis rotation. Also the effective steering radius r_t at an inclined sphere can easily be obtained as:

$$r_t(\alpha) = r_s \sin \alpha \tag{3}$$

Note that $r_t = 0$ when $\alpha = 0$ which is the case shown in Fig.5a. The wheel rotational motion about the axis-2 is provided by the rotation of the driven crown-gear by the driving-pinion-2. The gear ratio between the crown gear and the pinion (N_c) governs the torque and speed transmission. Here $N_c = 12/60$ and since the number of teeth on the crown gear is greater than the pinion, it gives the ability to transmit high torque. The wheel rotation is denoted by θ_w (Fig. 5) and in Case C with $\theta_s = \pi$ the traverse ceases (Case C of Fig. 5) as the spherical section rotates about a point.

B. Single Ospheel kinematics

Modeling the constraints due to the motion of each wheel is necessary step for Kinematic modeling of the robot. In this section the constraints are written for *Ospheel*. *Ospheel* has two axes of rotations as shown in Fig. 2. Fig. 6 depicts the *Ospheel* and O_P is the position of the wheel relative to the local robot frame (say attached with chassis) $\{X_R Y_R Z_R\}$. The position is expressed using the polar coordinates with length l and angle η . The orientation of the wheel w.r.t., chassis is denoted by ν . The rolling constraint for *Ospheel* enforces that all motion along the direction of the wheel plane must be accompanied by the appropriate amount of wheel axis (axis-2) spin and assuming pure rolling the constraint is given by:

$$[S(\eta+\mu) - C(\eta+\mu) - lC(\mu)]\mathbf{R}(\theta_{IR})\dot{\boldsymbol{\xi}}_r = r_w(\theta_s, \alpha)\dot{\theta}_w \quad (4)$$



Fig. 5: Different inclination angle of the *Ospheel* that can be used to connect it with the vehicle chassis

where S and C are sine and cosine respectively. $\mathbf{R}(\theta_{IR})$ is the rotation matrix about the Z-axis by the angle between the inertial frame and robotic vehicle frame. Velocity vector of the robot is defined as $\dot{\boldsymbol{\xi}}_r = [\dot{x}_r \quad \dot{y}_r \quad \dot{\theta}_r]^T$ and the above equation maps the contribution of $\dot{\boldsymbol{\xi}}_r$ for motion along the wheel plane Π_W which is perpendicular to the ground plane Π_G as shown in Fig. 6.

Similarly *Ospheel* will traverse in the inclined or slanted plane denoted by Π_S (Fig. 6) is by the appropriate amount of steering angle change (about axis-1) and assuming pure rolling the constraint equation is given by:

$$[C(\eta + \mu) \ S(\eta + \mu) \ lS(\mu)]\mathbf{R}(\theta_{IR})\dot{\boldsymbol{\xi}}_r = r_t(\alpha)\dot{\theta}_s \qquad (5)$$

With single wheel only two degrees of freedom are controllable, i.e., v_x and v_y which are the velocity components of the platform with respect to the body frame of the wheel. The rotational velocity for orienting the robot frame is not controllable. Defining vector $\mathbf{v} \equiv [v_x \quad v_y]^T$ and for mono wheel the vector $\boldsymbol{\phi}_m$ to be $[\theta_w \quad \theta_s]^T$, the forward kinematics can be represented as:

$$\mathbf{v} = \begin{bmatrix} r_w(\theta_s, \alpha)S(\theta_s) & r_t(\alpha) \\ r_w(\theta_s, \alpha)C(\theta_s) & 0 \end{bmatrix} \mathbf{N}_m \dot{\mathbf{\phi}}_m \tag{6}$$



Fig. 6: Single Ospheel and its parameter

where,
$$\mathbf{N}_m = \begin{bmatrix} N_c & -N_s N_c \\ 0 & N_s \end{bmatrix}$$
 (7)

where $r_w(\theta_s, \alpha)$ and $r_t(\alpha)$ are the wheel radius and steering radius defined in Eq. 2 and 3. N_m is 2×2 matrix of gear ratios which accounts the mapping from motor space to the joint space as generally there are gear box attached with the motors and so is here. The gear reduction ratio N_s and N_c are defined in Section II-B. The inverse kinematics of the single *Ospheel* can then be obtained from Eq. 6 when α is not equals to 0. For $\alpha = 0$, it is equal to the singularity condition where the rolling due to changing θ_s seizes to only steering, and the inverse kinematics should be separately derived.

C. Two Oshpheeled Vehicle

The two-wheeled vehicle here consists of two Ospheels arranged as depicted in Fig. 7. Subscripts 1 and 2 are used to indicate the two wheels symbols while the notations remain the same as used in previous section. For example, the steering angle of the front wheel is denoted as θ_{s1} , whereas the one of rear wheel is θ_{s2} . The distance between the two wheels is denoted as $2h_y$. Hence, for two wheel system the velocity vector $\dot{\Phi}_d$ is equivalent to $[\dot{\theta}_{w1} \ \dot{\theta}_{s1} \ \dot{\theta}_{w2} \ \dot{\theta}_{s2}]^T$. The velocity vector of the robot is $\dot{\xi}_r = [\dot{x}_r \ \dot{y}_r \ \dot{\theta}_r]^T$ consists of the linear and angular velocities of the double wheel system. Assuming no slipping and sliding of the wheels, the forward kinematics was derived and is given by:

$$\dot{\boldsymbol{\xi}}_{r} = \frac{1}{2} \begin{bmatrix} r_{w1}C_{s1} & 0 & r_{w2}C_{s2} & 0\\ r_{w1}S_{s1} & r_{t1} & r_{w2}S_{s2} & r_{t2}\\ -h_{y}r_{w1}S_{s1} & -h_{y}r_{t1} & h_{y}r_{w2}s_{s2} & h_{y}r_{t2} \end{bmatrix} \mathbf{N}_{d}\dot{\boldsymbol{\varphi}}_{d}$$
(8)

For brevity r_{w1} and r_{w2} are used in the above equation to represent $r_{w1}(\theta_{s1}, \alpha_1)$ and $r_{w2}(\theta_{s2}, \alpha_2)$ respectively. Also, C_{s1} and S_{s1} represents $\cos(\theta_{s1})$ and $\sin(\theta_{s1})$ respectively. Similarly for the second wheel. The 4×4 diagonal matrix $N_d \equiv diag[N_m \ N_m]$.

Here the robotic platform is an over-actuated system with four inputs and three outputs. With one set of $\dot{\xi}_r$ there are always more than one set of solutions. To solve it we first determined the motion due to the wheel rotation (along Y_R) and then the steering motion (along X_R). Since \dot{y}_r can only be achieved by changing $\dot{\theta}_{wi}$, the desired wheel velocities are solved first based on the desired \dot{y}_r .

$$\dot{\theta}_{wi} = \frac{\dot{y}_r C_{si}}{r_w} \pm \frac{\dot{\theta}_r h_y S_{si}}{r_w} \tag{9}$$



Fig. 7: Mobile robot with two Ospheels

Based on the position of the steering angles, the wheel velocities also contribute to \dot{x}_r and $\dot{\theta}_r$. Hence, the steering velocities $\dot{\theta}_{si}$ are determined by subtracting the components of $\dot{\theta}_{wi}$ along X-direction and rotation.

$$\dot{\theta}_{si} = \frac{\dot{x}_r}{r_{ti}} - \frac{\dot{y}_r S_{si}}{r_{ti} C_{si}} \pm \frac{h_y \theta_r}{r_{ti}} - \frac{h_y \theta_r}{r_{wi}}, i \in \{1, 2\}$$
(10)

Arranging Eq. 9 and 10 in the matrix form and considering the effect of the gear ratio, the inverse kinematics is written as:

$$\dot{\mathbf{\Phi}}_{d} = \mathbf{N}_{d}^{-1} \begin{bmatrix} 0 & \frac{1}{C_{s1}r_{w1}} & -\frac{\dot{\theta}_{r}h_{y}S_{si}}{r_{w1}} \\ \frac{1}{r_{s1}} & \frac{S_{s1}}{C_{s1}r_{t1}} & -\frac{h_{y}}{r_{t1}} - \frac{h_{y}}{r_{w1}} \\ 0 & \frac{1}{C_{s2}r_{w2}} & \frac{\dot{\theta}_{r}h_{y}S_{si}}{r_{w}} \\ \frac{1}{r_{s2}} & -\frac{S_{s2}}{C_{s2}r_{t2}} & \frac{h_{y}}{r_{s2}} - \frac{h_{y}}{r_{w2}} \end{bmatrix} \dot{\boldsymbol{\xi}}_{r}$$
(11)

where $\dot{\boldsymbol{\xi}}_r = [\dot{x}_r \quad \dot{y}_r \quad \dot{\theta}_r]^T$ is the velocity vector in robots frame. In this way, the locomotion mainly relies on the rotation of the wheel while the rotation of the steering axis. Steering velocity is only to provide the initial velocity along X-axis. This makes the platform able to instantly move to any direction. In the next section the experiments were performed using the mobile robot system developed using *Ospheel*.

IV. EXPERIMENTS AND DISCUSSION

The experiments are performed on the prototyped robotic vehicle with two *Ospheels* as shown in Fig. 7. The system specifications are: distance between the two wheels is equal to 250mm, the clearance of the acrylic sheet base from the ground in 25 mm and both the wheels were inclined with $\alpha_1 = \alpha_2 = 45^\circ$. Two Roboclaws for two wheels were used which are connected with the microcontroller, 12 volts power supply was used to power the motors. Initially homing of both the wheels were done and the orientation after homing is shown in Fig. 7 with both steering angles set as $\theta_{s1} = \theta_{s2} = 0$. Motor encoder values are used to measure θ_s from



(a)



Fig. 9: Mobile robot with two Ospheels tracing letters "S", "U", "T" and "D"

the steering motors on the two wheels and are then used to estimate the effective wheel radius $r_w(\theta_s, \alpha)$ as in Eq.2. The objective of these experiments are to demonstrate the omnidirectional and holonomic behaviour of the.

The trajectory tests were performed in the indoor environment as shown in Fig. 8a. The two *ospheeled* vehicle (Fig.7) is mounted with the LIDAR to obtain the position of the robot with respected to the generated map using the readily available packages in Robot Operating System (ROS). Fig. 8b shows the linear where the robot is commanded to move 5 meters and the trajectory traced. The circular trajectory traced by the robot with the diameter of 1 meter is shown in Fig. 8c. Moreover, the robot was also commanded to trace the letters "S", "U", "T" and "D" as shown in Fig. 9. During this experiment the robot was manually placed to its initial starting position which was marked on the floor.

Fig. 10a shows the trajectory of the robot and the overlaid traced trajectory. The test trajectory selected consists of a combination of orthogonal and angular segments. The black sticker pasted on the ground is the desired path. From the tracking results it is observed that the platform follows the path with slight deviation. The velocity plot as shown in Fig. 10b shows the ability of instantaneously changing the motion direction of the vehicle. However, at some time instances when the first command and the last command is given, there are delays observed in the plot.



Fig. 10: The trajectory of the two *Ospheeled* vehicle (with constant orientation)

The reason of delay associated are the acceleration of the motor and unsynchronized steering velocities. Another factor that causes the deviation is the material of the wheels are printed using Delrin which has less friction force with ground surface. In addition, the shaking or vibration of the platform observed in the supplemental video is due to the gaps on the wheel surface. Despite the deficiencies, the tracking results still demonstrate the working of the kinematic model and the holonomic behaviour of the robot. In future we target to estimate the power consumption by logging the current data as detailed in [24] and also by estimating torque using the dynamic model. Also the circle point method as reported in [25] will be used for identifying the axis of rotation and steering. The drain inspection robot named as Tarantula reported in [26] were modified and the developed Ospheel were assembled in [27].

V. CONCLUSIONS

In this paper, we presented the design of omnidirectional wheel which has a spherical sectioned geometry named as Ospheel. The geometry of the wheel is actuated by the motors which drive the two perpendicular gear trains to obtain the two rotations which also are orthogonal. The kinematic analysis of single *Ospheel* shows that it places no directional constraints on the motion. The proposed design is adequate for practical applications into the omnidirectional vehicle. The same is demonstrated with experiments using the two Ospheeled vehicle with its kinematics model derived and implemented. The tracking video and the velocity plots show the ability of the vehicle to instantaneously change the motion direction without stopping first to steer and then move. In future we aim to cover the outer shell of the wheel with soft-rubber tires to increase the traction force and avoid the slippage, and also perform extensive experiments for demonstrating its holonomic ability.

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