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Transferability in an 8-DoF Parallel Robot with a Configurable Platform*

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Abstract— **Parallel robots with configurable platforms (PRCPs) combine the benefits of parallel robots with additional functionalities such as grasping and cutting. However, some of the theoretical tools used to study classical parallel robots do not apply to parallel robots with configurable platforms. This paper uses screw theory to study the transferable wrenches from the robot's limbs to the configurable platform of an 8-DoF parallel robot. Deriving the transferable wrenches allows one to construct the screw system that is applied to each part of the configurable platform. Based on the analytical expressions of the limb and platform wrenches that have been derived and numerically validated, the mathematical tools that are used to study parallel kinematic structures, such as Grassmann line geometry, can thus be applied to the presented parallel robot with a configurable platform.**

I. INTRODUCTION

It is widely accepted that parallel robots have many advantages such as high stiffness, high payload to weight ratio, and high speed. Thanks to these properties, several parallel architectures have been successful in industry for pick-and-place applications [1], [2], large payload lifting [3], machining [4], and micro-robotics [5], [6]. Such robots have been studied for several decades and many methods have been developed to analyse and optimize their structure [7], [8].

Although most parallel robots have a rigid platform, a subclass of these robots possess an articulated platform. Some of the first architectures to have this property were the 4-DoF H4 [9] and the Par4 robots [10]. The configurable platform of these robots, whose kinematics was close to that of the delta robot, was used to produce an additional rotational motion around a vertical axis, thereby allowing the tool to perform the scara motions (Schönflies motions). This robotic architecture was highly successful in industry under the Adept Quattro™ robot and inspired many other works.

In [11], a 5-DoF parallel structure with a configurable platform offering three translations, one rotation and a grasping capability was proposed. A similar kinematically redundant version having 7 DoFs was developed as a haptic interface [12]. In [13], a redundant planar parallel robot was proposed, where the robot's redundancy was used to actuate a gripper and to eliminate singularities so that unlimited planar rotations are possible.

In most of the spatial parallel robots with configurable platform (PRCP), the platform is composed of a planar closed-loop chain (typically a parallelogram) that provides one additional DoF to the robot [9], [10], [11], [12]. In [14], a 4-DoF parallel wrist able to perform 3 rotations and grasping was proposed for surgical applications. In this structure, the grasping is not obtained by a parallelogram but by a simple folding platform. With similar principles, a 7-DoF redundant PRCP then an 8-DoF version were proposed in [15] and [16]. One of the advantages of these robotic structures is that they are composed of a single type of joint (spherical), which allows to miniaturise them by using soft joints [17].

Even though some PRCP exhibit interesting properties and have already been successful in industry, more theoretical and methodological work has to be devoted to this type of structures. Indeed, most analytical methods that have been developed for classical parallel robots are not directly applicable to PRCP and require either adaptation or extension [18].

In [19], a systematic approach was proposed to analyse the singularities of parallel robots with two end-effectors. This method is complete, which allows finding all the singular configurations. However, this algebraic approach is based on the analytical study of the Jacobian matrix. For a large number of DoFs, such an analysis becomes complex and the interpretation of the solutions might become difficult to understand and to represent physically.

Geometric methods such as Grassmann line geometry [20] have been applied to the analysis and synthesis of classical parallel structures but, to the best of our knowledge, these methods have never been used to analyse the kinematics of PRCP. Indeed, these methods have been developed to analyse the motion of rigid bodies, which excludes configurable platforms when considered as a whole.

One possible approach to use these geometric methods is to decompose the platform and analyse the screw system that applies to each PRCP's end-effector. Recently, Kang and Dai [21] proposed an approach to analyse the wrenches that are transferred to the platform's components. This method has been applied to two robotic structures, each having 3 DoFs.

In this paper, we develop a method to construct the wrench systems that are applied to the two end-effectors of an 8-DoF PRCP. The considered robot has a configurable platform

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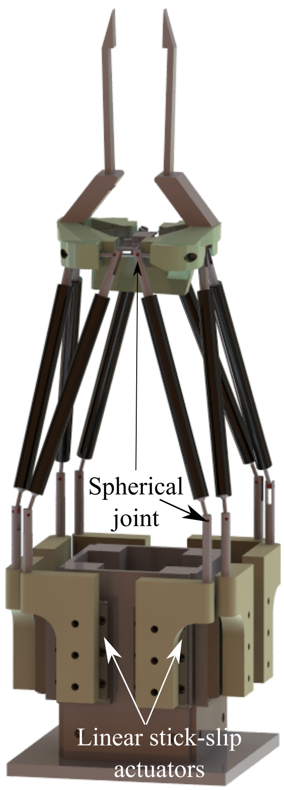

Fig. 1. Design of the 8-DoF dual fingers parallel robot actuated by linear piezoelectric linear stages. The actuators' motions are transmitted to the configurable platform through limbs that are connected with spherical joints to the actuators and the configurable platform [16].

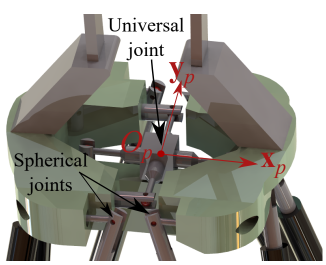

Fig. 2. Structure of the foldable platform of the robot that is composed of two parts. The two parts are connected through a custom made universal joint composed of spherical joints, each part of the platform being connected to four limbs through spherical joints [16].

consisting of two bodies connected to a universal joint, the whole structure being actuated with 8 linear stages (see Fig. 1 and Fig. 2) [16]. In this approach, we first model the wrench systems that are applied to each part of the platform by the robot's limbs. Then, we derive the wrench systems that each part of the platform applies to the other. The objective of this paper is thus the construction of the 6-screw system that applies to each part of the platform, thereby allowing the application of the mathematical methods that have been developed for classical parallel robots.

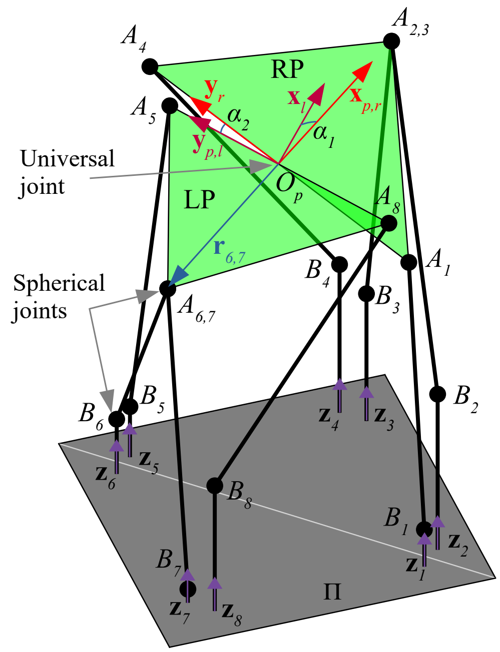

Fig. 3. Kinematic structure of the 8-DoF parallel robot with configurable platform.

II. Modelling of the 8-DoF parallel robot

In this section, we analyse the wrench system that is directly applied by the robot's limbs to each part of the platform, the left-hand part (LP) and the right-hand part (RP). As presented in a previous paper [16], each part of the platform is attached to four limbs through spherical joints at points A_i, $i = 1,\ldots,4$ for RP and $i = 5,\ldots,8$ for LP. Each limb of the robot is itself attached at the other end to a vertically translating actuator through a spherical joint at points B_i. The two sides of the platform are connected through a universal joint. The whole robot forms thus a (2(4($\underline{P}$SS)))U structure (see Fig. 3).

The frames $\mathscr{F}_r = O_p\mathbf{x}_r\mathbf{y}_r\mathbf{z}_r$ and $\mathscr{F}_l = O_p\mathbf{x}_l\mathbf{y}_l\mathbf{z}_l$ are respectively attached to the right and left parts of the platform. The frame attached to the platform $\mathscr{F}_p = O_p\mathbf{x}_p\mathbf{y}_p\mathbf{z}_p$ is attached to the central body of the universal joint whose first rotation axis is $\mathbf{x}_p$, which is collinear with $\mathbf{x}_r$ and its second rotation axis is $\mathbf{y}_p$, which is collinear with $\mathbf{y}_l$ as shown in Fig. 3. For the sake of simplicity, we consider that all A_i points are equidistant to O_p and all limbs lengths are equal. In the home configuration, frames $\mathscr{F}_r$, $\mathscr{F}_l$ and $\mathscr{F}_p$ are superimposed and all points B_i lie in the plane Π.

The configuration of the platform is given by $X = [\theta_x, \theta_y, \theta_z, \alpha_r, \alpha_l, x, y, z]$, where θ_x, θ_y and θ_z are the three components of Rodrigues's vector representing the platform orientation with respect to the base frame; α_r is the rotation angle of RP about $\mathbf{x}_p$; α_l is the rotation angle of LP about $\mathbf{y}_p$ and finally; x, y and z are the platform's Cartesian position in the robot's base frame.

The wrench system applied by the robot's limbs to each part of the platform can be derived by calculating the inverse Jacobian matrix related to RP and LP. Consider that $\mathscr{F}_r$, respectively, $\mathscr{F}_l$ are undergoing the rotational and translational velocities ω_r, $\mathbf{v}_r$ respectively ω_l, $\mathbf{v}_l$. By representing all the vectors and coordinates in the platform's frame, the velocities of the points A_i can then be expressed as follows:

$$\dot{A}_i = \mathbf{v}_r + \mathbf{r}_i \times \omega_r \quad \texttt{for} \quad i = 1, \ldots, 4 \tag{1}$$
$$\dot{A}_i = \mathbf{v}_l + \mathbf{r}_i \times \omega_l \quad \texttt{for} \quad i = 5, \ldots, 8 \tag{2}$$

where $\mathbf{r}_i = O_pA_i$ and the operator $\times$ denotes the vector cross product.

Since the lengths of the limbs are fixed, the amplitudes of the vectors AB_i are constant and their derivative with respect to time vanishes:

$$\frac{d|AB_i|}{dt} = 0 \tag{3}$$

which means that the velocities of the points A_i and B_i projected on $\mathbf{u}_i = \frac{AB_i}{|AB_i|}$ are equal:

$$\dot{A}_i \cdot \mathbf{u}_i = \dot{B}_i \cdot \mathbf{u}_i \tag{4}$$
$$= \dot{q}_i\, \mathbf{z}_q \cdot \mathbf{u}_i \tag{5}$$

where $\dot{q}_i$ is the scalar velocity of the i^{th} actuator and $\mathbf{z}_q$ a unitary vector corresponding to its axis.

By substituting (1) and (2) into (5) and rewriting it in matrix form, we obtain:

$$\mathbf{J}_{r;l}^{-1}\mathbf{t}_{r;l} = \mathbf{H}_{r;l}\dot{\mathbf{q}} \tag{6}$$

with

$$\mathbf{t}_{r;l} = [\mathbf{v}_{r;l}^t, \omega_{r;l}^t]^t$$

$$\mathbf{J}_r^{-1} = \begin{bmatrix} \mathbf{u}_1^t & (\mathbf{r}_1 \times \mathbf{u}_1)^t \\ \mathbf{u}_2^t & (\mathbf{r}_2 \times \mathbf{u}_2)^t \\ \mathbf{u}_3^t & (\mathbf{r}_3 \times \mathbf{u}_3)^t \\ \mathbf{u}_4^t & (\mathbf{r}_4 \times \mathbf{u}_4)^t \end{bmatrix}, \quad \mathbf{J}_l^{-1} = \begin{bmatrix} \mathbf{u}_5^t & (\mathbf{r}_5 \times \mathbf{u}_5)^t \\ \mathbf{u}_6^t & (\mathbf{r}_6 \times \mathbf{u}_6)^t \\ \mathbf{u}_7^t & (\mathbf{r}_7 \times \mathbf{u}_7)^t \\ \mathbf{u}_8^t & (\mathbf{r}_8 \times \mathbf{u}_8)^t \end{bmatrix} \tag{7}$$

$$\mathbf{H}_r = \begin{bmatrix} \mathbf{u}_1^t\mathbf{z}_q & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{u}_4^t\mathbf{z}_q \end{bmatrix}, \quad \mathbf{H}_l = \begin{bmatrix} \mathbf{u}_5^t\mathbf{z}_q & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{u}_8^t\mathbf{z}_q \end{bmatrix} \tag{8}$$

where the notation $\bullet_{r;l}$ refers to either $\bullet_r$ or $\bullet_l$.

One can notice that each row of $\mathbf{J}_{r;l}^{-1}$ is a normalized Plücker line that represents the constraints imposed on each part of the platform and that each row of the 4×6 Jacobian matrices is a zero pitch screw. The wrenches that the robot limbs apply to each part of the platform are thus given by:

$$\mathbf{S}_i = \mathbf{J}_r^{-1}(i, \bullet)^t \quad \texttt{for} \quad i = 1, \ldots, 4 \tag{9}$$
$$\mathbf{S}_i = \mathbf{J}_l^{-1}(i, \bullet)^t \quad \texttt{for} \quad i = 5, \ldots, 8 \tag{10}$$

where $\mathbf{J}_r(i, \bullet)$ and $\mathbf{J}_l(i, \bullet)$ represent the i^{th} row of the corresponding Jacobian matrix.

When the actuators are locked, it is clear from these definitions that the wrench system $\mathbb{S}_r = \texttt{span}(\mathbf{S}_1, \mathbf{S}_2, \mathbf{S}_3, \mathbf{S}_4)$ directly constrains RP while the wrench system $\mathbb{S}_l = \texttt{span}(\mathbf{S}_5, \mathbf{S}_6, \mathbf{S}_7, \mathbf{S}_8)$ constrains LP as shown in Fig. 4. The constraints applied by each part of the platform to the other are analysed in the next section.

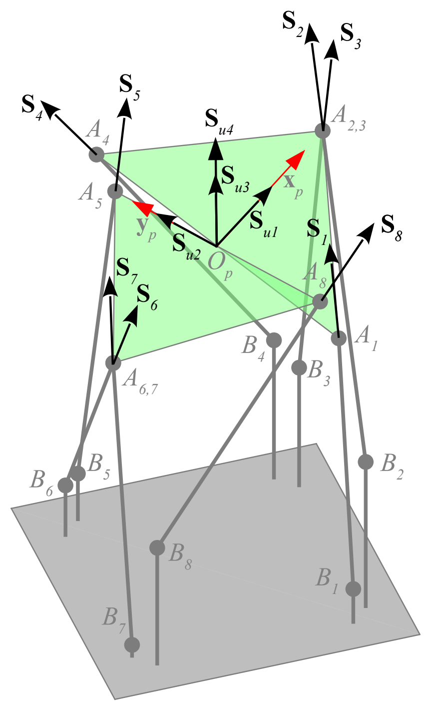

Fig. 4. The screws that are applied by the robot's limbs and the universal joint to the robot's platform.

III. The transferable wrenches

A. Wrench systems intersection

To analyse the robot's kinematics, one approach consists in analysing its full 8×8 Jacobian matrix [16]. However, such an approach is analytically complicated because of the high dimension of the Jacobian matrix. Thus, it would be judicious to find a way to use tools and methods that are readily available from the literature such as Grassmann line geometry, which requires constructing the wrench systems that apply to each part of the platform.

From the preceding section, one can see that each part of the platform is directly constrained by four zero pitch wrenches applied by the robot's limbs. To construct the entire screw systems that apply to each part of the platform, it is necessary to calculate the intersection between the 4-screw systems from the limbs ($\mathbf{S}_r$ respectively $\mathbf{S}_l$) and the platform's screw system ($\mathbf{S}_u$) that corresponds to the connection between the two parts of the platform [21].

The wrench system induced by the universal joint is a 4-screw system $\mathbb{S}_u = \texttt{span}(\mathbf{S}_{u1}, ..., \mathbf{S}_{u4})$ that prevents three translations and one rotation (about $\mathbf{z}_p$) and can be represented in the platform's frame as follows:

$$\mathbf{S}_{u1} = (1, 0, 0, 0, 0, 0)^t \tag{11}$$
$$\mathbf{S}_{u2} = (0, 1, 0, 0, 0, 0)^t \tag{12}$$
$$\mathbf{S}_{u3} = (0, 0, 1, 0, 0, 0)^t \tag{13}$$
$$\mathbf{S}_{u4} = (0, 0, 0, 0, 0, 1)^t \tag{14}$$

The dimension of the wrench system that constrains each part of the platform can then be calculated using the following formula [22]:

$$\dim(\mathbb{S}_{r;l} \cup \mathbb{S}_u) = \dim(\mathbb{S}_{r;l}) + \dim(\mathbb{S}_u) - \dim(\mathbb{S}_{r;l} \cap \mathbb{S}_u) \quad (15)$$

However, it seems difficult to calculate directly the intersection between the two 4-screw systems. Nevertheless, an indirect method that exploits the reciprocal screws of the platform wrench system $\mathbb{S}_u$ seems more practical.

The reciprocal space of the platform's wrench system is a 2-screw system, which represents the two twists allowing the 2 DoFs of the universal joint. This space is simply constructed by the two pure rotations about $\mathbf{x}_p$ and $\mathbf{y}_p$ axes of the platform whose corresponding twists can be written as follows:

$$\mathbf{S}_{u1}^r = (1,0,0,0,0,0)^t \quad (16)$$

$$\mathbf{S}_{u2}^r = (0,1,0,0,0,0)^t \quad (17)$$

Let us now define $\mathbb{S}_{rp}$ respectively $\mathbb{S}_{lp}$ as the intersection spaces between the limb wrenches $\mathbb{S}_r$ respectively $\mathbb{S}_l$ with the universal joint wrenches ($\mathbb{S}_{rp;lp} = \mathbb{S}_{r;l} \cap \mathbb{S}_u$) and let $\mathbb{S}_{r;p}^r$ be the space generated by the screws reciprocal to $\mathbb{S}_{r;p}$ screws. Since $\mathbb{S}_{rp;lp} \subset \mathbb{S}_u$, then $\mathbb{S}_{rp;lp}$ is also reciprocal to $\mathbb{S}_u^r$. It is thus possible to find $\mathbb{S}_{rp;lp}$ by calculating the subspace of $\mathbb{S}_{r;l}$ that is reciprocal to the twist system $\mathbb{S}_u^r$.

Since $\mathbb{S}_{r;p}$ and $\mathbb{S}_u^r$ are generated by zero pitch screws, a sufficient condition for a screw that belongs to $\mathbb{S}_{r;p}$ to be reciprocal to $\mathbb{S}_u^r$ is that it intersects both $\mathbf{S}_{u1}^r$ and $\mathbf{S}_{u2}^r$ or intersects one and be parallel to the other [22]. Since $\mathbf{S}_{u1}^r$ and $\mathbf{S}_{u2}^r$ are perpendicular and intersect at the platform's origin O_p, $\mathbb{S}_{rp;lp}$ can be constructed from linearly independent screws that belong to $\mathbb{S}_{r;l}$ and that are either coplanar with $\mathbf{S}_{u1}^r$ and $\mathbf{S}_{u2}^r$ or cross the platform's origin. Such screws can be obtained by performing linear combinations of the screws that constitute $\mathbb{S}_{r;l}$.

B. Wrench derivation

To construct the wrench system that constrains the motion of each platform, we need first to calculate the intersection between $\mathbb{S}_{r;l}$ and $\mathbb{S}_u$, which consists in calculating independent screws from $\mathbb{S}_r$ and $\mathbb{S}_l$ that are reciprocal to $\mathbf{S}_{u1}^r$ and $\mathbf{S}_{u2}^r$. As determined in the preceding section, this leads to calculating bases of $\mathbb{S}_{r,p}$ by finding the screws that are either coplanar with $\mathbf{S}_{u1;u2}$ or intersect with both screws at the platform's origin O_p.

For a screw to be coplanar with both $\mathbf{S}_{u1}^r$ and $\mathbf{S}_{u2}^r$, it must have the following form:

$$\mathbf{S}_{r1;l1} = (\bullet,\bullet,0,0,0,\bullet)^t \quad (18)$$

where the $\bullet$ marks represent arbitrary real numbers.

Similarly, to intersect with the platform's origin, the screws $\mathbf{S}_{r2}$ and $\mathbf{S}_{l2}$ must have the following form:

$$\mathbf{S}_{r2;l2} = (\bullet,\bullet,\bullet,0,0,0)^t \quad (19)$$

From the screw systems $\mathbb{S}_r$ and $\mathbb{S}_l$, it is possible to construct such screws by performing successive eliminations through linear operations between the wrenches. Given the platform described in Fig. 3, the coordinates of the points A_i represented in the platform frame are given by:

$$\mathbf{r}_1 = r(0,-c_r,-s_r)^t \quad (20)$$

$$\mathbf{r}_2 = r(1,0,0)^t \quad (21)$$

$$\mathbf{r}_3 = r(1,0,0)^t \quad (22)$$

$$\mathbf{r}_4 = r(0,c_r,s_r)^t \quad (23)$$

$$\mathbf{r}_5 = r(0,1,0)^t \quad (24)$$

$$\mathbf{r}_6 = r(-c_l,0,c_l)^t \quad (25)$$

$$\mathbf{r}_7 = r(-c_l,0,c_l)^t \quad (26)$$

$$\mathbf{r}_8 = r(0,-1,0)^t \quad (27)$$

where r is the distance between the points A_i and the platform's origin and s_r respectively c_r represent $\sin(\alpha_r)$ respectively $\cos(\alpha_r)$, where $\alpha_r = X(4)$ is the angle between the platform's frame attached to the central body of the universal joint and the frame attached to the right part (about the $\mathbf{x}_p$ axis). Similarly, $s_l = \sin(\alpha_l)$, $c_l = \cos(\alpha_l)$, where $\alpha_l = X(5)$ is the angle between the platform's frame and the frame attached to the left part (about the $\mathbf{y}_p$ axis).

By substituting these values into (7), we obtain the following wrenches represented in the platform's frame:

$$\mathbf{S}_1 = \left(\mathbf{u}_1^t, -rc_r\mathbf{u}_{1z} + rs_r\mathbf{u}_{1y}, rs_r\mathbf{u}_{1x}, rc_r\mathbf{u}_{1x}\right)^t \quad (28)$$

$$\mathbf{S}_2 = \left(\mathbf{u}_2^t, 0, -r\mathbf{u}_{2z}, r\mathbf{u}_{2y}\right)^t \quad (29)$$

$$\mathbf{S}_3 = \left(\mathbf{u}_3^t, 0, -r\mathbf{u}_{3z}, r\mathbf{u}_{3y}\right)^t \quad (30)$$

$$\mathbf{S}_4 = \left(\mathbf{u}_4^t, rc_r\mathbf{u}_{4z} - rs_r\mathbf{u}_{4y}, rs_r\mathbf{u}_{4x}, -rc_r\mathbf{u}_{4x}\right)^t \quad (31)$$

$$\mathbf{S}_5 = \left(\mathbf{u}_5^t, r\mathbf{u}_{5z}, 0, -r\mathbf{u}_{5x}\right)^t \quad (32)$$

$$\mathbf{S}_6 = \left(\mathbf{u}_6^t, -rs_l\mathbf{u}_{6y}, rs_l\mathbf{u}_{6x} + rc_l\mathbf{u}_{6z}, -rc_l\mathbf{u}_{6y}\right)^t \quad (33)$$

$$\mathbf{S}_7 = \left(\mathbf{u}_7^t, -rs_l\mathbf{u}_{7y}, rs_l\mathbf{u}_{7x} + rc_l\mathbf{u}_{7z}, -rc_l\mathbf{u}_{7y}\right)^t \quad (34)$$

$$\mathbf{S}_8 = \left(\mathbf{u}_8^t, -r\mathbf{u}_{8z}, 0, r\mathbf{u}_{8x}\right)^t \quad (35)$$

Finally, the constraint wrench forms shown in (19) and (18) can be built by successive eliminations using linear combinations of the wrenches (similarly to Gauss-Jordan elimination). Since both screw forms contain three zero values, these forms can be obtained, in the general case, only if $\dim(\mathbb{S}_r) = \dim(\mathbb{S}_l) = 4$ which means that the wrenches that the limbs apply to each platform are linearly independent. The pairs of constraint wrenches applied by each part of the platform to the other are presented in the Appendix. As a result, each platform is subjected to six wrenches; four of them are directly applied by the robot's limbs while the other two are transferred by the other part of the platform. All the wrenches have the particularity of being zero pitch screws, which might make the singularity analysis of the mechanism easier.

C. Validation

Fig. 5, illustrates the transferred wrenches from one part of the platform to the other for a randomly generated configuration of the platform given by:

$$X = (-4.74, \quad -25.80, \quad 23.07, \quad 25.48, \quad -0.52, \quad -0.42\text{E}{-2}, \quad -6.49\text{E}{-2}, \quad 16.0\text{E}{-2}) \quad (36)$$

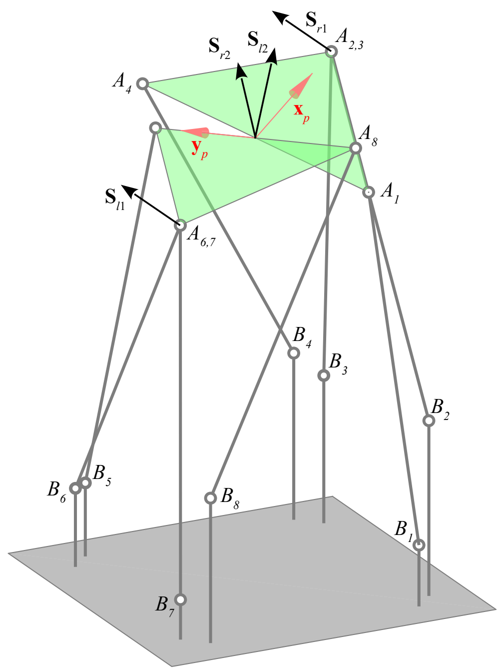

Fig. 5. The wrenches that are transferred from each part of the platform to the other. The wrenches $\mathbf{S}_{r1}$ and $\mathbf{S}_{r2}$ are transferred from RP to LP and the wrenches $\mathbf{S}_l1$ and $\mathbf{S}_l2$ are transferred from LP to RP.

where the angles are given in degrees and the positions values are normalized by r, namely the distance between point A_i and the origin of the platform frame. Thus, all X parameters are dimensionless.

The values of the screws expressed in the platform frame are given by:

$$\mathbf{S}_{r1} = \left(-0.39, \quad -0.92, \quad 0, \quad 0, \quad 0, \quad -6.90\right)^t \tag{37}$$

$$\mathbf{S}_{r2} = \left(-0.002, \quad 0.27, \quad -0.96, \quad 0, \quad 0, \quad 0\right)^t \tag{38}$$

$$\mathbf{S}_{l1} = \left(0.39, \quad 0.92, \quad 0, \quad 0, \quad 0, \quad -6.85\right)^t \tag{39}$$

$$\mathbf{S}_{l2} = \left(-0.71, \quad 0.10, \quad -0.69, \quad 0, \quad 0, \quad 0\right)^t \tag{40}$$

Several random configurations of the robot's platform were computed and the wrench values obtained analytically were the same as the values obtained numerically (with a relative precision of 1E−13), which validates the expressions obtained in (41) to (53).

IV. CONCLUSIONS

This paper presents a method for deriving the transferable wrenches in an 8-DoF parallel robot with a configurable platform. This method uses the platform's twists to calculate the intersection between the robot's limbs wrench system and the platform's wrench system. The result is the two 6-wrench systems that are applied to each part of the platform both by the limbs and by the other part of the configurable platform. Such a result makes it possible to use the same analytical and geometric tools that are used to study classical parallel robots. In future work, Grassmann line geometry will be applied to analyse the kinematics of the presented robot using the analytical expressions of the screws obtained here. The extension of the developed approach to other PRCP structures will also be studied.

APPENDIX

Expressed in the platform's frame, the first motion constraint applied by RP to LP is the zero pitch wrench:

$$\begin{aligned}\mathbf{S}_{r1} = (&\mathbf{u}_{3x}\mathbf{u}_{2z} - \mathbf{u}_{3z}\mathbf{u}_{2x}, \mathbf{u}_{2z}\mathbf{u}_{3y} \\ &- \mathbf{u}_{2y}\mathbf{u}_{3z}, 0, 0, 0, r(\mathbf{u}_{2z}\mathbf{u}_{3y} - \mathbf{u}_{2y}\mathbf{u}_{3z}))^t\end{aligned} \tag{41}$$

The components of the second zero pitch wrench $\mathbf{S}_{r2}$ transferred to LP are given by:

$$\begin{aligned}\mathbf{S}_{r2}(1) = &-{c_r}^2\mathbf{u}_{1x}\mathbf{u}_{2x}\mathbf{u}_{3z}\mathbf{u}_{4z} + {c_r}^2\mathbf{u}_{1x}\mathbf{u}_{2z}\mathbf{u}_{3x}\mathbf{u}_{4z} \\ &+ {c_r}^2\mathbf{u}_{1z}\mathbf{u}_{2x}\mathbf{u}_{3z}\mathbf{u}_{4x} - {c_r}^2\mathbf{u}_{1z}\mathbf{u}_{2z}\mathbf{u}_{3x}\mathbf{u}_{4x} \\ &+ c_r s_r\mathbf{u}_{1x}\mathbf{u}_{2x}\mathbf{u}_{3y}\mathbf{u}_{4z} + c_r s_r\mathbf{u}_{1x}\mathbf{u}_{2x}\mathbf{u}_{3z}\mathbf{u}_{4y} \\ &- c_r s_r\mathbf{u}_{1x}\mathbf{u}_{2y}\mathbf{u}_{3x}\mathbf{u}_{4z} - c_r s_r\mathbf{u}_{1x}\mathbf{u}_{2z}\mathbf{u}_{3x}\mathbf{u}_{4y} \\ &- c_r s_r\mathbf{u}_{1y}\mathbf{u}_{2x}\mathbf{u}_{3z}\mathbf{u}_{4x} + c_r s_r\mathbf{u}_{1y}\mathbf{u}_{2z}\mathbf{u}_{3x}\mathbf{u}_{4x} \\ &- c_r s_r\mathbf{u}_{1z}\mathbf{u}_{2x}\mathbf{u}_{3y}\mathbf{u}_{4x} + c_r s_r\mathbf{u}_{1z}\mathbf{u}_{2y}\mathbf{u}_{3x}\mathbf{u}_{4x} \\ &- {s_r}^2\mathbf{u}_{1x}\mathbf{u}_{2x}\mathbf{u}_{3y}\mathbf{u}_{4y} + {s_r}^2\mathbf{u}_{1x}\mathbf{u}_{2y}\mathbf{u}_{3x}\mathbf{u}_{4y} \\ &+ {s_r}^2\mathbf{u}_{1y}\mathbf{u}_{2x}\mathbf{u}_{3y}\mathbf{u}_{4x} - {s_r}^2\mathbf{u}_{1y}\mathbf{u}_{2y}\mathbf{u}_{3x}\mathbf{u}_{4x} \\ &+ c_r\mathbf{u}_{1x}\mathbf{u}_{2y}\mathbf{u}_{3z}\mathbf{u}_{4z} - c_r\mathbf{u}_{1x}\mathbf{u}_{2z}\mathbf{u}_{3y}\mathbf{u}_{4z} \\ &+ c_r\mathbf{u}_{1z}\mathbf{u}_{2y}\mathbf{u}_{3z}\mathbf{u}_{4x} - c_r\mathbf{u}_{1z}\mathbf{u}_{2z}\mathbf{u}_{3y}\mathbf{u}_{4x} \\ &- s_r\mathbf{u}_{1x}\mathbf{u}_{2y}\mathbf{u}_{3z}\mathbf{u}_{4y} + s_r\mathbf{u}_{1x}\mathbf{u}_{2z}\mathbf{u}_{3y}\mathbf{u}_{4y} \\ &- s_r\mathbf{u}_{1y}\mathbf{u}_{2y}\mathbf{u}_{3z}\mathbf{u}_{4x} + s_r\mathbf{u}_{1y}\mathbf{u}_{2z}\mathbf{u}_{3y}\mathbf{u}_{4x}\end{aligned} \tag{42}$$

$$\begin{aligned}\mathbf{S}_{r2}(2) = -\big(&{c_r}^2\mathbf{u}_{1x}\mathbf{u}_{4z} - {c_r}^2\mathbf{u}_{1z}\mathbf{u}_{4x} - c_r s_r\mathbf{u}_{1x}\mathbf{u}_{4y} \\ &+ c_r s_r\mathbf{u}_{1y}\mathbf{u}_{4x} - c_r\mathbf{u}_{1y}\mathbf{u}_{4z} - c_r\mathbf{u}_{1z}\mathbf{u}_{4y} \\ &+ 2 s_r\mathbf{u}_{1y}\mathbf{u}_{4y}\big)(\mathbf{u}_{2y}\mathbf{u}_{3z} - \mathbf{u}_{2z}\mathbf{u}_{3y})\end{aligned} \tag{43}$$

$$\begin{aligned}\mathbf{S}_{r2}(3) = -\big(&c_r s_r\mathbf{u}_{1x}\mathbf{u}_{4z} - c_r s_r\mathbf{u}_{1z}\mathbf{u}_{4x} - {s_r}^2\mathbf{u}_{1x}\mathbf{u}_{4y} \\ &+ {s_r}^2\mathbf{u}_{1y}\mathbf{u}_{4x} - 2 c_r\mathbf{u}_{1z}\mathbf{u}_{4z} + s_r\mathbf{u}_{1y}\mathbf{u}_{4z} \\ &+ s_r\mathbf{u}_{1z}\mathbf{u}_{4y}\big)(\mathbf{u}_{2y}\mathbf{u}_{3z} - \mathbf{u}_{2z}\mathbf{u}_{3y})\end{aligned} \tag{44}$$

$$\mathbf{S}_{r2}(4,5,6) = (0,0,0)^t \tag{45}$$

Similarly, the 6 components of the first wrench transferred by LP to RP are the following:

$$\begin{aligned}\mathbf{S}_{l1}(1) = &-s_l\mathbf{u}_{5z}\mathbf{u}_{6x}\mathbf{u}_{7z}\mathbf{u}_{8x} - {s_l}^2\mathbf{u}_{5z}\mathbf{u}_{6x}\mathbf{u}_{7y}\mathbf{u}_{8x} \\ &- c_l s_l\mathbf{u}_{5z}\mathbf{u}_{6z}\mathbf{u}_{7y}\mathbf{u}_{8x} + s_l\mathbf{u}_{5z}\mathbf{u}_{6z}\mathbf{u}_{7x}\mathbf{u}_{8x} \\ &+ {s_l}^2\mathbf{u}_{5z}\mathbf{u}_{6y}\mathbf{u}_{7x}\mathbf{u}_{8x} + c_l s_l\mathbf{u}_{5z}\mathbf{u}_{6y}\mathbf{u}_{7z}\mathbf{u}_{8x} \\ &- s_l\mathbf{u}_{5x}\mathbf{u}_{6x}\mathbf{u}_{7z}\mathbf{u}_{8z} + {s_l}^2\mathbf{u}_{5x}\mathbf{u}_{6x}\mathbf{u}_{7y}\mathbf{u}_{8z} \\ &+ c_l s_l\mathbf{u}_{5x}\mathbf{u}_{6z}\mathbf{u}_{7y}\mathbf{u}_{8z} + s_l\mathbf{u}_{5x}\mathbf{u}_{6z}\mathbf{u}_{7x}\mathbf{u}_{8z} \\ &- {s_l}^2\mathbf{u}_{5x}\mathbf{u}_{6y}\mathbf{u}_{7x}\mathbf{u}_{8z} - c_l s_l\mathbf{u}_{5x}\mathbf{u}_{6y}\mathbf{u}_{7z}\mathbf{u}_{8z} \\ &+ 2\mathbf{u}_{8z}\mathbf{u}_{7x}\mathbf{u}_{5z}c_l\mathbf{u}_{6z} - 2\mathbf{u}_{8z}c_l\mathbf{u}_{7z}\mathbf{u}_{6x}\mathbf{u}_{5z}\end{aligned} \tag{46}$$

$$
\begin{aligned}
\mathbf{S}_{l1}(2) = & -s_l \mathbf{u}_{8y} \mathbf{u}_{5z} \mathbf{u}_{7z} \mathbf{u}_{6x} - \mathbf{u}_{8y} \mathbf{u}_{5z} {s_l}^2 \mathbf{u}_{6x} \mathbf{u}_{7y} \\
& - s_l \mathbf{u}_{8y} \mathbf{u}_{5z} c_l \mathbf{u}_{6z} \mathbf{u}_{7y} + s_l \mathbf{u}_{8y} \mathbf{u}_{5z} \mathbf{u}_{7x} \mathbf{u}_{6z} \\
& + \mathbf{u}_{8y} \mathbf{u}_{5z} {s_l}^2 \mathbf{u}_{6y} \mathbf{u}_{7x} + s_l \mathbf{u}_{8y} \mathbf{u}_{5z} c_l \mathbf{u}_{6y} \mathbf{u}_{7z} \\
& - s_l \mathbf{u}_{8z} \mathbf{u}_{5y} \mathbf{u}_{7z} \mathbf{u}_{6x} + \mathbf{u}_{8z} \mathbf{u}_{5y} {s_l}^2 \mathbf{u}_{6x} \mathbf{u}_{7y} \\
& + s_l \mathbf{u}_{8z} \mathbf{u}_{5y} c_l \mathbf{u}_{6z} \mathbf{u}_{7y} + s_l \mathbf{u}_{8z} \mathbf{u}_{5y} \mathbf{u}_{7x} \mathbf{u}_{6z} \\
& - \mathbf{u}_{8z} \mathbf{u}_{5y} {s_l}^2 \mathbf{u}_{6y} \mathbf{u}_{7x} - s_l \mathbf{u}_{8z} \mathbf{u}_{5y} c_l \mathbf{u}_{6y} \mathbf{u}_{7z} \\
& - 2 \mathbf{u}_{8z} \mathbf{u}_{5z} c_l \mathbf{u}_{6y} \mathbf{u}_{7z} + 2 \mathbf{u}_{8z} \mathbf{u}_{5z} c_l \mathbf{u}_{6z} \mathbf{u}_{7y} \\
& + 2 \mathbf{u}_{8z} \mathbf{u}_{5z} \mathbf{u}_{7y} s_l \mathbf{u}_{6x} - 2 \mathbf{u}_{8z} \mathbf{u}_{5z} \mathbf{u}_{6y} s_l \mathbf{u}_{7x}
\end{aligned} \tag{47}
$$

$$
\mathbf{S}_{l1}(3,4,5) = (0,0,0)^t \tag{48}
$$

$$
\begin{aligned}
\mathbf{S}_{l1}(6) = r \big(& 2 {c_l}^2 \mathbf{u}_{5z} \mathbf{u}_{6y} \mathbf{u}_{7z} \mathbf{u}_{8z} - 2 {c_l}^2 \mathbf{u}_{5z} \mathbf{u}_{6z} \mathbf{u}_{7y} \mathbf{u}_{8z} \\
& + c_l s_l \mathbf{u}_{5x} \mathbf{u}_{6y} \mathbf{u}_{7z} \mathbf{u}_{8z} - c_l s_l \mathbf{u}_{5x} \mathbf{u}_{6z} \mathbf{u}_{7y} \mathbf{u}_{8z} \\
& - 2 c_l s_l \mathbf{u}_{5z} \mathbf{u}_{6x} \mathbf{u}_{7y} \mathbf{u}_{8z} + 2 c_l s_l \mathbf{u}_{5z} \mathbf{u}_{6y} \mathbf{u}_{7x} \mathbf{u}_{8z} \\
& + c_l s_l \mathbf{u}_{5z} \mathbf{u}_{6y} \mathbf{u}_{7z} \mathbf{u}_{8x} - c_l s_l \mathbf{u}_{5z} \mathbf{u}_{6z} \mathbf{u}_{7y} \mathbf{u}_{8x} \\
& - {s_l}^2 \mathbf{u}_{5x} \mathbf{u}_{6x} \mathbf{u}_{7y} \mathbf{u}_{8z} + {s_l}^2 \mathbf{u}_{5x} \mathbf{u}_{6y} \mathbf{u}_{7x} \mathbf{u}_{8z} \\
& - {s_l}^2 \mathbf{u}_{5z} \mathbf{u}_{6x} \mathbf{u}_{7y} \mathbf{u}_{8x} + {s_l}^2 \mathbf{u}_{5z} \mathbf{u}_{6y} \mathbf{u}_{7x} \mathbf{u}_{8x} \\
& + s_l \mathbf{u}_{5x} \mathbf{u}_{6x} \mathbf{u}_{7z} \mathbf{u}_{8z} - s_l \mathbf{u}_{5x} \mathbf{u}_{6z} \mathbf{u}_{7x} \mathbf{u}_{8z} \\
& - s_l \mathbf{u}_{5z} \mathbf{u}_{6x} \mathbf{u}_{7z} \mathbf{u}_{8x} + s_l \mathbf{u}_{5z} \mathbf{u}_{6z} \mathbf{u}_{7x} \mathbf{u}_{8x} \big)
\end{aligned} \tag{49}
$$

Finally, the components of the second wrench transferred to RP can be written as:

$$
\begin{aligned}
\mathbf{S}_{l2}(1) = & {c_l}^2 \mathbf{u}_{5x} \mathbf{u}_{6y} \mathbf{u}_{7z} \mathbf{u}_{8z} - {c_l}^2 \mathbf{u}_{5x} \mathbf{u}_{6z} \mathbf{u}_{7y} \mathbf{u}_{8z} \\
& + {c_l}^2 \mathbf{u}_{5z} \mathbf{u}_{6y} \mathbf{u}_{7z} \mathbf{u}_{8x} - {c_l}^2 \mathbf{u}_{5z} \mathbf{u}_{6z} \mathbf{u}_{7y} \mathbf{u}_{8x} \\
& - c_l s_l \mathbf{u}_{5x} \mathbf{u}_{6x} \mathbf{u}_{7y} \mathbf{u}_{8z} + c_l s_l \mathbf{u}_{5x} \mathbf{u}_{6y} \mathbf{u}_{7x} \mathbf{u}_{8z} \\
& + 2 c_l s_l \mathbf{u}_{5x} \mathbf{u}_{6y} \mathbf{u}_{7z} \mathbf{u}_{8x} - 2 c_l s_l \mathbf{u}_{5x} \mathbf{u}_{6z} \mathbf{u}_{7y} \mathbf{u}_{8x} \\
& - c_l s_l \mathbf{u}_{5z} \mathbf{u}_{6x} \mathbf{u}_{7y} \mathbf{u}_{8x} + c_l s_l \mathbf{u}_{5z} \mathbf{u}_{6y} \mathbf{u}_{7x} \mathbf{u}_{8x} \\
& - 2 {s_l}^2 \mathbf{u}_{5x} \mathbf{u}_{6x} \mathbf{u}_{7y} \mathbf{u}_{8x} + 2 {s_l}^2 \mathbf{u}_{5x} \mathbf{u}_{6y} \mathbf{u}_{7x} \mathbf{u}_{8x} \\
& - c_l \mathbf{u}_{5x} \mathbf{u}_{6x} \mathbf{u}_{7z} \mathbf{u}_{8z} + c_l \mathbf{u}_{5x} \mathbf{u}_{6z} \mathbf{u}_{7x} \mathbf{u}_{8z} \\
& + c_l \mathbf{u}_{5z} \mathbf{u}_{6x} \mathbf{u}_{7z} \mathbf{u}_{8x} - c_l \mathbf{u}_{5z} \mathbf{u}_{6z} \mathbf{u}_{7x} \mathbf{u}_{8x}
\end{aligned} \tag{50}
$$

$$
\begin{aligned}
\mathbf{S}_{l2}(2) = & \big(c_l \mathbf{u}_{5y} \mathbf{u}_{8z} + c_l \mathbf{u}_{5z} \mathbf{u}_{8y} + s_l \mathbf{u}_{5x} \mathbf{u}_{8y} + s_l \mathbf{u}_{5y} \mathbf{u}_{8x} \\
& - \mathbf{u}_{5x} \mathbf{u}_{8z} + \mathbf{u}_{5z} \mathbf{u}_{8x} \big) \big(c_l \mathbf{u}_{6y} \mathbf{u}_{7z} - c_l \mathbf{u}_{6z} \mathbf{u}_{7y} \\
& - s_l \mathbf{u}_{6x} \mathbf{u}_{7y} + s_l \mathbf{u}_{6y} \mathbf{u}_{7x} \big)
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
\mathbf{S}_{l2}(3) = & 2 {c_l}^2 \mathbf{u}_{5z} \mathbf{u}_{6y} \mathbf{u}_{7z} \mathbf{u}_{8z} - 2 {c_l}^2 \mathbf{u}_{5z} \mathbf{u}_{6z} \mathbf{u}_{7y} \mathbf{u}_{8z} \\
& + c_l s_l \mathbf{u}_{5x} \mathbf{u}_{6y} \mathbf{u}_{7z} \mathbf{u}_{8z} - c_l s_l \mathbf{u}_{5x} \mathbf{u}_{6z} \mathbf{u}_{7y} \mathbf{u}_{8z} \\
& - 2 c_l s_l \mathbf{u}_{5z} \mathbf{u}_{6x} \mathbf{u}_{7y} \mathbf{u}_{8z} + 2 c_l s_l \mathbf{u}_{5z} \mathbf{u}_{6y} \mathbf{u}_{7x} \mathbf{u}_{8z} \\
& + c_l s_l \mathbf{u}_{5z} \mathbf{u}_{6y} \mathbf{u}_{7z} \mathbf{u}_{8x} - c_l s_l \mathbf{u}_{5z} \mathbf{u}_{6z} \mathbf{u}_{7y} \mathbf{u}_{8x} \\
& - {s_l}^2 \mathbf{u}_{5x} \mathbf{u}_{6x} \mathbf{u}_{7y} \mathbf{u}_{8z} + {s_l}^2 \mathbf{u}_{5x} \mathbf{u}_{6y} \mathbf{u}_{7x} \mathbf{u}_{8z} \\
& - {s_l}^2 \mathbf{u}_{5z} \mathbf{u}_{6x} \mathbf{u}_{7y} \mathbf{u}_{8x} + {s_l}^2 \mathbf{u}_{5z} \mathbf{u}_{6y} \mathbf{u}_{7x} \mathbf{u}_{8x} \\
& + s_l \mathbf{u}_{5x} \mathbf{u}_{6x} \mathbf{u}_{7z} \mathbf{u}_{8z} - s_l \mathbf{u}_{5x} \mathbf{u}_{6z} \mathbf{u}_{7x} \mathbf{u}_{8z} \\
& - s_l \mathbf{u}_{5z} \mathbf{u}_{6x} \mathbf{u}_{7z} \mathbf{u}_{8x} + s_l \mathbf{u}_{5z} \mathbf{u}_{6z} \mathbf{u}_{7x} \mathbf{u}_{8x}
\end{aligned} \tag{52}
$$

$$
\mathbf{S}_{l2}(4,5,6) = (0,0,0)^t \tag{53}
$$

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