Robust Internal Model Control for Motor Systems Based on Sliding Mode Technique and Extended State Observer*

Ping Li, Member, IEEE, Kaiqi Guo, Student Member, IEEE, Chenyang Sun and Mingming Zhang, Member, IEEE

Abstract- Electric motors have been widely used as the actuators of robot and automation systems. This paper aims at achieving the high-precision position control of motor drive systems. For this purpose, a robust control scheme is presented by combining the internal model principle, the sliding mode technique and the extended state observer (ESO). The PID-type controller is firstly designed by using the internal model control (IMC) rules. Since the analysis of the IMC system is performed via a sliding surface, a robust sliding mode control (SMC) law is then synthesized to enhance the control ability of the system to uncertainties. However, this robust solution should make a trade-off between the chattering attenuation and the control accuracy. To handle this drawback, a linear ESO is employed to compensate the modeling errors for a higher control accuracy. The stability analysis is provided via a Lyapunov-based method, and the superiority of the proposed approach was validated by comparative experiments on a motor drive platform.

I. INTRODUCTION

Electric motors have been extensively utilized for robot and automation systems in industry and healthcare [1-3], owing to their merits such as low noise, excellent control capability, high torque production, high-dynamic response, etc. [4, 5]. However, it is still a challenging problem to design highperformance controller for motor systems. Many factors such as modeling errors and unexpected disturbances inevitably exist in actual systems, which may have a great impact on the control accuracy, even excite unstable dynamics [6].

Among existing control approaches, internal model control (IMC) establishes a model-based framework for the controller design and analysis [7]. In general, there are two directions for the development of IMC. One is applying a higher order IMC filter to change the dynamics of the closed-loop [8]. This usually leads to a complex control structure where more control parameters need to be tuned and the frequency-domain performance analysis becomes increasingly difficult. The other is the application of intelligent algorithms, such as immune algorithm [9], neural network method [10], and fuzzy

*This research was supported in part by the National Natural Science Foundation of China under Grant 61903181, the National Natural Science Foundation of Guangdong Province under Grant 2020A151501401, the China Postdoctoral Science Foundation under Grant 2020M672700, and the Guangdong Innovative and Entrepreneurial Research Team Program under Grant 2019ZT08Y191.

P. Li, K. Guo, and C. Sun are with the Department of Biomedical Engineering, Southern University of Science and Technology, Shenzhen 518055, Guangdong, China (e-mail: <u>lip3@sustech.edu.cn</u>; <u>11930454@mail.sustech.edu.cn</u>).

adaptive law [11]. However, their control performances greatly depend on the adaptive values, which has limited their performance improvement, especially under unmodeled dynamics. Besides, more efforts are required for the convergence of the algorithms, leading to the difficulty increase of their applications.

To deal with the control problems of uncertainties, sliding mode control (SMC) is a powerful method for achieving more excellent robustness of the electro-mechanical systems [12]. In the standard SMC structure, a discontinuous switching control law, generally a signum function, is applied to eliminate the effect of uncertainties. However, due to the unmodeled dynamics and imperfect implementation of actual systems, this switching control activity will yield harmful chattering, which may lead to degradation of control accuracy [13]. To suppress the chattering, a robust continuous control scheme combining the IMC and the SMC was proposed for the boundary layer solution of servo motor systems [4]. However, this continuous solution enables the system states move around the desired manifold, leading to a certain bounded tracking error. Since the boundary layer solution is designed based on the boundary conditions, large control gains are generally employed for the systems with large uncertainties, which may limit the control accuracy with the consideration of stability.

Recently, active disturbance rejection control (ADRC) presented in [14] has gained a lot of attention. It is an effective tool to cope with the large disturbances. The core of the ADRC scheme is to treat all the uncertain parts of the system model as an augmented system state, and then utilize an extended state observer (ESO) to estimate it for compensation [15]. To overcome the practical issues of the nonlinear ADRC, Prof. Gao proposed a linear ADRC to simplify the design process based on bandwidth parameterization [16]. However, the linearized version encounters the selection limitation of bandwidth. Therefore, the effect of the disturbances cannot be eliminated completely, resulting in a limited control accuracy for that the feedback controller comprised of proportional (P) and differential (D) terms has a poor disturbance rejection.

Motivated by the above difficulties and limitations, a robust control scheme by combining the IMC [17], the SMC [12], and the ESO [18] is proposed in this paper for the high-precision motion control system. The proposed approach employs the IMC rules with a 2-degree-of-freedom (2DOF) structure to design the linear feedback controller, aiming at achieving a specified tracking performance. By establishing the sliding mode dynamics of the IMC system, a SMC-based robust controller is introduced to enhance the robust performance. To avoid excessive control gains of the

M. Zhang is an assistant professor at the Department of Biomedical Engineering, Southern University of Science and Technology, Shenzhen, 518055, Guangdong, China (Corresponding author; email: <u>zhangmm@</u> <u>sustech.edu.cn</u>).



Fig. 1. The schematic diagram of the motor drive system.

continuous SMC, a linear ESO is constructed to diminish the gap between the controlled plant and the system model by disturbance compensation. The controller analysis is performed via a Lyapunov approach. Theoretical results illustrate the specified tracking performance and the uniformly ultimately stable property of the proposed approach.

This novel control strategy elegantly integrates the IMC, the SMC and the ESO for the controller design and performance analysis, and its effectiveness was validated by experiments conducted on a motor drive platform. That is rarely reported in the literature and can be viewed as the major contribution of this article.

The remainder of this paper is arranged as follows. In section II, the modeling of motion system is presented and the control objective is formulated. The proposed controller design with theoretical analysis is presented in section III. Comparative experiment results are provided in section IV. Conclusion is finally included in Section V

II. MODELING AND PROBLEM FORMULATION

The motion system studied in this paper is a servo motor directly driving an inertia load, since the motor is the basic component in the robot and automation systems. Fig. 1 shows the schematic diagram of the motor drive system. In the system modeling, the dynamics of current-loop can be ignored for its much faster response than the outer position-loop. The dynamic model of the system can be simplified as

$$J\ddot{\theta}(t) = u(t) - B\dot{\theta}(t) - D(t) \tag{1}$$

where, J and B are the system parameters related to the inertia load and the friction damping factor, respectively, θ is the angular position of the motor shaft, u denotes the control torque, D(t) denotes the unknown part of system dynamics.

System (1) can be presented in a state-space expression as

$$\begin{cases} \dot{x}_{1} = x_{2} \\ \dot{x}_{2} = \phi_{1}u - \phi_{2}x_{2} - d(t) \end{cases}$$
(2)

where, $x_1 = \theta$ and $x_2 = \dot{\theta}$ denote the system states, $\phi_1 = 1/J$ $\phi_2 = B/J$, and d(t) = D(t)/J.

The control objective is to develop a control law for the high-precision position tracking control of the system (2) regardless of the existence of bounded uncertainties and unknown disturbances. The control approach is expected to be simple and intuitive, as well as easy implementation.

Assumption 1. The system parameters are positive constant satisfying

$$0 < \phi_{1\min} \le \phi_1 \le \phi_{1\max} < +\infty; \ 0 < \phi_{2\min} \le \phi_2 \le \phi_{2\max} < +\infty$$
(3)

Assumption 2. The disturbance dynamics is unknown but bounded by

$$|d(t)| \le d_{\max} < +\infty; \tag{4}$$

III. CONTROLLER DESIGN

A. PID control based on IMC rules

According to the IMC rules, the first step for PID controller design is to obtain the nominal model $G_n(s)$ of the controlled plant (1), which can be expressed as

$$G_n(s) = \frac{1}{(J_n s + B_n)s} \tag{5}$$

where J_n and B_n denote the estimated or nominal parameters. The desired trajectory x_{1d} is defined as

$$x_{1d}(s) = f(s)R(s); f(s) \triangleq 1/(\lambda s + 1)$$
 (6)

where, *R* denotes the reference trajectory, f(s) represents the specified low-pass filter with a filter constant λ .

As presented in [4] and [5], the IMC filter $f_D(s)$ is given by

$$f_D(s) = (2\lambda s + 1) / (\lambda s + 1)^2 \tag{7}$$

The PID controller via the IMC rules with a 2DOF structure is designed as

$$C(s) = \frac{f_D(s)G_n^{-1}(s)}{1 - f_D(s)} = \frac{2J_n}{\lambda s}(s + \frac{B_n}{J_n})(s + \frac{1}{2\lambda})$$
(8)

To reduce the overshoot, the set-point filter F(s) is given by

$$F(s) = \frac{f(s)}{f_D(s)} = \frac{\lambda s + 1}{2\lambda s + 1}$$
(9)

The filtered reference $R_{\rm F}$ is defined as

$$R_F(s) = F(s)R(s) \tag{10}$$

B. Robust control scheme combining IMC and SMC We can define an integral sliding surface as

$$z_{2} = \dot{z}_{1} + (k_{a} + 2k_{b})z_{1} + 2k_{a}k_{b}\int z_{1}dt = x_{2} - x_{2eq};$$

$$x_{2eq} \triangleq \dot{x}_{1d} - (k_{a} + 2k_{b})z_{1} - 2k_{a}k_{b}\int z_{1}dt$$
(11)

where, $z_1 = x_1 - x_{1d}$ denotes the tracking error, $k_a = B_n/J_n$ and $k_b = 1/(2\lambda)$ are control gains.

By defining the controller input error $e_1 = x_1 - \Delta R$, one has

$$e_1 = z_1 - \Delta R; \ \Delta R \triangleq R_F - x_{1d}; \tag{12}$$

The robust controller combining the IMC and the SMC technique is designed as

$$u = u_{IMC} + u_R / \phi_{1n};$$

$$u_{IMC} = -k_c [\dot{e}_1 + (k_a + k_b)e_1 + k_a k_b \int e_1 dt]$$
(13)

where, u_{IMC} denotes the output torque of the IMC-based PID controller (8) and $k_c = 2J_n/\lambda$, u_R is the SMC-based robust control law and will be designed later.

By differentiating (11) and applying (2), (12) and (13), the dynamics of z_2 can be given by

$$\dot{z}_{2} = -\frac{1}{\lambda}z_{2} + \frac{2}{\lambda}[\Delta \dot{R} + (k_{a} + k_{b})\Delta R + k_{a}k_{b}\int\Delta Rdt]$$

$$-\ddot{x}_{1d} - k_{a}x_{1d} + u_{R} + \varphi^{T}\tilde{\phi} - d$$
(14)

where, $\varphi \triangleq [u, -x_2]^T$, $\tilde{\phi}$ is defined as

$$\tilde{\phi} = [\phi_1 - \phi_{1n}, \phi_2 - \phi_{2n}]^{\mathrm{T}}$$
 (15)

Based on the standard SMC theory, the robust control law u_{R} is designed as a signum function.

$$u_R = -k_g sign(z_2) \tag{16}$$

where, k_g is a positive control gain which is large enough such that the following condition is satisfied.

$$k_g \ge \left\| \phi_M \right\| \left\| \varphi \right\| + d_{\max} \tag{17}$$

where, $\phi_{\rm M} = [\phi_{\rm 1max} - \phi_{\rm 1min}, \phi_{\rm 2max} - \phi_{\rm 2min}]^T$.

Theorem 1: Given that assumptions $1 \sim 2$ holds, the control law (13) with (16) ensures the globally exponential stability of the system (2). The Lyapunov function selected as

$$V_s(t) \triangleq \frac{1}{2} z_2^2 \tag{18}$$

Satisfies

$$V_s(t) \le V_s(0)e^{-\gamma t}; \quad \forall t \ge 0 \tag{19}$$

where $\gamma = 2/\lambda$.

proof: From (6), (9) and (10), we have

$$x_{1d}(s) = \frac{f(s)}{F(s)} R_F(s) = \frac{2\lambda s + 1}{(\lambda s + 1)^2} R_F(s)$$
(20)

By calculating the inverse Laplace transform of (20), and noting that $\Delta R \triangleq R_F - x_{1d}$, one has

$$\lambda^2 \ddot{x}_{1d} = 2\lambda \Delta \dot{R} + \Delta R \tag{21}$$

$$\lambda^2 \dot{x}_{1d} = 2\lambda \Delta R + \int \Delta R dt \tag{22}$$

Applying $k_a = \phi_{2n}$ and $k_b = 1/(2\lambda)$, we have

$$\frac{2}{\lambda} [\Delta \dot{R} + (k_a + k_b)\Delta R + k_a k_b \int \Delta R dt] = \ddot{x}_{1d} + k_a \dot{x}_{1d} \qquad (23)$$

Substituting (23) into (14), we have

$$\dot{z}_2 = -\frac{1}{\lambda} z_2 + u_R + \varphi^T \tilde{\phi} - d \tag{24}$$

From (18) and (24), the time derivative of V_s is given by

$$\dot{V}_s = -\frac{1}{\lambda}z_2^2 + z_2(u_R + \varphi^T\tilde{\phi} - d)$$
⁽²⁵⁾

Applying (16) and noting the condition (17), we have

$$\dot{V}_s \le -\frac{2}{\lambda} V_s \tag{26}$$

Integrating (26) infers to (19), which ensures the globally exponential stability of the control system.

Remark 1: Theorem 1 illustrates that the robust IMC scheme (13) based on the SMC achieves a globally exponential stability of the control system. However, due to the drawbacks of actual implementation, the signum function (16) causes a harmful chattering and may deteriorate the control accuracy. To attenuate the chattering, a continuous approximation is usually applied, e.g., the control law u_R is revised as a simple saturation function.

$$u_{R} = -k_{g}sat(z_{2}); \ sat(z_{2}) \triangleq \begin{cases} z_{2} \ / \varepsilon & if \ |z_{2}| \le \varepsilon \\ sign(z_{2}) & others \end{cases}$$
(27)

This continuous solution only achieves the globally uniform ultimate boundedness of the tracking error, and its upper bound depends on the parameter ε . However, too small ε leads to a high-feedback control gain in the boundary layer, which will intensify the chattering. If a large ε is employed, this control law will do little for the control problem of uncertainties. The tracking accuracy is thusly limited.

C. Linear ESO design

The lumped disturbance of the system model is defined as $\Delta(t) = (\phi_1 - \phi_{1n})u - (\phi_2 - \phi_{2n})x_2 - d$, where $\phi_{1n} = 1/J_n$ and $\phi_{2n} = B_n/J_n$. Define $x_3 = \Delta(t)$ as the extended system state, and let $h(t) = \dot{\Delta}(t)$, then, we can extend the dynamics (2) as

$$\begin{cases} \dot{x}_{1} = x_{2} \\ \dot{x}_{2} = \phi_{1n}u - \phi_{2n}x_{2} + x_{3} \\ \dot{x}_{3} = h(t) \end{cases}$$
(28)

It is easy to know that the above state-space equation is observable. We can design a linear ESO as

$$\begin{cases} \dot{\hat{x}}_{1} = \hat{x}_{2} - 3\omega_{0}(\hat{x}_{1} - x_{1}) \\ \dot{\hat{x}}_{2} = \phi_{1n}u - \phi_{2n}x_{2} + \hat{x}_{3} - 3\omega_{0}^{2}(\hat{x}_{1} - x_{1}) \\ \dot{\hat{x}}_{3} = -\omega_{0}^{3}(\hat{x}_{1} - x_{1}) \end{cases}$$
(29)

where, \hat{x}_1 , \hat{x}_2 and \hat{x}_3 denote the estimates of system states, ω_0 denotes the bandwidth parameter of the ESO.

Subtracting (29) from (28) infers to

$$\begin{aligned}
& \left\{ \begin{aligned} \dot{\tilde{x}}_{1} &= \tilde{x}_{2} - 3\omega_{0}\tilde{x}_{1} \\
& \dot{\tilde{x}}_{2} &= \tilde{x}_{3} - 3\omega_{0}^{2}\tilde{x}_{1} \\
& \dot{\tilde{x}}_{3} &= h(t) - \omega_{0}^{3}\tilde{x}_{1} \end{aligned} \right.
\end{aligned}$$
(30)

where, $\tilde{x}_i = x_i - \hat{x}_i$ (*i* = 1, 2, 3).

Lemma 1 [18]: Assuming that h(t) is bounded, there exist constants $\sigma_i > 0$ and finite time $T_1 > 0$ such that

$$\left|\tilde{x}_{i}\right| \leq \sigma_{i}, \ \sigma_{i} = \mathcal{O}(\frac{1}{\omega_{0}^{c}}), \ i = 1, 2, 3, \ \forall t \geq T_{1}$$
(31)



Fig. 2. The diagram block of the composite controller combining IMC, SMC and ESO

for some positive integer *c*.

Remark 2: We can know from Lemma 1 that in the presence of the disturbances with bounded differential, the upper limits of the estimation errors \tilde{x}_i can be made sufficiently small by tuning ω_0 . Since the estimated state \tilde{x}_3 is applied to suppress the effect of various disturbances, the ESO can compensate most of the deviations between the process and the nominal model. Due to the model-based control structure of the robust IMC scheme, the addition of the ESO will greatly alleviate the burden on the IMC, and thusly enhance the control ability to disturbances.

D. Robust control scheme combining IMC, SMC and ESO

Shown in Fig. 2, the composite controller based on IMC, SMC and ESO is proposed as

$$u = u_{IMC} + (u_R + u_{ESO}) / \phi_{1n}; \quad u_{ESO} = -\hat{x}_3; \quad (32)$$

where, u_{ESO} denotes the compensation law from the extended state estimation of (29).

By differentiating (11) and applying (2), (12) and (32), the dynamics of z_2 is deduced as

$$\dot{z}_{2} = -\frac{1}{\lambda}z_{2} + \frac{2}{\lambda}[\Delta \dot{R} + (k_{a} + k_{b})\Delta R + k_{a}k_{b}\int\Delta Rdt] - \ddot{x}_{1d} - k_{a}x_{1d} + u_{R} + \tilde{x}_{3}$$
(33)

Since the exact bound of \tilde{x}_3 cannot be known a prior, the constraint like (17) cannot be pre-specified. Thus, a robust linearized control law satisfying the following condition

$$z_2(u_r + \tilde{x}_3) \le \varepsilon \sigma_3^2 \tag{34}$$

is employed by

$$u_r = -z_2 / (4\varepsilon) \tag{35}$$

Theorem 2: Considering the bounded h(t), the composite controller (32) with (29) and (35) ensures that all the system signals are uniformly ultimately bounded. Furthermore, after the finite time T_1 , z_2 satisfies

$$z_{2}(t) \leq e^{-\eta \Delta t} z_{2}(T_{1}) + \frac{\sigma_{3}}{\eta} (1 - e^{-\gamma \Delta t}), \ \forall t \geq T_{1}$$
(36)

and the Lyapunov function V_s ($V_s = z_2^2/2$) is bounded by

$$V_s(t) \le e^{-\gamma \Delta t} V_s(T_1) + \frac{\varepsilon \sigma_3^2}{\gamma} (1 - e^{-\gamma \Delta t}), \ \forall t \ge T_1$$
(37)



Fig. 3. Experiment platform

where, $\eta = 1/\lambda + 1/(4\varepsilon)$.

proof: By substituting (23) into (33), and applying (35), we have

$$\dot{z}_2 = -(\frac{1}{\lambda} + \frac{1}{4\varepsilon})z_2 + \tilde{x}_3 \tag{38}$$

By applying Lemma 1, one has

$$\dot{z}_2 \le -(\frac{1}{\lambda} + \frac{1}{4\varepsilon})z_2 + \sigma_3, \ \forall t \ge T_1$$
(39)

Integrating (39) from T_1 to t infers to (36).

From (39), the time derivative of the Lyapunov function (18) can be derived as

$$\dot{V}_{s} \leq -\frac{1}{\lambda} z_{2}^{2} - z_{2} (\frac{1}{4\varepsilon} z_{2} - \sigma_{3}) \leq -\frac{2}{\lambda} V_{s} + \varepsilon \sigma_{3}^{2}$$
(40)

Integrating (40) from T_1 to t yields (37). Therefore, the uniformly ultimately stable property can be ensured.

Remark 3: By comparing the results in (19) and (37). It seems that the controller (32) with (35) achieves a worse performance than the controller (13) with (16). However, in the developed controller (32), most of the lumped disturbance is compensated, which can avoid high-gain feedback control for high tracking accuracy when large disturbances are presented.

Since the ESO specifies the behaviors of \tilde{x}_3 and the IMC combining the SMC defines the closed-loop dynamics, all the virtues of the three control techniques (IMC, SMC and ESO), such as simplicity, intuition and effectiveness, can be accessible in the proposed composite controller. In this way, a novel control framework can be established for the control system design and analysis.

IV. EXPERIMENT VERIFICATION

A. Experiment Setup

The experimental verification platform is shown in Fig. 3. The detailed configuration can be found in Ref. [4] and [5]. The plant parameters are estimated as $J_n = 1.5 \times 10^{-4} \text{ kg} \cdot \text{m}^2$ and $B_n = 1.8 \times 10^{-3} \text{ Nm/(rad/s)}$, leading to the nominal model as



Fig. 4. The S-curve motion trajectory

 TABLE I

 TRACKING PERFORMANCE UNDER DISTURBANCES (MAE AND IAE)

Schemes	Step disturbance		Sine disturbance	
	MAE	IAE	MAE	IAE
2DOF-IMC-PID	6.1×10 ⁻³	6.4×10 ⁻³	9.1×10 ⁻³	1.1×10 ⁻²
RIMC-SMC	4.5×10 ⁻³	4.6×10 ⁻³	6.5×10 ⁻³	7.3×10 ⁻³
RIMC-SMC-ESO	5.8×10 ⁻⁴	4.6×10 ⁻⁴	9.6×10 ⁻⁴	1.0×10 ⁻³

$$G_n(s) = \frac{1}{(J_n s + B_n)s} = \frac{1}{(1.5 \times 10^{-4} s + 1.8 \times 10^{-3})s}$$
(41)

The low-pass filter f(s) = 1/(0.0025s+1) is selected to prescribe the set-point tracking. The following controllers are design for comparison:

- 1) **2DOF-IMC-PID:** This control strategy is designed by the 2DOF-IMC rules, which is presented in (8), leading to the PID control gains as $k_a = 12$, $k_b = 200$, $k_c = 0.12$, and the reference prefilter as F(s) = (0.0025s+1)/(0.005s+1).
- 2) **RIMC-SMC:** This robust controller (16) is designed by combining IMC and SMC, where the same control gains to the 2DOF-IMC-PID are applied for fair comparison. The boundary conditions are designed as $[\phi_{1\min}, \phi_{2\min}]^T = [3000, 1]^T$ and $[\phi_{1\max}, \phi_{2\max}]^T = [15000, 120]^T$ for parameter uncertainties, and $d_{\max} = 0.5/J$ for the added disturbance.
- 3) **RIMC-SMC-ESO:** This is the proposed control strategy (32) combining IMC, SMC and ESO, in which the same controller gains and boundary conditions to the RIMC-SMC are applied. The bandwidth of the ESO is selected as $\omega_0 = 300$.

Two performance indices, including maximum absolute error (MAE = max{ $|z_1|$ }) and integral absolute error (IAE =

 $\int |z_1| dt$), are applied to evaluate the controller performance.

B. Position tracking experiments under disturbances

Shown in Fig. 4, the S-curve motion trajectory is employed in this testing case. Two types of disturbances (ramp and sine) are added for the performance test of the three controllers. The slope of the ramp disturbance is selected as 1.0 Nm/s. The sine type is given by $0.2[1 - \cos(4\pi t)]$ Nm.



Fig. 5. Position tracking experiments under ramp disturbance



Fig. 6. Position tracking experiments under sine disturbance

Fig. 5 shows the tracking error curves under the ramp disturbance and Fig. 6 is under the sine disturbance, with their performance indices (MAE and IAE) presented in table I. We can know that comparing with the 2DOF-IMC-PID, the RIMC-SMC has a relatively better tracking accuracy. This implies that adding the SMC law to the IMC scheme can better enhance the closed-loop control ability to reject disturbance. However, when the robust control law improves the tracking accuracy, or introduces the chattering.

Figs. 5~6 also shows that the time-varying disturbances can be exactly observed by the ESO. It suggests that most of the added disturbance can be compensated with the observed value. As a result, when the ESO is integrated into the RIMC-SMC for ADRC, a better tracking accuracy is obtained without the increase of chattering. These comparative results demonstrate the effectiveness of the proposed controller by combining IMC SMC and ESO on the disturbance rejection capability.

C. Position tracking experiments under uncertainties

The motion trajectory shown in Fig. 7 is applied to make the effect of the uncertainties on the tracking performance more obvious. To evaluate the robust performance, the following parametric uncertainties are designed.

- 1) **Case A**: The control input *u* in this case is modified as 0.5u. From the model (2), this case equivalently alters ϕ_1 into $0.5\phi_1$ for the robustness test.
- 2) **Case B**: By adding $-0.01x_2$ to *u*, this case equivalently alters ϕ_2 into $0.01\phi_1 + \phi_2$ for another parameter variation.



Fig. 7. Motion trajectory for robustness test

TABLE II TRACKING PERFORMANCE UNDER UNCERTAINTIES (MAE AND IAE)

Schemes	Case A		Case B	
	MAE	IAE	MAE	IAE
2DOF-IMC-PID	0.0120	0.0254	0.0433	0.0846
RIMC-SMC	0.0089	0.0173	0.0290	0.0566
RIMC-SMC-ESO	0.0019	0.0025	0.0042	0.0075

The tracking error curves of case A is depicted in Fig. 8 and case B is in Fig. 9, with the MAE and the IAE summarized in table II. As seen, the RIMC-SMC obtains a relative higher tracking accuracy comparing with the 2DOF-IMC-PID. This implies that the former can better reduce the effect of parameter variations than the later, thus can better improve the closed-loop robustness. Even though, the RIMC-SMC-ESO yields a much higher tracking precision by adding the ESO to the RIMC-SMC. Since the parameter variations is treated a part of the lumped disturbance, the ESO can alleviate the burden of the robust controller by disturbance compensation, such that the control system robustness to uncertainties can be further improved. Due to the effective combination of IMC, SMC and ESO, the tracking accuracy of the proposed RIMC-SMC-ESO is the best among the comparative controllers.

V. CONCLUSION

In this paper, by combining the IMC rules, the SMC technique and the linear ESO, a novel, simple, effective and intuitive framework is established for motion control systems driven by electric motors. In the proposed approach, the SMC and the linear ESO, which are expected to achieve an improved robustness and disturbance rejection, respectively, are applied to the IMC scheme for a higher tracking control accuracy. The analysis of the error dynamics is performed via the Lyapunov method, which ensures the uniformly ultimately bounded stability regardless of internal uncertainties and external disturbances. Experimental results suggest that a more excellent tracking accuracy can be obtained by applying the SMC and the ESO to the IMC scheme, which has verified the superiority of the proposed composite controller.

REFERENCES

- P. M. Wensing, A. Wang, S. Seok, D. Otten, J. Lang, and S. Kim, [1] "Proprioceptive Actuator Design in the MIT Cheetah: Impact Mitigation and High-Bandwidth Physical Interaction for Dynamic Legged Robots," IEEE Trans. Robot., vol. 33, no. 3, pp. 509-522, Jun 2017
- Q. Miao, A. McDaid, M. Zhang, P. Kebria, and H. Li, "A three-stage [2] trajectory generation method for robot-assisted bilateral upper limb training with subject-specific adaptation," Robot. Auton. Syst., vol. 105, pp. 38-46, Jul. 2018.



Fig. 8. Tracking error curves under case A



Fig. 9. Tracking error curves under case B

- M. Zhang, S. Zhang, A. McDaid, C. Davies, and S. Xie, "Automated [3] objective robot-assisted assessment of wrist passive ranges of motion," J. Biomech., vol. 73, pp. 223-226, May 2018.
- P. Li and G. Zhu, "Robust internal model control of servo motor based [4] on sliding mode control approach," ISA Trans., vol. 93, pp. 199-208, Oct. 2019
- [5] P. Li and G. Zhu, "IMC-based PID control of servo motors with extended state observer," Mechatronics, vol. 62, no. 102252, Oct. 2019.
- [6] J. Yao, Z. Jiao, and D. Ma, "Adaptive Robust Control of DC Motors With Extended State Observer," IEEE Trans. Ind. Electron., vol. 61, no. 7, pp. 3630-3637, Jul 2014.
- S. Saxena and Y. V. Hote, "Advances in Internal Model Control [7] Technique: A Review and Future Prospects," IETE Tech. Rev., vol. 29, no. 6, pp. 461-472, Nov-Dec 2012.
- [8] Q. Jin and Q. Liu, "Analytical IMC-PID design in terms of performance/robustness tradeoff for integrating processes: From 2-Dof to 1-Dof," J. Process Control, vol. 24, no. 3, pp. 22-32, Mar. 2014.
- Q. Zhu, Z. Yin, Y. Zhang, J. Niu, Y. Li, and Y. Zhong, "Research on [9] Two-Degree-of-Freedom Internal Model Control Strategy for Induction Motor Based on Immune Algorithm," IEEE Trans. Ind. Electron., vol. 63, no. 3, pp. 1981-1992, Mar 2016.
- [10] X. Sun, L. Chen, H. Jiang, Z. Yang, J. Chen, and W. Zhang, "High-Performance Control for a Bearingless Permanent-Magnet Synchronous Motor Using Neural Network Inverse Scheme Plus Internal Model Controllers," IEEE Trans. Ind. Electron., vol. 63, no. 6, pp. 3479-3488, Jun 2016.
- [11] S. Li and H. Gu, "Fuzzy Adaptive Internal Model Control Schemes for PMSM Speed-Regulation System," IEEE Trans. Ind. Inform., vol. 8, no. 4, pp. 767-779, Nov. 2012.
- V. Utkin, J. Guldner, and J. Shi, Sliding mode control in [12] electro-mechanical systems, 2 ed. Boca Raton, FL: CRC press, 2009.
- X. Zhang, L. Sun, K. Zhao, and L. Sun, "Nonlinear Speed Control for PMSM System Using Sliding-Mode Control and Disturbance Compensation Techniques," IEEE Trans. Power Electron., vol. 28, no. 3, pp. 1358-1365, Mar. 2013.
- [14] J. Han, "From PID to Active Disturbance Rejection Control," IEEE Trans. Ind. Electron., vol. 56, no. 3, pp. 900-906, Mar. 2009.
- R. Madonski and P. Herman, "Survey on methods of increasing the [15] efficiency of extended state disturbance observers," ISA Trans., vol. 56, pp. 18-27, May. 2015.
- [16] Z. Gao, "Scaling and bandwidth-parameterization based controller tuning," in Proc. Amer. Control Conf, 2003, vol. 6, pp. 4989-4996.
- [17] D. E. Rivera, M. Morari, and S. Skogestad, "Internal model control. 4. PID controller design," Ind. Eng. Chem. Process Des. Dev., vol. 25, no. 1, pp. 252-265, Jan. 1986.
- [18] Q. Zheng, L. Gao, and Z. Gao, "On stability analysis of active disturbance rejection control for nonlinear time-varying plants with unknown dynamics," in Proc. IEEE Conf. Decision Control, 2007, pp. 3501-3506.