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Nonlinear Balance Control of an Unmanned Bicycle: Design and Experiments

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***Abstract*— In this paper, nonlinear control techniques are exploited to balance an unmanned bicycle with enlarged stability domain. We consider two cases. For the first case when the autonomous bicycle is balanced by the flywheel, the steering angle is set to zero, and the torque of the flywheel is used as the control input. The controller is designed based on the Interconnection and Damping Assignment Passivity Based Control (IDA-PBC) method. For the second case when the bicycle is balanced by the handlebar, the bicycle's velocity is high, and the flywheel is turned off. The angular velocity of the handlebar is used as the control input and the balance controller is designed based on feedback linearization. In these cases, the global stability of the closed-loop unmanned bicycle is theoretically proved based on Lyapunov theory. The experiments are conducted to validate the efficacy of the proposed nonlinear balance controllers.**

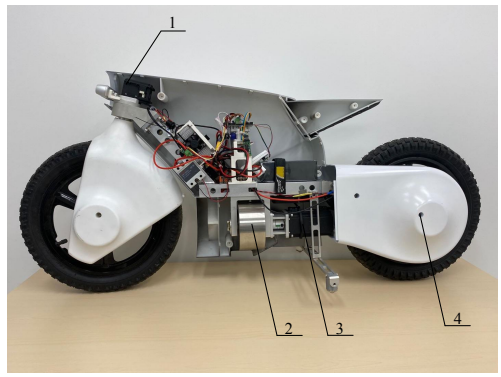

Fig. 1. The unmanned bicycle.

I. INTRODUCTION

Compared with other ground vehicles, bicycles are environmentally friendly, cheap and versatile. Therefore, they are one of the most popular transportation means over the last two centuries. During the 20th century, most efforts were concentrated on making them easier to ride. Moving into the 21st century, the rapid development of computing and sensing technologies makes autonomous riding a popular and significant research topic. The huge market for bicycles creates tremendous opportunities for unmanned bicycles. Because there are only two contact points between the bicycle and the support road, as an inverted pendulum, bicycles are unstable systems. Therefore, balance control is fundamentally important, yet challenging, for autonomous riding of bicycles, and many researchers have made some progresses on this topic.

The dynamic modeling and balance control of bicycles are reviewed in [1]–[4]. As stated in [1], it is difficult to balance a low speed bicycle via steering handlebar. In this case, the steering angle is set to zero, and other auxiliary balance equipment, such as flywheels, are required to balance the autonomous bicycle. However, when the autonomous bicycle is moving forward at a sufficiently high speed, the bicycle can be balanced by steering the handlebar [5], [6]. In this case, the steering angular velocity is the control input. Therefore, the balance problem can be divided into two cases.

When the autonomous bicycle is balanced by other auxiliary balance equipment, in [7]–[12], gyroscopic balance is applied. Yetkin *et al.* [7] design a controller to regulate the gimbal angle based on sliding mode control. In [8], the dynamic model is linearized and discretized. Then a model predictive controller is designed to balance the bicycle. In [9], based on the linear dynamic model of bicycles with a gyroscope, a mixed H_2/H_∞ controller is designed, and the particle swarm optimization algorithm is applied to determine the coefficients in the controller. In [10], the root locus method is applied to design a balance controller. In [11], the dynamic model is also linearized and the pole-zero placement method is applied to design the balance controller. In [12], a nonlinear controller is also designed. However, it also cannot guarantee the global stability of the closed-loop autonomous bicycle. Among the aforementioned references, all the designed controllers can only give local stability results. Besides, two flywheels, spinning in opposite direction, are needed to cancel the reactive torques on the yaw dynamics of the bicycle [8]. For each flywheel, two actuators are required to regulate the angular velocities around two orthogonal axes. These properties would increase the complexity of the balance equipment. In this paper, to overcome the first aforementioned problem, we propose a nonlinear controller based on IDA-PBC method, which is detailed in [13] and [14]. In order to simplify the auxiliary balance equipment, only one flywheel is used, and the reactive torque, the direction of which is opposite to the active torque of the flywheel, is applied to balance the bicycle.

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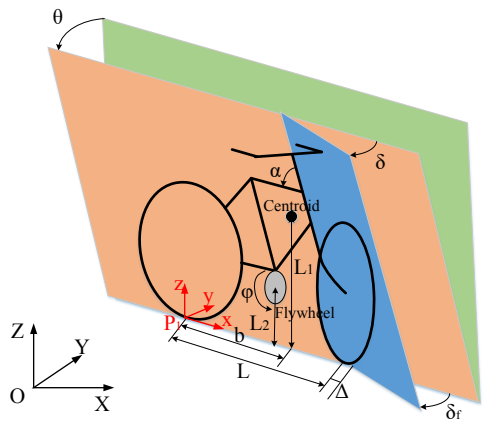


Fig. 2. Sketch of the bicycle.

When the autonomous bicycle is balanced by steering the handlebar, controllers are designed in [15]–[19]. In [15], based on a linearized bicycle model, the controller is designed via gain-scheduling techniques. Defoort *et al.* [16] apply second-order sliding mode control and disturbance observer to balance the bicycle. In [17], a PD controller is designed to track a straight line. In [18], a fuzzy controller is designed based on a linearized bicycle model. In [19], a linear controller combined with disturbance rejection term is designed. Among the aforementioned papers, most of them employ linear controllers, which can only give local results for stability and performance analysis. Although [16] designs a nonlinear controller, the effect of acceleration on the balance of the bicycle is not considered. In this paper, to address the aforementioned limitations, based on both bicycle's roll and acceleration dynamics, a nonlinear controller is designed via feedback linearization, which can guarantee the global stability. Here the global stability means that the bicycle can be balanced when the initial roll angle is within the range from $-\pi/2$ to $\pi/2$ radian.

The structure of the following contents is as follows. In Section II, the problem is formulated and the dynamic model of the bicycle is derived. When the autonomous bicycle is balanced by a flywheel, a nonlinear controller is designed based on IDA-PBC in Section III. The global stability of the closed-loop autonomous bicycle is also proved theoretically. In Section IV, when the autonomous bicycle is balanced by steering the handlebar, a nonlinear controller is designed, which can also guarantee the global stability. In Section V, experiments are conducted to show the performance of the proposed controllers. Some concluding remarks are made in Section VI.

II. PROBLEM FORMULATION AND DYNAMIC MODELING

As shown in Fig. 1, component 1 is a motor which can regulate the angular velocity of the handlebar. Component 2 is a flywheel, the torque of which can be controlled by motor 3. Component 4 is a motor which can control the angular velocity or torque of the rear wheel. In Fig. 2, $O-XYZ$ denotes the base frame. P_1-xyz is the frame, where P_1 is the contact point between the rear wheel and the ground, P_1x is the direction of the bicycle, and P_1z is vertical and upward. θ denotes the roll angle, ϕ denotes the rotation angle of the flywheel, δ denotes the steering angle and δ_f is the effective steering angle. V_x and V_y are longitudinal and lateral velocities respectively. V is the forward velocity of the bicycle. Let m_1 and m_2 denote the mass of the bicycle and the flywheel separately, and $m = m_1 + m_2$. I_1 and I_2 are moment of inertia of the bicycle and the flywheel. g denotes the gravity acceleration. Let $u_\delta = \dot{\delta}$ denote angular velocity of the steering angle, u_ϕ denote the torque applied to the flywheel, and u_v denote the propulsive force applied to the bicycle. Given the dynamic parameters of the bicycle, the nonlinear balance controllers will be designed in the following cases.

Case 1 (Balancing by the Flywheel): When $\delta = 0$, design a nonlinear controller to balance the roll angle of the bicycle to zero via regulating the torque applied to the flywheel u_ϕ.

Case 2 (Balancing by the Handlebar): When $V > 0$, design a nonlinear controller to balance the roll angle of the bicycle to θ_{eq} and regulate the velocity V to a desired value V_d by means of the angular velocity of the steering angle u_δ and the propulsive force u_v.

A. Dynamic Modeling

When δ is constant, the path of the bicycle is a circle. Let σ denote the curvature of this circle, which can be expressed as

$$\sigma = \tan(\delta_f)/L. \tag{1}$$

The relationship between δ and δ_f is

$$\tan(\delta_f)\cos(\theta) = \tan(\delta)\sin(\alpha). \tag{2}$$

Combining (1) and (2), one can obtain the following expression

$$\sigma = \frac{\tan(\delta)\sin(\alpha)}{\cos(\theta)L}. \tag{3}$$

Define u_σ as follows

$$\begin{aligned} u_\sigma &= \dot{\sigma} \\ &= \frac{\sin(\alpha)}{L}\left(\frac{\sec^2(\delta)u_\delta\cos(\theta) + \tan(\delta)\sin(\theta)\dot{\theta}}{\cos^2(\theta)}\right). \end{aligned} \tag{4}$$

Then we establish the dynamic model of the bicycle by means of Euler-Lagrange equation

$$\mathscr{L} = T - U, \tag{5a}$$

$$\frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathscr{L}}{\partial q_i} = \tau_i. \tag{5b}$$

In (5), T denotes the kinetic energy, U denotes the potential energy and τ_i denotes the external forces, which are described below.

The motion of a rigid body can be split into two parts: translational and rotational. For translational motion , the velocities of the bicycle's centroid and the flywheel's centroid

are

$$V_{xi} = V, \tag{6a}$$
$$V_{yi} = -V\sigma b - L_i\dot{\theta}\cos(\theta), \tag{6b}$$
$$V_{zi} = -L_i\dot{\theta}\sin(\theta) \quad (i = 1,2). \tag{6c}$$

The rotational velocities of the bicycle and the flywheel are

$$\omega_1 = \dot{\theta}, \tag{7a}$$
$$\omega_2 = \dot{\theta} + \dot{\phi}. \tag{7b}$$

Therefore, with (6) and (7), the kinetic energy can be expressed as

$$\begin{aligned} T = &\frac{1}{2}m_1(V_{x1}^2 + V_{y1}^2 + V_{z1}^2) + \frac{1}{2}m_2(V_{x2}^2 + V_{y2}^2 + V_{z2}^2) \\ &+ \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2. \end{aligned} \tag{8}$$

Further, the potential energy can be expressed as

$$U = (m_1L_1 + m_2L_2)g(\cos(\theta) + 1), \tag{9}$$

and the external force τ can be expressed as

$$\tau_\theta = -(m_1L_1 + m_2L_2)\cos(\theta)\sigma V^2 + \frac{mgb\Delta\sin(\alpha)}{L}\cos(\theta)\delta_f, \tag{10a}$$
$$\tau_\phi = u_\phi, \tag{10b}$$
$$\tau_v = u_v, \tag{10c}$$

where the first term of (10a) denotes the centrifugal force and the last term of (10a) denotes the torque produced by the effect of trail Δ [20].

Define the last term in (10a) as $\tau_\Delta(\theta, \delta_f)$. Substituting (8), (9) and (10) into (5), one can get the following dynamic model

$$\begin{aligned} &(m_1L_1^2 + m_2L_2^2 + I_1 + I_2)\ddot{\theta} + I_2\ddot{\phi} + (m_1L_1 + m_2L_2)b\sigma\cos(\theta)\dot{V} \\ &= (m_1L_1 + m_2L_2)g\sin(\theta) - (m_1L_1 + m_2L_2)\cos(\theta)\sigma V^2 \\ &+ \tau_\Delta(\theta, \delta_f) - (m_1L_1 + m_2L_2)bV\cos(\theta)u_\sigma, \end{aligned} \tag{11a}$$
$$I_2\ddot{\theta} + I_2\ddot{\phi} = u_\phi, \tag{11b}$$
$$\begin{aligned} &(m_1L_1 + m_2L_2)b\sigma\cos(\theta)\ddot{\theta} + (m + mb^2\sigma^2)\dot{V} = \\ &-(2mb^2V\sigma + (m_1L_1 + m_2L_2)b\cos(\theta)\dot{\theta})u_\sigma \\ &+ (m_1L_1 + m_2L_2)b\sigma\sin(\theta)\dot{\theta}^2 + u_v. \end{aligned} \tag{11c}$$

In the next two sections, the nonlinear balance controllers are designed in the following two cases: balancing by the flywheel and balancing by the handlebar.

III. BALANCING BY THE FLYWHEEL

When the bicycle is balanced by the flywheel, the steering angle δ is set to zero, which means $\sigma = 0$. In this case, the dynamic model (11) can be simplified into the following equation

$$(m_1L_1^2 + m_2L_2^2 + I_1 + I_2)\ddot{\theta} + I_2\ddot{\phi} = (m_1L_1 + m_2L_2)g\sin(\theta), \tag{12a}$$
$$I_2\ddot{\theta} + I_2\ddot{\phi} = u_\phi. \tag{12b}$$

Define $I_\theta = m_1L_1^2 + m_2L_2^2 + I_1$, $I_\phi = I_2$, $\lambda = (m_1L_1 + m_2L_2)g$, $M = diag(I_\theta, I_\phi)$, $q = [\theta, \theta + \phi]^{\mathrm{T}}$, $G = [-1, 1]^{\mathrm{T}}$ and $p = M\dot{q}$. Then the Hamiltonian form of (12) is

$$H = \frac{1}{2}p^{\mathrm{T}}M^{-1}p + \lambda(\cos(q_1) + 1), \tag{13a}$$
$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & I_{2\times2} \\ -I_{2\times2} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial q} \\ \frac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0_{2\times1} \\ G \end{bmatrix} u_\phi. \tag{13b}$$

In (13b), H is the total mechanical energy of the bicycle with the flywheel. $I_{2\times2}$ denotes the identity matrix of dimension 2.

We will employ the IDA-PBC control method of [13]. The IDA-PBC controller design method can be partitioned into two parts: energy shaping and damping injection [13], which means $u_\phi = u_{\phi,s} + u_{\phi,i}$. For energy shaping, the goal is to design $u_{\phi,s}$, such that the Hamiltonian form of the closed-loop system is as follows

$$H_d = \frac{1}{2}p^{\mathrm{T}}M_d^{-1}p + U_d(q), \tag{14a}$$
$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & M^{-1}M_d \\ -M_dM^{-1} & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H_d}{\partial q} \\ \frac{\partial H_d}{\partial p} \end{bmatrix} + \begin{bmatrix} 0_{2\times1} \\ G \end{bmatrix} u_{\phi,i}. \tag{14b}$$

In (14b), $M_d = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2 & \alpha_3 \end{bmatrix}$, $\alpha_1 > 0, \alpha_1\alpha_3 - \alpha_2^2 > 0$. Comparing (13) with (14), one can find that the following conditions hold

$$Gu_{\phi,s} = \frac{\partial H}{\partial q} - M_dM^{-1}\frac{\partial U_d(q)}{\partial q}, \tag{15a}$$
$$G^{\perp}(\frac{\partial H}{\partial q} - M_dM^{-1}\frac{\partial U_d(q)}{\partial q}) = 0. \tag{15b}$$

In (15b), $G^{\perp}$ is a vector that is orthogonal to G, i.e. $G^{\perp}G = 0$. From (15b), one can get the following partial differential equation equation for $U_d(q)$

$$\frac{\alpha_1 + \alpha_2}{I_\theta}\frac{\partial U_d(q)}{\partial q_1} + \frac{\alpha_2 + \alpha_3}{I_\phi}\frac{\partial U_d(q)}{\partial q_2} = -\lambda\sin(q_1). \tag{16}$$

The solution to (16) is

$$U_d(q) = \frac{\lambda I_\theta}{\alpha_1 + \alpha_2}(\cos(q_1) - 1). \tag{17}$$

In (17), in order to guarantee that $q_1 = 0$ is the minimum of $U_d(q)$, $\alpha_1 + \alpha_2 < 0$. According to (15a) and (17), $u_{\phi,s}$ can be expressed as

$$u_{\phi,s} = \frac{\lambda\alpha_2}{(\alpha_1 + \alpha_2)}\sin(q_1). \tag{18}$$

For damping injection, the aim is to design $u_{\phi,i}$, such that $\dot{H}_d \leq 0$. According to (14), after designing $u_{\phi,s}$ as (18) the expression of $\dot{H}_d$ is

$$\dot{H}_d = \left(\frac{\partial H_d}{\partial p}\right)^{\mathrm{T}} Gu_{\phi,i}. \tag{19}$$

Therefore, $u_{\phi,i}$ is designed as

$$u_{\phi,i} = -k_vG^{\mathrm{T}}\left(\frac{\partial H_d}{\partial p}\right) = -k_vG^{\mathrm{T}}M_d^{-1}p \tag{20}$$

Theorem 1: For the system (12) with the controller (18) and (20), we have the following convergence properties

$$\lim_{t\to+\infty}\theta(t)=0,\ \lim_{t\to+\infty}\dot{\phi}(t)=0. \tag{21}$$

Proof: Let H_d in (14) denote the candidate Lyapunov function. According to (19) and (20), $\dot{H}_d$ can be expressed as

$$\dot{H}_d=-k_v\left(G^{\mathrm{T}}M_d^{-1}p\right)^2\leq 0. \tag{22}$$

When $\dot{H}_d=0$, $q_1=0, p_1=0$ and $p_2=0$ can be inferred. According to LaSalle's invariance principle [21], (21) can be proved. ■

IV. BALANCING BY THE HANDLEBAR

When the autonomous bicycle is balanced by the handlebar, $V>0$ and the flywheel is turned off. Ignore the effect of trail $\tau_\Delta(\theta,\delta_f)$. Define the following matrices and vectors

$$M_l=\begin{bmatrix}(m_1L_1^2+m_2L_2^2+I_1+I_2) & (m_1L_1+m_2L_2)b\sigma\cos(\theta)\\ (m_1L_1+m_2L_2)b\sigma\cos(\theta) & (m+mb^2\sigma^2)\end{bmatrix},$$

$$G=\begin{bmatrix}(m_1L_1+m_2L_2)g\sin(\theta)-(m_1L_1+m_2L_2)\cos(\theta)\sigma V^2\\ (m_1L_1+m_2L_2)b\sigma\sin(\theta)\dot{\theta}^2\end{bmatrix},$$

$$B=\begin{bmatrix}-(m_1L_1+m_2L_2)bV\cos(\theta) & 0\\ -(2mb^2V\sigma+(m_1L_1+m_2L_2)b\cos(\theta)\dot{\theta}) & 1\end{bmatrix}.$$

In the case, according to (11), the dynamic model can be simplified as

$$M_l\begin{bmatrix}\ddot{\theta}\\ \dot{V}\end{bmatrix}=G+B\begin{bmatrix}u_\sigma\\ u_v\end{bmatrix}. \tag{24}$$

It is obvious that when $V>0$, the matrix B is invertible. In order to control the roll angle θ and the velocity V to the desired value θ_d and V_d, where V_d is positive and $\theta_d\in(-\pi/2,\pi/2)$, the following controller is designed

$$\begin{bmatrix}u_\sigma\\ u_v\end{bmatrix}=B^{-1}\left(-G+M_l\begin{bmatrix}v_1\\ v_2\end{bmatrix}\right), \tag{25a}$$

$$v_1=-k_{d1}\dot{\theta}-k_p(\theta-\theta_d), \tag{25b}$$

$$v_2=-k_{d2}(V-V_d). \tag{25c}$$

In (25), k_p, k_{d1}, and k_{d2} are positive. Combining (24) and (25), the dynamics of the closed-loop system is

$$\ddot{\theta}=-k_{d1}\dot{\theta}-k_p(\theta-\theta_d), \tag{26a}$$

$$\dot{V}=-k_{d2}(V-V_d) \tag{26b}$$

Theorem 2: For the system (24), with the controller (25), we have the following convergence properties

$$\begin{aligned}&\lim_{t\to+\infty}V(t)=V_d,\ \lim_{t\to+\infty}\theta(t)=\theta_d,\\ &\lim_{t\to+\infty}u_\sigma(t)=0,\ \lim_{t\to+\infty}u_v(t)=0.\end{aligned} \tag{27}$$

Proof: Define the candidate Lyapunov function as

$$L_y=\frac{k_p}{2}(\theta-\theta_d)^2+\frac{1}{2}\dot{\theta}^2+\frac{1}{2}(V-V_d)^2. \tag{28}$$

According to (26), the time derivative of (28) is

$$\dot{L}_y=-k_{d1}\dot{\theta}^2-k_{d2}(V-V_d)^2\leq 0. \tag{29}$$

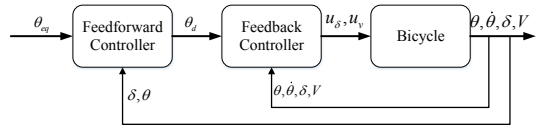


Fig. 3. Control scheme when the bicycle is balanced by the handlebar.

When $\dot{L}_y=0$, $V=V_d$, $\theta=\theta_d$, $\dot{\sigma}=0$ and $u_v=0$ can be inferred. According to the generalization of LaSalle's invariance principle [22], the theorem can be proved. ■

From (4) and (25), we can get the expression of u_δ as

$$u_\delta=\frac{\cos^2(\delta)}{\cos(\theta)}\left(\frac{L\cos^2(\theta)}{\sin(\alpha)}u_\sigma-\tan(\delta)\sin(\theta)\dot{\theta}\right). \tag{30}$$

Remark 1: In some cases, if the effect of $\tau_\Delta(\theta,\delta_f)$ cannot be ignored, the feedforward controller can be applied to eliminate the steady-state error, which can be expressed as

$$\theta_d=\frac{-\tau_\Delta(\theta_{eq},\delta_f)+k_p(m_1L_1^2+m_2L_2^2+I_1+I_2)\theta_{eq}}{k_p(m_1L_1^2+m_2L_2^2+I_1+I_2)}. \tag{31}$$

The control scheme is described in Fig.3.

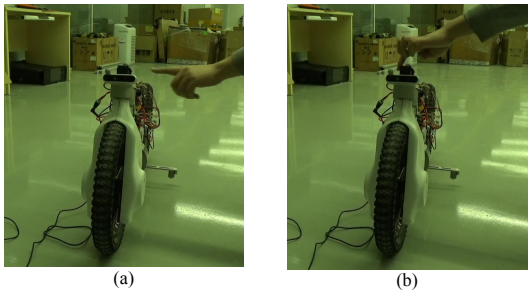
V. EXPERIMENTS

In this section, several experiments are conducted to show the performance of the proposed nonlinear controllers when the autonomous bicycle is balanced by the flywheel and by the handlebar respectively. For clarity, the controller designed in Section III is named FC and the controller designed in Section IV is named HC.

As shown in Fig.1, the handlebar is driven by a steering motor, and the steering angular velocity can be regulated. The flywheel is driven by a servo motor, the torque of which can be regulated. The rear wheel is also driven by a servo motor, both the velocity and the torque of which can be regulated. The roll angle of the bicycle is measured by the installed IMU. The embedded chip STM32H7 is applied as a calculator. The reference signal is sent by a infrared remote controller. In order to move along a clockwise circle, θ_{eq} should be positive, and vice versa. Parameters of the bicycle are listed as follows: $m_1=11kg, m_2=3.5kg, I_1=0.12418kg\cdot m^2, I_2=0.007882kg\cdot m^2, L_1=0.2316m, L_2=0.15m, L=0.7485m, b=0.3642m, \alpha=75^\circ, \Delta=4.6cm$.

A. Experiments for Balancing by the Flywheel

In order to validate the efficacy of the proposed FC, the following two experiments are conducted. Firstly, when the autonomous bicycle keeps stationary, the FC is applied. Fig. 4, (a) and (b) show that two opposite external disturbances are exerted to the bicycle. The results of this experiment are shown in Fig. 5. From Fig. 5 (b), one can find two disturbances at 20s and 40s. In Fig. 5 (d), in order to balance the bicycle, two torques are exerted to the flywheel simultaneously. Then the reactive torques are produced to balance the bicycle. From Fig. 5 (e), one can find that the angular velocity of the flywheel eventually converges to zero.



(a) (b)

Fig. 4. Experiment for the stationary bicycle. (a) External disturbance towards one direction. (b) External disturbance towards the opposite direction.

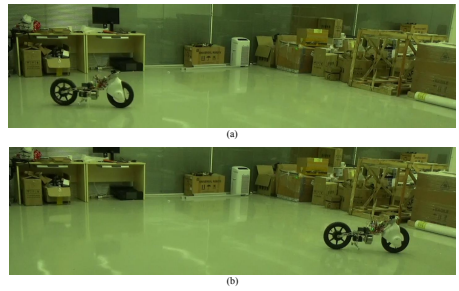


Fig. 6. Experiment for accelerating and braking. (a) The initial position (b) The final position.

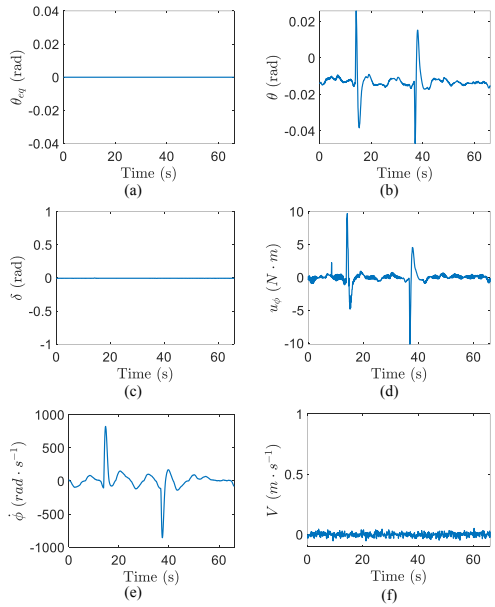


Fig. 5. The results of the experiment for the stationary scenario

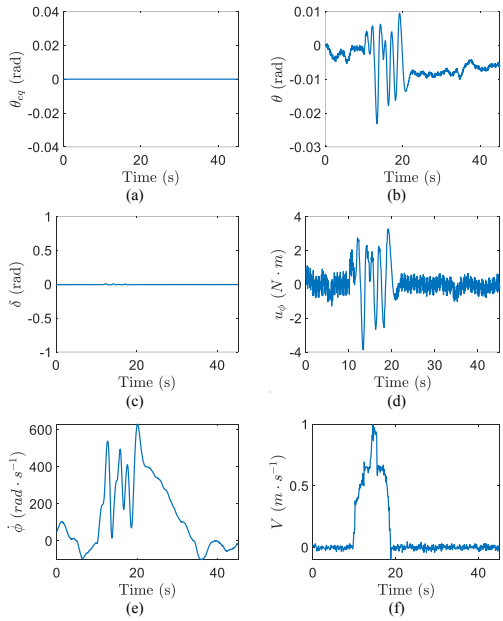


Fig. 7. The results of the experiment for accelerating and braking.

Secondly, the autonomous bicycle accelerates and brakes along a straight line, and the FC is applied. The velocity of the rear wheel is controlled by the velocity command of the servo motor. As shown in Fig. 6, the bicycle accelerates and brakes from the initial position to the final position. From Fig. 7 (b) and (f), when the bicycle is accelerating and braking, there are some disturbances moving the bicycle away from the equilibrium point. From Fig. 7 (d), when the bicycle is away from the equilibrium point, the torque of the flywheel changes correspondingly, balancing the bicycle to the equilibrium point. From Fig. 7 (d), one can find that the angular speed of the flywheel converges to zero finally.

From the aforementioned two experiments, one can conclude that when there are some external disturbances or the bicycle is accelerating and braking, the proposed nonlinear controller can balance the autonomous bicycle.

B. Experiments for Balancing by the Handlebar

For the FC, the desired roll angle can only be zero. For the HC, it can balance the bicycle to a non-zero desired roll angle, but the velocity of the bicycle must satisfy that $V > V_c$, where V_c is a positive velocity threshold. The higher the forward velocity, the more effective the proposed HC. Therefore, if the desired roll angle $\theta_{eq} = 0$, the FC is applied. If $\theta_{eq} \neq 0$ and $V > V_c$, the HC is applied. According to this control logic, we design the following accelerating-circle-braking experiment to test the performance of the proposed HC.

Firstly, the bicycle accelerates from standstill, and the FC is applied. Then a positive desired roll angle is sent to the bicycle, during which the HC is applied. Finally, the desired roll angle is reset to zero, the bicycle brakes and the FC is applied. Fig. 8 shows different moments when the bicycle is accelerating, moving along a circle and braking. The results of this experiment are shown in Fig. 9. As shown in 9 (a), (b) and (c), when a positive desired roll angle is sent to the bicycle, with the HC, the steering angle turns to the positive direction, and the roll angle is balanced to the desired position. As shown in Fig. 9 (d) and (e), during this time, the flywheel is turned off. Therefore, it shows that the autonomous bicycle can be balanced by the proposed nonlinear controller via steering the handlebar.

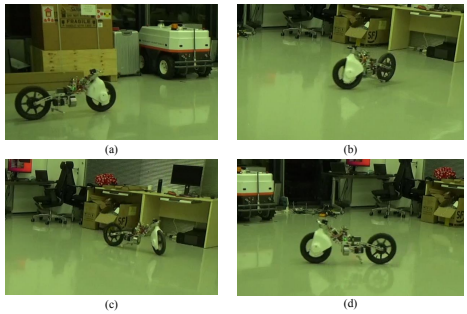


Fig. 8. Experiment for the accelerating-circle-braking. (a) Accelerating (b) Moving along a circle. (c) Moving along a circle. (d) Braking

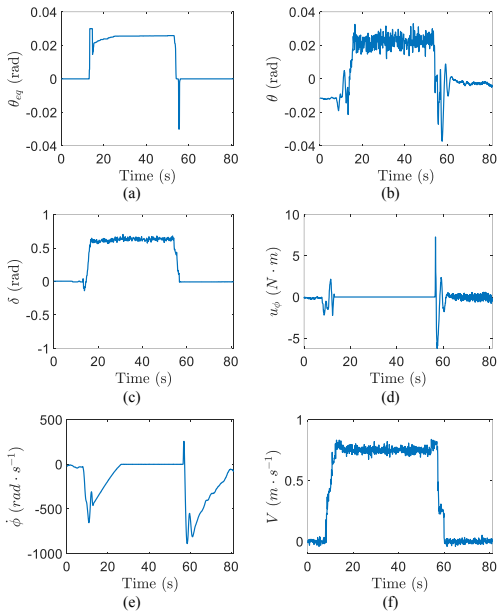


Fig. 9. The results of the experiment for accelerating-circle-braking.

VI. CONCLUSIONS

In this paper, two nonlinear controllers are presented to balance an unmanned bicycle with non-local stability guarantee. When the steering angle of the bicycle is zero, the flywheel is applied and the corresponding nonlinear controller is designed to balance the bicycle based on IDA-PBC. When the bicycle is moving forward, the nonlinear controller is designed based on the feedback linearization. The stability of the closed-loop bicycle system is also guaranteed by Lyapunov's direct method. The efficacy of the proposed nonlinear controllers has been validated by experiments. Our future work will be directed at generalizing the presented results to formation control of a group of unmanned bicycles using techniques in distributed nonlinear control [23]. Besides, the continuous-time robust dynamic programming [24] can be applied to handle the uncertain disturbances in the dynamics of the bicycle.

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