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Hand-Object Contact Force Synthesis for Manipulating Objects by Exploiting Environment

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***Abstract*— In this paper, we study the problem of computing grasping forces for quasi-static manipulation of large and heavy objects, by exploiting object-environment contacts. We present a general formulation of this problem as a Second-Order Cone Program (SOCP) that considers (i) contact friction constraints at the object-manipulator contacts and object-environment contacts, (ii) force/moment equilibrium constraints, and (iii) manipulator joint torque constraints. The SOCP formulation implies that the optimal grasping forces for manipulating objects with the help of the environment can be computed efficiently. Different optimization objectives like minimizing contact forces at the object-manipulator contacts or minimizing joint torques of manipulators can be considered. We evaluated our method by simulations of single-handed and dual-handed manipulation scenarios.**

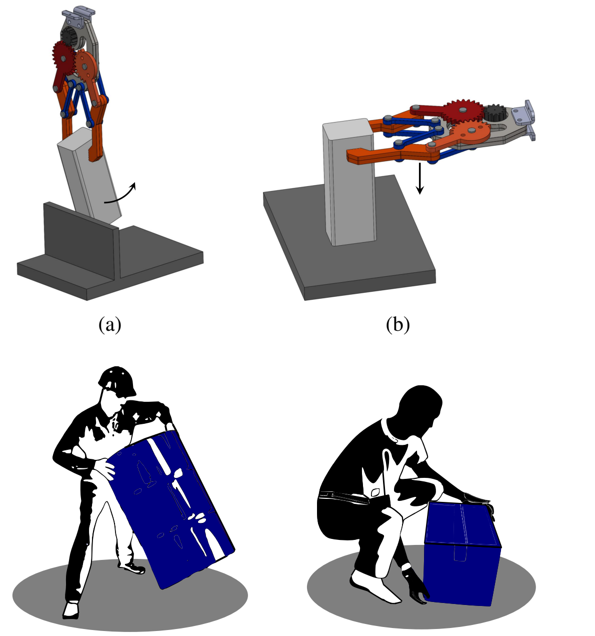

Fig. 1. (a)-(b) Examples of repositioning and reorienting an object grasped by a gripper. (c)-(d) Examples of reorienting and manipulating heavy objects.

I. Introduction

In the past few decades, significant advances have been made in the design and construction of dexterous robotic hands. Nevertheless, it is still difficult to utilize these hands for complex manipulation tasks except for pick-and-place grasping tasks. To increase ability of the robotic hands and manipulators to grasp and manipulate a wider range of objects, environment can be effectively exploited. Successful accomplishment of many tasks involve exploitation of contacts of the grasped objects with the environment [1]. In general, environment contact can be exploited in two main ways: (1) environment contacts as external forces for re-positioning and reorienting an object which is already grasped by a robotic hand or gripper (Fig. 1-a,b), [2],[3],[4] (2) environment contacts as essential supports for reorienting a heavy object or moving it from one position to another (Fig. 1-c,d). The former task can enable simple affordable grippers to do dexterous manipulation [2]. The latter task can enable the robotic arms to manipulate heavy or large objects effectively.

For moving heavy or large objects, a common strategy used (by humans) is to lift the object by pivoting on one edge and then, move it, while maintaining contact with the ground. This strategy exploits the contact of the object with the ground so that the arms do not need to carry the full weight, yet the object can be manipulated. Thus, one of the key technical issues in synthesis of such grasps is the development of algorithms to compute the optimum contact forces to be applied at the object-robot contacts. We call this problem the *grasping force synthesis problem* for manipulating heavy objects and it is the main focus of this paper. Intuitively, for any manipulator, an object is considered heavy, if the joint motors are not powerful enough to balance the weight of the object.

In grasping force synthesis, the grasping force has to be computed while considering some essential constraints, namely, (i) contact constraints, which are the friction constraints at the object-manipulator contacts and object-environment contacts, (ii) equilibrium constraints so that the object does not fall when being manipulated, and (iii) manipulator joint torque constraints, which are bounds on the maximum amount of torque that the manipulator actuators can apply to generate the contact forces.

In this paper, we present a general optimization formulation of the grasp synthesis problem (for heavy objects) that considers all the three constraints stated above. We show that the grasping force synthesis problem while considering the physical constraints and hand capabilities can be formulated as a *Second-Order Cone Program* (SOCP), which is a convex optimization problem [5] and this is a key contribution of the paper. The SOCP formulation implies that we can obtain the optimal solution for the grasping force synthesis problem

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efficiently. Our problem formulation is general enough to consider different objective functions like minimizing the maximum torque in the joints of the manipulators or minimizing the maximum contact force. We present simulation results for grasping force synthesis for single-handed and dual-handed manipulation of objects (using the Baxter robot from ReThink Robotics), where the objective is to reconfigure the object. The simulation scenarios are set up such that the physical constraints of the robot (e.g., limits on motor torques) imply that the robot cannot pick and place the objects. Our simulation results show that the robot could compute a sequence of grasping forces and/or joint torques to reconfigure the object by exploiting the contact with the environment, while ensuring that it's physical constraints are satisfied.

II. Related Work

A key requirement of any grasp is the ability to resist the externally applied wrenches. If there exist sufficient forces at each contact to compensate every external wrench, the grasp is known as a *force-closure* grasp [6], [7]. Among the contact forces that can hold the object in equilibrium, we may seek one with the minimum force. The problem of finding such a set of contact forces is known as the *Force Optimization Problem* (FOP). Since the contact friction models are nonlinear, early works for solving FOPs use a linear approximation of the friction cone and formulate the FOP as a *Linear Programming* (LP) problem, which is solved using the Simplex algorithm [8], [9]. However, the LP-based formulation is not exact as it approximates the friction force constraints as linear inequalities. Yoshikawa and Nagai [10], [11], [12] proposed a heuristic-based scheme for determining the internal grasping forces for multi-fingered hands and, later on, Nakamura [13] solved the problem as a nonlinear programming problem using Lagrange's multipliers for obtaining a solution.

The friction cone constraint can be expressed as a positive semi-definiteness constraint on a matrix [14]. Thus, the problem of finding the optimal contact forces has been formulated using Linear Matrix Inequalities (LMI) [15], [16]. The resulting optimization problem is a semidefinite programming problem (SDP). Buss [17] developed a Dikin-type algorithm for solving the FOP formulated as a SDP. Helmke [15] proposed a new compact formulation for the SDP arising from the FOP, and developed a quadratically convergent algorithm. Lobo *et al.* [18] formulated the FOP for the first time as a *Second-Order Cone Program* (SOCP). Later, Boyd and Wegbreit [19] used this conic formulation and proposed a fast interior-point algorithm for solving the FOP, with the point-contact friction model as the cone constraint. However, the SOCP formulation in [19] does not take into account the constraints imposed by the object-environment contact and joints motor limitations. Note that the SOCP is a convex optimization problem with linear objective and equality constraints plus one or more quadratic (or second order) cone constraints.

While grasping an object using robotic manipulators, determining the joint torques required to produce the necessary contact forces is also very important. Therefore, the FOP can be extended and expressed as *Torque Optimization Problem* (TOP), wherein, one optimizes the torques applied at the motors that are used to generate the desired contact forces while satisfying the torque constraints. Lippiello *et al.* [20], [21] developed a new algorithm for online TOP of a dextrous robotic hand based on [17], [15] and considering the joint torque constraints. Dai *et al.* [22] have formulated the problem of simultaneous computation of grasping force and grasping locations as a Bilinear Matrix Inequality (BMI). Although solving a BMI is NP-hard, the authors proposed efficient solution techniques using sequential SDP.

The research described above is focused on manipulating an object using only the capabilities of the hand. This is sometimes referred to as *intrinsic dexterity* [4]. The use of environmental factors and their effects in manipulation was studied by Brock in [23]. He studied reorienting an already grasped object with the help of controlled slippage at object-environment contact. The use of the environment and other factors (like gravity and dynamics) for in-hand manipulation has been studied in great detail by Chavan-Dafle *et al.* [2], by providing a repertoire of regrasps used to navigate the grasp taxonomy. They coined the term, *extrinsic dexterity*, referring to the external factors like gravity, external contacts, and dynamic effects which can be used to reorient or regrasp an object as desired. This work was further extended by Chavan-Dafle and Rodriguez [3], where they introduced the notion of *prehensile pushing* which can be used for in-hand manipulation by pushing an object against the environment. They presented the force analysis as a quasi-dynamic formulation and solved it as a linear complementarity problem using quadratic optimization techniques. The action of pivoting or tilting has been analyzed as a type of extrinsic regrasping in [4], [24] as a more faster and efficient alternative to the traditional *pick and place* grasps. In [4], the pivoting action has been analyzed as a rotation along the axis of the object-robot contact normals and an open-loop trajectory has been planned to achieve this rotation. Manipulating an object while maintaining external contact with the object has been referred to as 'shared grasping' in [25]. In this work, the authors have provided a general framework to determine the contact mode for a manipulation task involving external contact while controlling the motion of the object using Hybrid Force Velocity Control (HFVC). A stability analysis has also been provided to determine the robustness of the contact mode.

Our goal in this paper is to make use of the object-environment contact to manipulate (reposition and reorient) heavy objects without lifting the objects off the environment. For simplicity of exposition, we assume that at all contacts (1) the grasped object does not rotate about the axis of the contact normal as in [4], and (2) the object continuously remains in contact with the environment without slippage. However, our formulation is general enough that these constraints can be applied or removed on a per contact basis, depending on the application. When manipulating a heavy object, we tilt the object using one of its edges as a support. The reaction forces produced at the contact along the edge can be used to partly balance the weight of the object. This, in turn, reduces the force required to grasp the object against the effect of its own weight. We have evaluated our method in simulation, for (1) tilting a large hollow cylindrical object using a parallel jaw gripper, and (2) tilting a box about its edge using two arms of the Baxter robot.

III. Problem Formulation

Consider a rigid object which is in contact with the environment at m points and grasped by n manipulators at n positions as shown in Fig. 2. Contact coordinate frames $\{C_i\}$ and $\{E_j\}$ (where $i = 1, .., n$ and $j = 1, ..., m$) are attached to the object at each manipulator and environment contact, respectively, such that $\boldsymbol{n}$-axis of the frames is normal (inward) to the object surface and two other axes, $\boldsymbol{t}$ and $\boldsymbol{o}$, are tangent to the surface. Positions of the contact frames in the inertial coordinate system (X,Y,Z) are represented by $\boldsymbol{p}_{C_i} \in \mathbb{R}^3$ and $\boldsymbol{p}_{E_j} \in \mathbb{R}^3$.

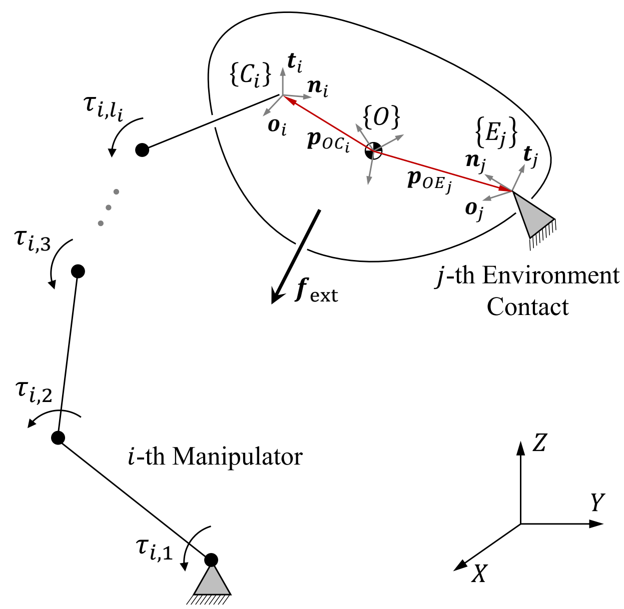


Fig. 2. A rigid object grasped by environment and manipulators.

A. Contact Constraints

A *contact model* can be used to impose constraints on the forces that arise at the contact locations. In general, there are two main contact models; *point contact with friction* (PCWF) and *soft finger contact with elliptic approximation* (SFCE). In this paper, we assume that the contacts between the object and environment are PCWF and the contacts between the object and the manipulator tips are SFCE. In the PCWF, the contact wrench expressed in the contact frame has three components of frictional and normal forces, $\boldsymbol{f}_{E_j} = [f_{E_{t,j}}, f_{E_{o,j}}, f_{E_{n,j}}, 0, 0, 0]^{\mathrm{T}}$, which satisfy the *friction cone constraint* as

$$\frac{1}{\mu_{Ei}}\sqrt{\left(\frac{f_{E_{t,j}}}{e_{E_{t,j}}}\right)^2 + \left(\frac{f_{E_{o,j}}}{e_{E_{o,j}}}\right)^2} \leq f_{E_{n,j}}, \tag{1}$$

where the parameters μ_{Ei}, $e_{E_{t,j}}$, and $e_{E_{o,j}}$ are positive constants defining the friction coefficients at the jth environment contact point and $j = 1, ..., m$. The SFCE is a generalization of the Coulomb's friction law and derived from the principle of maximum power dissipation. In this contact model, the contact wrench expressed in the contact frame has four components of frictional and normal forces and frictional moment, $\boldsymbol{f}_{C_i} = [f_{C_{t,i}}, f_{C_{o,i}}, f_{C_{n,i}}, 0, 0, m_{C_{n,i}}]^{\mathrm{T}}$, which satisfy the elliptic constraint as

$$\frac{1}{\mu_{Ci}}\sqrt{\left(\frac{f_{C_{t,i}}}{e_{C_{t,i}}}\right)^2 + \left(\frac{f_{C_{o,i}}}{e_{C_{o,i}}}\right)^2 + \left(\frac{m_{C_{n,i}}}{e_{C_{n,i}}}\right)^2} \leq f_{C_{n,i}}, \tag{2}$$

where the parameters μ_{Ci}, $e_{C_{t,i}}$, $e_{C_{o,i}}$, and $e_{C_{n,i}}$ are positive constants defining the friction coefficients at the ith manipulator contact and $i = 1, ..., n$. Note that $e_{C_{t,i}}$ and $e_{C_{o,i}}$ are dimensionless and $e_{C_{n,i}}$ has a dimension of length. Note that the contact constraints (1) and (2) are valid for $f_{E_{n,i}} \geq 0$ and $f_{C_{n,i}} \geq 0$. Moreover, the contact constraints (1) and (2) are *Second-Order Cone* (SOC) constraints [5]. For any $\boldsymbol{x} \in \mathbb{R}^6$, we define the friction cones $\mathcal{K}_{Ej}$ and $\mathcal{K}_{Ci}$ as

$$\mathcal{K}_{Ej} = \left\{ \boldsymbol{x} \;\middle|\; \frac{1}{\mu_{Ei}}\sqrt{\frac{x_1^2}{e_{E_{t,j}}^2} + \frac{x_2^2}{e_{E_{o,j}}^2}} \leq x_3,\ x_{4,5,6} = 0 \right\}, \tag{3}$$

$$\mathcal{K}_{Ci} = \left\{ \boldsymbol{x} \;\middle|\; \frac{1}{\mu_{Ci}}\sqrt{\frac{x_1^2}{e_{C_{t,i}}^2} + \frac{x_2^2}{e_{C_{o,i}}^2} + \frac{x_6^2}{e_{C_{n,i}}^2}} \leq x_3, x_{4,5} = 0 \right\}, \tag{4}$$

where x_i, $i = 1, \ldots, 6$ is the ith component of $\boldsymbol{x}$. The contact constraints (1) and (2) can be written respectively in a concise form as

$$\boldsymbol{f}_{E_j} \in \mathcal{K}_{Ej}, \quad j = 1, \cdots, m, \tag{5}$$

$$\boldsymbol{f}_{C_i} \in \mathcal{K}_{Ci}, \quad i = 1, \cdots, n. \tag{6}$$

Furthermore, to guarantee that the object is not damaged by the *internal forces* at the contacts between the object and the manipulator tips, we can consider an upper bound constraint for the normal contact forces. Let's define the vector of normal contact forces applied by the manipulators to the object as $\boldsymbol{F}_n = [f_{C_{n,1}}, \cdots, f_{C_{n,n}}]^{\mathrm{T}}$. Therefore, this constraint can be written as

$$\boldsymbol{F}_n \leq \boldsymbol{F}_{n,\max}, \tag{7}$$

where $\boldsymbol{F}_{n,\max}$ is the upper limit for the normal contact forces. Note that a set of contact forces that result in no net force on the object is known as *internal force*.

B. Equilibrium Constraints

In this paper, we assume that the manipulation is *quasi-static*, therefore, the inertia forces and dynamic effects are very small and can be ignored. Let $\mathbf{R}_{OC_i} \in \mathrm{SO}(3)$ and $\mathbf{R}_{OE_j} \in \mathrm{SO}(3)$ be 3×3 orthogonal matrices that transforms the manipulator and environment contact forces from the contact coordinate frames to object coordinate frame $\{O\}$, respectively. Hence, each contact wrench, $\boldsymbol{f}_{C_i}$ and $\boldsymbol{f}_{E_j}$, can be expressed in the object coordinate frame $\{O\}$ as

$$\boldsymbol{F}_{OC_i} = \mathbf{G}_{C_i}\boldsymbol{f}_{C_i} = \begin{bmatrix} \mathbf{R}_{OC_i} & \mathbf{0} \\ \mathbf{S}(\boldsymbol{p}_{OC_i})\mathbf{R}_{OC_i} & \mathbf{R}_{OC_i} \end{bmatrix} \boldsymbol{f}_{C_i}, \quad (8)$$

$$\boldsymbol{F}_{OE_j} = \mathbf{G}_{E_j}\boldsymbol{f}_{E_j} = \begin{bmatrix} \mathbf{R}_{OE_j} & \mathbf{0} \\ \mathbf{S}(\boldsymbol{p}_{OE_j})\mathbf{R}_{OE_j} & \mathbf{R}_{OE_j} \end{bmatrix} \boldsymbol{f}_{E_j}, \quad (9)$$

where $\mathbf{S}(\boldsymbol{p}_{OC_i}) \in \text{skew}(3)$ and $\mathbf{S}(\boldsymbol{p}_{OE_j}) \in \text{skew}(3)$ are 3×3 skew-symmetric matrices that satisfy $\mathbf{S}(\boldsymbol{p}_{OC_i})\boldsymbol{z} = \boldsymbol{p}_{OC_i} \times \boldsymbol{z}$ and $\mathbf{S}(\boldsymbol{p}_{OE_j})\boldsymbol{z} = \boldsymbol{p}_{OE_j} \times \boldsymbol{z}$ ($\forall \boldsymbol{z} \in \mathbb{R}^3$) where $\boldsymbol{p}_{OC_i}$ and $\boldsymbol{p}_{OE_j}$ are positions of $\{C_i\}$ and $\{E_j\}$ with respect to $\{O\}$ and expressed in $\{O\}$, respectively.

Therefore, by computing the net manipulator and environment contact wrenches, the force and toque equilibrium conditions can be written as

$$\sum_{i=1}^{n} \mathbf{G}_{C_i}\boldsymbol{f}_{C_i} + \sum_{j=1}^{m} \mathbf{G}_{E_j}\boldsymbol{f}_{E_j} + \boldsymbol{f}_{\text{ext}} = \mathbf{0}, \quad (10)$$

where $\boldsymbol{f}_{\text{ext}} \in \mathbb{R}^6$ is the total external (force and moment) wrench acting on the object (including the object weight) expressed in the object coordinate frame $\{O\}$.

By introducing $\boldsymbol{F}_C \in \mathbb{R}^{6n}$ and $\boldsymbol{F}_E \in \mathbb{R}^{6m}$ as

$$\boldsymbol{F}_C = \begin{bmatrix} \boldsymbol{f}_{C_1} \\ \boldsymbol{f}_{C_2} \\ \vdots \\ \boldsymbol{f}_{C_n} \end{bmatrix}, \quad \boldsymbol{F}_E = \begin{bmatrix} \boldsymbol{f}_{E_1} \\ \boldsymbol{f}_{E_2} \\ \vdots \\ \boldsymbol{f}_{E_m} \end{bmatrix}. \quad (11)$$

Equation (10) can be represented in more compact form as

$$\mathbf{G}_C\boldsymbol{F}_C + \mathbf{G}_E\boldsymbol{F}_E + \boldsymbol{f}_{\text{ext}} = \mathbf{0}, \quad (12)$$

where $\mathbf{G}_C = [\mathbf{G}_{C_1}, \cdots, \mathbf{G}_{C_n}] \in \mathbb{R}^{6\times6n}$ is the manipulator contact matrix, which is also known as the *grasp map* or *grasp matrix*, and $\mathbf{G}_E = [\mathbf{G}_{E_1}, \cdots, \mathbf{G}_{E_m}] \in \mathbb{R}^{6\times6m}$ is the environment contact matrix.

C. Manipulator Joint Torque Constraints

While performing any manipulation or grasping task using robotic manipulators (or a robotic hand), we are primarily interested in computing the joint torques required to produce the necessary wrenches at the contact of manipulators with the object. Therefore, we need to establish a relationship between the joint torques and the contact wrenches, $\boldsymbol{F}_C$, exerted on the object. Let l_i be the number of joints of the i-th manipulator, $\boldsymbol{q}_i = [q_{i,1}, \cdots, q_{i,l_i}]^{\text{T}}$ be vector of joint variables, and $\boldsymbol{\tau}_i = [\tau_{i,1}, \cdots, \tau_{i,l_i}]^{\text{T}}$ be vector of joint torques of this manipulator. The relationship between the contact wrench, $\boldsymbol{f}_{C_i}$, and the joint torques, $\boldsymbol{\tau}_i$, for manipulators is written as

$$\boldsymbol{\tau}_i = -\mathbf{J}_i^{\text{T}}(\boldsymbol{q}_i)\boldsymbol{f}_{C_i} + \boldsymbol{\tau}_{g\,i}(\boldsymbol{q}_i), \quad i = 1, \cdots, n, \quad (13)$$

where $\mathbf{J}_i(\boldsymbol{q}_i) \in \mathbb{R}^{6\times l_i}$ is the Jacobian matrix of the i-th manipulator expressed in the contact frames $\{C_i\}$ and $\boldsymbol{\tau}_{g\,i}(\boldsymbol{q}_i) \in \mathbb{R}_i^l$ is the torque due to gravitational forces of the i-th manipulator. The negative sign appears in (13) because the reaction wrench of $\boldsymbol{f}_{C_i}$ is applied to the manipulator tip. Moreover, since it is assumed that the manipulation is *quasi-static*, the inertia forces and dynamic effects of the manipulator can be ignored. By defining $\boldsymbol{\tau} \in \mathbb{R}^l$, $\boldsymbol{\tau}_g \in \mathbb{R}^l$, and $\boldsymbol{q} \in \mathbb{R}^l$ ($l = l_1 + \cdots + l_n$) as

$$\boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{\tau}_1 \\ \boldsymbol{\tau}_2 \\ \vdots \\ \boldsymbol{\tau}_n \end{bmatrix}, \quad \boldsymbol{\tau}_g = \begin{bmatrix} \boldsymbol{\tau}_{g\,1} \\ \boldsymbol{\tau}_{g\,2} \\ \vdots \\ \boldsymbol{\tau}_{g\,n} \end{bmatrix}, \quad \boldsymbol{q} = \begin{bmatrix} \boldsymbol{q}_1 \\ \boldsymbol{q}_2 \\ \vdots \\ \boldsymbol{q}_n \end{bmatrix}. \quad (14)$$

Equation (13) can be represented in a compact form as

$$\boldsymbol{\tau} = -\mathbf{J}^{\text{T}}(\boldsymbol{q})\boldsymbol{F}_C + \boldsymbol{\tau}_g(\boldsymbol{q}), \quad (15)$$

where $\mathbf{J} = \text{diag}(\mathbf{J}_1, \cdots, \mathbf{J}_n) \in \mathbb{R}^{6n\times l}$ is the overall Jacobian matrix of manipulators. For simplicity, it is presumed that the manipulators are serial and fully-actuated.

To ensure that the joint actuators of the manipulators are able to generate the required joint torques without saturation, the joint torque constraints need to be considered as

$$\boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau} \leq \boldsymbol{\tau}_{\max}, \quad (16)$$

where $\boldsymbol{\tau}_{\min}$ and $\boldsymbol{\tau}_{\max}$ are the lower and the upper limits of the manipulators joint torques, respectively.

IV. Grasping Force Synthesis

When the equations (5), (6), (7), (12), (15), and (16) are satisfied simultaneously, the object can be stably grasped and quasi-statically manipulated by the manipulators and with the help of environment. In order to find the best solution from all the feasible solutions for the contact wrenches, we can define a general optimization problem with the objective function Φ which satisfies the equations (5), (6), (7), (12), (15), and (16) as

$$\begin{aligned} \underset{\boldsymbol{F}_C, \boldsymbol{F}_E}{\text{minimize}} \quad & \Phi \\ \text{subject to} \quad & \mathbf{G}_C\boldsymbol{F}_C + \mathbf{G}_E\boldsymbol{F}_E + \boldsymbol{f}_{\text{ext}} = \mathbf{0}, \\ & \boldsymbol{\tau} + \mathbf{J}^{\text{T}}\boldsymbol{F}_C - \boldsymbol{\tau}_g = \mathbf{0}, \\ & \boldsymbol{\tau}_{\min} \leq \boldsymbol{\tau} \leq \boldsymbol{\tau}_{\max}, \\ & \boldsymbol{F}_n \leq \boldsymbol{F}_{n,\max}, \\ & \boldsymbol{f}_{C_i} \in \mathcal{K}_{C\,i}, \quad i = 1, \cdots, n, \\ & \boldsymbol{f}_{E_j} \in \mathcal{K}_{E\,j}, \quad j = 1, \cdots, m. \end{aligned} \quad (17)$$

Note that in (17), the second term of the first constraint (i.e., $\mathbf{G}_E\boldsymbol{F}_E$) and the last constraint (i.e., $\boldsymbol{f}_{E_j} \in \mathcal{K}_{E\,j}$) represent the effect of the environment in the object grasping and manipulation.

If among all the sets of contact wrenches that satisfy the system constraints we seek the one that minimizes the grasping forces, we can define a Grasping Force Optimization Problem (GFOP). Moreover, if we seek the contact wrenches that minimize the manipulators joint torques, we can define a Torque Optimization Problem (TOP).

A. Grasping Force Optimization Problem (GFOP)

The GFOP can be defined as (1) minimizing the maximum magnitude of the manipulators normal contact forces, i.e., $\Phi = \max(\boldsymbol{F}_n)$, or (2) minimizing the Euclidean norm of the vector of normal contact forces, i.e., $\Phi = \|\boldsymbol{F}_n\|_2$, while satisfying the conditions expressed in equations (5), (6), (7), (12), (15), and (16). In some real world scenarios, it may be desirable to grasp a brittle object without damaging it. The GFOP formulation can be effectively used, with or without the object-environment contact constraint (5), in such situations to compute the optimal contact forces by specifying the maximum permissible normal contact force using the constraint (7).

B. Torque Optimization Problem (TOP)

Similarly, our goal in the TOP can be defined as (1) minimizing the value of maximum joint torque of the manipulators, i.e., $\Phi = \max(\boldsymbol{\tau})$, or (2) minimizing the Euclidean norm of the vector of manipulators joint torques, i.e., $\Phi = \|\boldsymbol{\tau}\|_2$, while satisfying the conditions expressed in equations (5), (6), (7), (12), (15), and (16). The TOP formulation may be used to compute the optimal joint torque required for a specific task. It can be also used to reduce the variation in actuator effort among the joints of a manipulator.

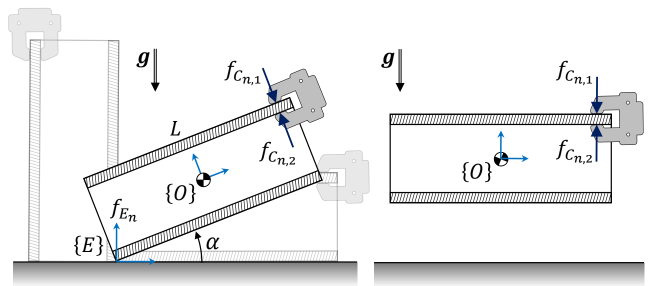
In order to solve (17) when $\Phi = \max(\boldsymbol{F}_n)$ or $\Phi = \max(\boldsymbol{\tau})$, we can add a constraint as $f_{C_{n,i}} \leq \lambda$ or $\tau_{i,k} \leq \lambda$ ($i = 1, \cdots, n$ and $k = 1, \cdots, l_i$), respectively, and changing the objective function to $\Phi = \lambda$ where λ is a scalar variable that bounds the magnitude of normal contact forces or joint torques. Note that the problem (17) is a *convex* optimization problem, since the objective functions are convex and the constraints are linear equalities/inequalities and convex SOCs; therefore, it can be solve by using the CVX toolbox [26].

V. Implementation and Results

In this section, we apply the grasping force synthesis formulation described in (17) to compute the effort (i.e., grasping force and/or joint torque) required for two different manipulation scenarios. Intuitively, in both examples (see Fig. 3 and Fig. 4), we want to reconfigure an object from a horizontal position on a support surface to a vertical position. In both examples, the joint effort constraints are such that it is not possible for the manipulator(s) to lift the objects off the support surface, hence the objects cannot be reconfigured by a pick and place strategy. We show that by exploiting contact with the support surface, we can obtain manipulation efforts such that the objects can be reconfigured without violating the actuator limits.

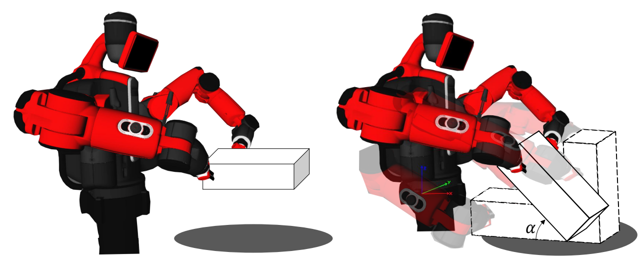
In the first example, we consider the reconfiguration of a hollow cylindrical object from a horizontal position to a vertical position (see Fig. 3). This exemplifies an application scenario where a single manipulator with a parallel jaw gripper is reconfiguring an object. For simplicity, in this example we consider only the maximum force constraint of the actuators at the parallel jaw gripper.

In the second example, we consider a heavy box to be reconfigured by a two-armed Baxter robot, with each arm having 7 degrees-of-freedom (DoF). This exemplifies an application scenario, where two robot arms need to cooperatively manipulate an object. In this example, we consider actuator limits at each joint. The optimization formulations for both the examples have been implemented and solved in MATLAB using the CVX toolbox [26] with the default solver, SDPT3, on a Dell XPS with Intel i7 1.8GHz processor and 16GB RAM.

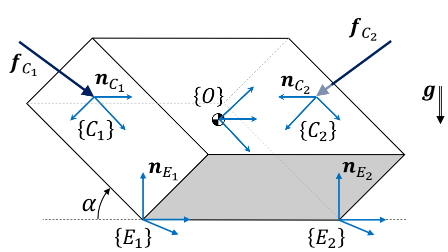


(a) Titling by exploiting the environment (b) Lifting off the environment

Fig. 3. A hollow cylinder being manipulated using a parallel jaw gripper (side view).



(a) Lifting off the environment (b) Titling by exploiting the environment



(c) Contact forces while titling by exploiting the environment

Fig. 4. A box being manipulated by using both arms of a Baxter robot.

A. Cylinder Tilting Using Parallel Jaw gripper

In the first example, as shown in Fig. 3-a, we consider two cylinder-gripper contact frames $\{C_1\}$ and $\{C_2\}$, one at each location where the gripper makes contact with the object, and one cylinder-environment contact frame $\{E\}$. We use the GFOP formulation with the contact forces at both the cylinder-gripper and cylinder-environment contacts, i.e., $\boldsymbol{f}_{C_1}$, $\boldsymbol{f}_{C_2}$, and $\boldsymbol{f}_E$, as the optimization variables, which are in total

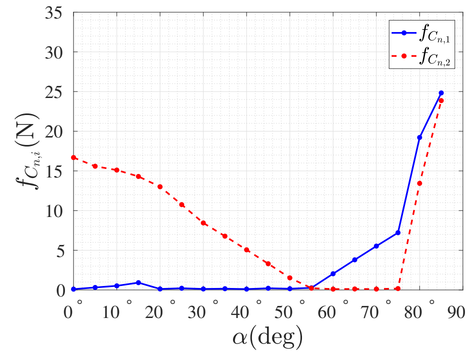
11 variables. The objective function used while implementing the GFOP is to minimize the maximum magnitude of the normal force at the cylinder-gripper contact, i.e., $\Phi = \max(\boldsymbol{F}_n)$, while the cylinder is being lifted quasi-statically. In this example, we do not consider the manipulator joint torque constraints specified in (15) and (16), but we use a constraint, as given in (7), on the maximum force at the gripper contact points to guarantee that the cylinder is not damaged by the internal forces. Therefore, we can solve the GFOP formulation to compute the optimal normal contact forces, $f_{C_{n,1}}$ and $f_{C_{n,2}}$, required at a particular angle α while lifting the cylinder with the help of the environment contact.

The maximum grip force which can be exerted by the parallel jaw gripper at a particular contact point, in the normal direction, is assumed to be 30 (N). Results of the simulation implementation using the parameters given in Table I are presented in Fig. 5. The computation time required to solve our GFOP formulation for each value of the cylinder tilting angle α is about 1.15 seconds. Initially, the cylinder is being grasped at two contact locations and the normal contact forces act along the same direction as the external gravitational wrench. As a result, as shown in Fig. 5-a, the magnitude of $f_{C_{n,2}}$ is significant and that of $f_{C_{n,1}}$ is negligible till the value of the tilting angle, α reaches approximately $55°$ where the line of action of the normal component of the cylinder-environment contact, f_{E_n}, passes through the cylinder center of mass. Beyond this angle, as the value of α increases, the magnitude of $f_{C_{n,1}}$ also increases. Moreover, as shown in Fig. 5-b, the normal component of the cylinder-environment contact, f_{E_n}, increases as the magnitude of $f_{C_{n,2}}$ decreases. Note that since at $\alpha = 90°$ the cylinder-environment contact is not a point contact anymore, the results are represented till $\alpha = 85°$.

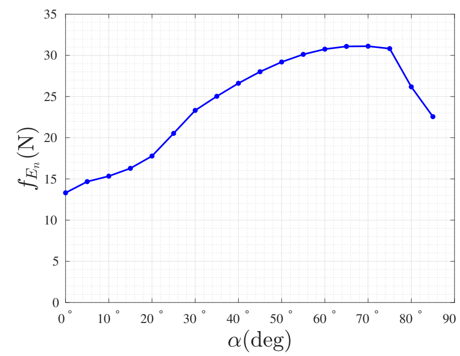
On the other hand, manipulating the cylinder by lifting it off the environment, as shown in Fig. 3-b, requires us to remove the environmental contact constraints (5) from the GFOP formulation. This modified formulation is infeasible because the upper bound constraint (7) does not allow us to exert a force greater than 30 (N) at each of the cylinder-gripper contact locations. Owing to the location of the contact reference frames, $\{C_1\}$ and $\{C_2\}$, as well as the geometry of the object, we need to exert a force greater than 30 (N) at the cylinder-gripper contacts to completely resist the weight of the cylinder and satisfy the equilibrium constraints (12) without the cylinder-environment contact. Thus, the parallel jaw gripper in this example is not capable of generating enough squeezing force to reconfigure the cylinder by lifting it off the environment.

TABLE I
SIMULATION PARAMETERS FOR TILTING A HOLLOW CYLINDER.

Parameter	Value
Weight	30 (N)
Dimensions	$\text{Ø}_{\text{out}} = 0.36$, $\text{Ø}_{\text{in}} = 0.3$, $L = 0.5$ (m)
Constants	$\mu_E = 0.2$, $\mu_{C1,2} = 0.15$, $e_{C_{n,1,2}} = 0.25$, $e_* = 1$



(a)



(b)

Fig. 5. (a) Variation of the cylinder-gripper contact forces in the normal direction, $f_{C_{n,1}}$ and $f_{C_{n,2}}$, with respect to the tilting angle, α. (b) Variation of the cylinder-environment contact force in the normal direction, f_{E_n}, with respect to the tilting angle, α.

B. Dual-armed Box Tilting

In the second example, as shown in Fig. 4-c, we consider two reference frames $\{C_1\}$ and $\{C_2\}$ at each box–end-effector contact. We model the edge contact between the box and the environment with two point contacts at the vertices of the contact edge and the reference frames $\{E_1\}$ and $\{E_2\}$. To compute the joint torques in both the arms, we need the corresponding joint angles when the box is grasped at a particular tilting angle, α, as shown in Fig. 4-b. By having the geometry of the object, the position and orientation of the contact reference frames $\{C_1\}$, $\{C_2\}$, $\{E_1\}$, and $\{E_2\}$ with respect to the Baxter base reference frame is computed for α ranging from $0°$ to about $90°$ with a step size of $5°$. The joint angles for both the left and the right arms of Baxter is obtained using inverse kinematics for each value of α. The end-effector reference frame has the same position and orientation as the contact reference frames.

The two objective functions used while implementing the optimization problem (17) are to minimize the maximum magnitude of the normal force at the box–end-effector contacts, i.e., the GFOP with $\Phi = \max(\boldsymbol{F}_n)$, and to minimize the magnitude of the maximum torque generated in a particular joint of the Baxter arms, i.e., the TOP with $\Phi = \max(\boldsymbol{\tau})$. Both the optimization problems are solved for all values of the box tilting angle, α. The optimization variables for the GFOP are the contact forces at box–end-effector contacts, $\boldsymbol{f}_{C_1}$ and $\boldsymbol{f}_{C_2}$, as well as the contact forces at the box-environment contacts, $\boldsymbol{f}_{E_1}$ and $\boldsymbol{f}_{E_2}$. Considering the individual components of these contact forces, we get a total of 14 optimization variables. Whereas for the TOP formulation, we have to additionally include two 7×1 joint torque vectors for both arms of Baxter, i.e., $\boldsymbol{\tau}_{\text{left}}$ and $\boldsymbol{\tau}_{\text{right}}$, giving us a total of 28 optimization variables. Note that, in this example, we do not consider any upper bound constraint for the normal contact forces as given in 7.

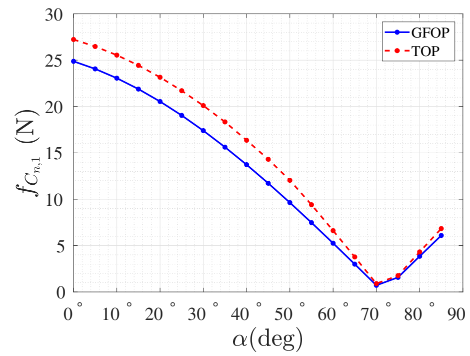
Results of the simulation using the parameters given in Table II are presented in Fig. 6 to Fig. 8. The computation time required to solve the GFOP and TOP, for each value of α, is 2.23 seconds and 2.41 seconds, respectively. As it is evident from Fig. 6, as the value of α increases, the magnitude of the normal contact forces, $f_{C_{n,1}}$ and $f_{C_{n,2}}$, decreases for both the GFOP and TOP formulations. The magnitude of the normal component of the box-environment contacts, $f_{E_{n,1}}$ and $f_{E_{n,2}}$, increases accordingly to contribute to the force balance condition. As seen in the previous case, the value of both $f_{C_{n,1}}$ and $f_{C_{n,2}}$ becomes negligible as the value of α approaches $70°$, since at this angle of the box, the weight of the box passes through its support edge. It should be noted that since the configuration (joint angles) of the both the Baxter arms while simulating the task of tilting the box is not the same, the magnitude of the joint torques will also be dissimilar. Thus, there is a slight variation between the computed values of the contact forces at $\{C_1\}$ and $\{C_2\}$, and also at $\{E_1\}$ and $\{E_2\}$ for the TOP formulation, since we are computing the optimal value for the joint torque vector, $\boldsymbol{\tau}$. Figure 8 compares the magnitude of the maximum joint torque for both the GFOP and TOP. The maximum joint torque occurs at the elbow joints, 3 or 4, for both arms of Baxter depending on the value of the box tilting angle α.

TABLE II

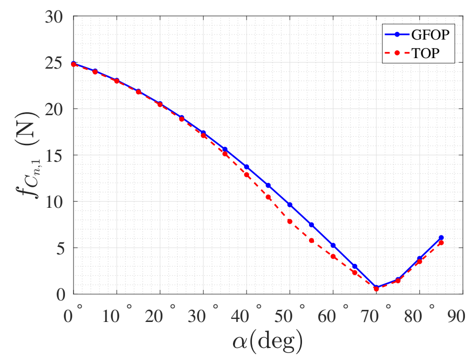
SIMULATION PARAMETERS FOR TILTING A BOX.

Parameter	Value
Weight	20 (N)
Dimensions	$0.3 \times 0.2 \times 0.1$ (m^3)
Constants	$\mu_{E1,2} = 0.25$, $\mu_{C1,2} = 0.1$, $e_{C_{n,1,2}} = 0.5$, $e_* = 1$

On the other hand, if we assume that the box is to be reconfigured from a horizontal position to a vertical position using a pick and place strategy as shown in Fig. 4-a, we have to eliminate the environmental contact constraints (5) from the GFOP formulation. Additionally, let us also, for the sake of convenience, remove the manipulator joint torque constraints, specified in (15) and (16), from our initial optimization formulation. By solving the GFOP, the magnitude of the optimal normal contact forces, $f_{C_{n,1}}$ and $f_{C_{n,2}}$, for lifting the box off the environment is 52.97 (N). This is greater than the maximum value of 27.22 (N) (refer to Fig. 6-a) computed while considering both the environmental contact constraints and manipulator joint torque constraints in the GFOP and TOP formulations. Furthermore, upon including the manipulator joint torque constraints without the environmental contact constraints, the GFOP formulation becomes infeasible. This means that the joint actuators

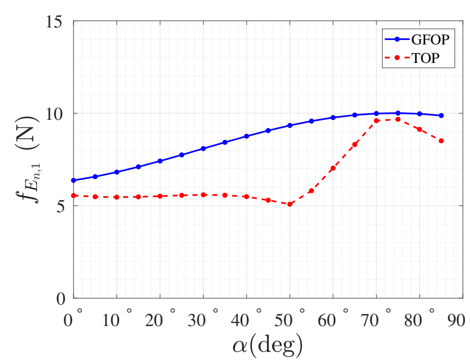


(a)

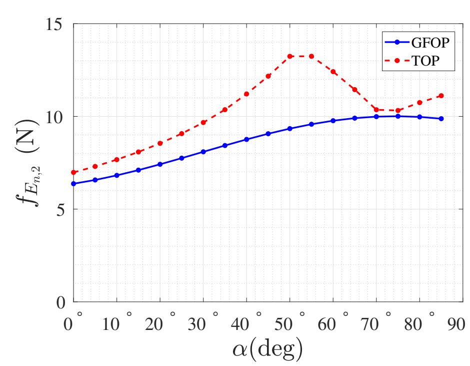


(b)

Fig. 6. (a)-(b) Variation of the box–end-effector contact forces in the normal direction, $f_{C_{n,1}}$ and $f_{C_{n,2}}$, with respect to the tilting angle, α, for both GFOP and TOP.



(a)



(b)

Fig. 7. (a)-(b) Variation of the box-environment contact forces in the normal direction, $f_{E_{n,1}}$ and $f_{E_{n,2}}$, with respect to the tilting angle, α, for both GFOP and TOP.

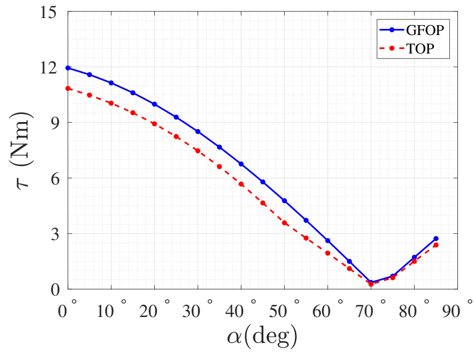


Fig. 8. Magnitude of the maximum joint torque for both the GFOP and TOP.

present in the arms of Baxter cannot generate the necessary effort to lift the box using the pick and place strategy.

VI. Conclusion and Future Work

In this paper, we presented a *Second-Order Cone Program* (SOCP) for computing the grasping force required for reconfiguring large and heavy objects with the help of object-environment contacts. Our formulation is general enough to incorporate different objective functions and constraints based on the physical limits or characteristics of the joint actuators and objects that are being manipulated. Furthermore, our formulation does not need to approximate the friction cone constraints at the contacts. We evaluate our proposed formulation by simulating the task of reconfiguring two different objects from an initial pose to a goal pose. Given the object pose, the manipulator configuration, and the object contacts, we can compute the optimal values of the contact wrenches required to grasp the object, and also the joint torques of the manipulators.

Future Work: In the current work, we have assumed that the path along which the object should move is known. Further, we assumed that the motion is quasistatic. In future work, we will consider combining path/trajectory planning for the pivoting motion along with the force synthesis problem. This may also involve extending the present formulation to incorporate the dynamics of the object as well as the manipulator. We plan to use recent work [27] on using screw-linear interpolation for motion planning with task space constraints for the path planning. Future work will also involve evaluation of the proposed algorithms using dynamic simulation [28] and experiments.

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