2020 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)
October 25-29, 2020, Las Vegas, NV, USA (Virtual)

Identification of Dynamic Parameters for Rigid Robots based on Polynomial Approximation

Alexander Lomakin[1] and Joachim Deutscher[2]

***Abstract*— In this paper an approach for the identification of the dynamic parameters, i.e. base parameters, of rigid robots is presented. By using the polynomial approximation operator, an equation is obtained for the identification of the parameters which solely depends on measurable signals and thereby contains no equation error. The resulting expressions can be evaluated online or offline by filtering the measurable signals with FIR filters. In order to identify the parameters on the basis of measurements, an algorithm is presented to calculate the parameters numerically stable, even if the data is obtained sequentially, without a singular value decomposition. The parameters can be determined meaningfully by considering box constraints in order to ensure physical feasibility. The presented methods are finally used to identify the dynamic parameters of a delta robot and compared to the standard approach.**

I. INTRODUCTION

For the application of model-based algorithms and for the interpretation of the physical behaviour of a robot it is essential to determine the physical parameters properly. However, since these parameters are not always available or can not be determined a priori for example due to aging and other influences, it is necessary to identify the parameters based on measurement. Therefore, parameter identification is an important topic in robotics. There exists a multitude of approaches and methods to identify parameters on the basis of measurements (see, e.g., [1],[2],[3],[4],[5]). Yet there are still problems in this context which make correct identification difficult (see [6]). The main problems are, on the one hand, that the time derivatives of the measured positions are not available, the numerical issues caused by the inadequacy of the data and the ensuring of the physical integrity of the identified parameters.

However, this paper presents methods which offer a solution for each of the three presented problems and which can be implemented with reasonable effort both offline based on recorded data and online on a controller. Therefore, the polynomial approximation already introduced in [7] is used for the evaluation of nonlinear differential algebraic expressions. This results in an equation for identification that depends solely on measurable signals and contains no equation error. Furthermore, an algorithm is presented to determine the parameters reasonably despite of inadequate data.

[1]Alexander Lomakin is with the Lehrstuhl für Regelungstechnik, Friedrich-Alexander-Universität, Erlangen-Nürnberg, Germany alexander.lomakin@fau.de

[2]Joachim Deutscher is with the Institut für Mess-, Regel- und Mikrotechnik, Universität Ulm, Germany joachim.deutscher@uni-ulm.de

This paper is structured as follows: In the following section a formulation of the considered parameter identification problem is set up. Then, the polynomial approximation presented in [7] is briefly reviewed and the application to the obtained parameter identification problem is provided thereafter. In section V, the modified recursive least squares algorithm is presented, which allows the parameters to be adjusted iteratively and thereby always numerically stable. Subsequently, the physical feasibility of the determined parameters is examined and ensured by adapting the identification. Finally, the proposed methods are applied to a closed-loop delta robot and the performance is of the presented methods is evaluated.

II. PROBLEM FORMULATION

In this paper, a generic rigid robot is considered as a general nonlinear mechanical system consisting of n_l links, whose motion can be determined by n fully actuated rigid joints. The motion of the robot can therefore be described in the joint space by the generalized coordinates $q(t) \in \mathbb{R}^n$, such as link positions that are assumed to be available for measurement, as well as their associated time derivatives are $\dot{q}(t) \in \mathbb{R}^n$ and $\ddot{q}(t) \in \mathbb{R}^n$. The dynamic behavior can therefore be represented by

$$M(q)\,\ddot{q} + C(q,\dot{q})\,\dot{q} + g(q) + r(\dot{q}) = \tau \tag{1a}$$

$$y = q \tag{1b}$$

with the initial condition $q(0), \dot{q}(0) \in \mathbb{R}^n$. The output $y(t) \in \mathbb{R}^n$ of (1) is available for measurement and equal to the link positions q. The system (1) is actuated by the input torque $\tau(t) \in \mathbb{R}^n$. In this equation, $M(q) \in \mathbb{R}^{n\times n}$ corresponds to the positive definite and symmetric inertia matrix. The vector $C(q,\dot{q})\,\dot{q} \in \mathbb{R}^n$ represents the Coriolis and centripetal components and $g(q) \in \mathbb{R}^n$ is regarded as the influence of gravitational force. Additionally, dissipative components such as Coulomb and viscous friction are taken into account by the vector $r(\dot{q}) \in \mathbb{R}^n$.

The respective matrices $M(q)$, $C(q,\dot{q})$ and the vectors $g(q)$ and $r(\dot{q})$ each depend on specific physical parameters which result from the physical interconnection of the n_l links. For each link $k = 1, ..., n_l$ the mass $m_k \in \mathbb{R}$, the first moment of inertia vector $\mathrm{col}(m_k r_{kx}, m_k r_{ky}, m_k r_{kz}) \in \mathbb{R}^3$, and the values $\mathrm{col}(J^k_{xx},\ J^k_{xy},\ J^k_{xz},\ J^k_{yy},\ J^k_{yz},\ J^k_{zz}) \in \mathbb{R}^6$ of the symmetric and positive definite inertia tensor J^k with respect to the k-th link frame can be defined by an inertial parameter vector $\theta_{k,I} = \mathrm{col}(m_k, m_k r_{kx}, m_k r_{ky}, m_k r_{kz}, J^k_{xx}, J^k_{xy}, J^k_{xz}, J^k_{yy}, J^k_{yz}, J^k_{zz}) \in \mathbb{R}^{10}$ (see [8]). Additionally for each actuated joint

$i = 1, ..., n$ the motor inertia J_{mi}, the Coulomb friction parameter $c_{fc,i}$ and the viscous friction parameter $c_{fv,i}$ are summarized in the dynamic parameter vector $\theta_{i,D} = \mathrm{col}(J_{mi}, c_{fc,i}, c_{fv,i}) \in \mathbb{R}^3$. This yields $n_p = 10n_l + 3n$ parameters for the whole rigid robot, which are combined in the parameter vector

$$\theta = \mathrm{col}(\theta_{1,I}, ..., \theta_{n_l,I}, \theta_{1,D}, ..., \theta_{n,D}) \in \mathbb{R}^{n_p}. \tag{2}$$

In order for the parameter values to be physically feasible, the conditions

$$m_k > 0 \tag{3a}$$

$$J^k = \begin{bmatrix} J^k_{xx} & J^k_{xy} & J^k_{xz} \\ J^k_{xy} & J^k_{yy} & J^k_{yz} \\ J^k_{xz} & J^k_{yz} & J^k_{zz} \end{bmatrix} \succ 0 \tag{3b}$$

$$J_{m_i} > 0 \tag{3c}$$

must be fulfilled for each link $k = 1, ..., n_l$ and each joint $i = 1, ..., n$.

The objective is to provide a reasonable estimate of the parameters θ based on the measured values q and τ. For this reason, the linearity of the equation of motion of the robot (1a) with respect to the parameters θ is exploited (see e.g. [8]), resulting in the equivalent representation

$$Y(q, \dot{q}, \ddot{q})\, \theta = \tau \tag{4}$$

for (1a) with $Y(q, \dot{q}, \ddot{q}) = \left[Y_1(q, \dot{q}, \ddot{q}), ..., Y_{n_p}(q, \dot{q}, \ddot{q})\right] \in \mathbb{R}^{n \times n_p}$. Due to the structure of the robot, $Y(q, \dot{q}, \ddot{q})$ inevitably has linear dependent columns, for all possible values $q, \dot{q}$ and $\ddot{q}$, which makes it difficult to identify unambiguously. For this reason, it is necessary to eliminate parameters that do not affect the equations of motion and simultaneously regroup linearly dependent parameters. The resulting regrouped parameters are called base parameters $\theta_B \in \mathbb{R}^{n_B}$ with $n_B \leq n_p$ (see [9]) and can either be derived directly from the robot structure (see, e.g., [2]) or derived from the symbolic interpretation of (1a). Thus, the equation (4) can be reformulated to

$$Y_B(q(t), \dot{q}(t), \ddot{q}(t))\, \theta_B = \tau(t), \tag{5}$$

with $Y_B(q(t), \dot{q}(t), \ddot{q}(t)) \in \mathbb{R}^{n \times n_B}$. Assuming $n < n_B$, it is still not possible to solve (5) for θ_B unambiguously. Therefore, by considering N_s sampling times $t_i, i = 1, ..., N_s, N_s \in \mathbb{N}$, and by stacking the sampled values of (5) and the torque in matrices and vectors the result

$$\bar{Y}_B\, \theta_B = \bar{\tau}, \tag{6}$$

yields, with the stacked regressor matrix

$$\bar{Y}_B = \begin{bmatrix} Y_B(q(t_1), \dot{q}(t_1), \ddot{q}(t_1)) \\ Y_B(q(t_2), \dot{q}(t_2), \ddot{q}(t_2)) \\ \vdots \\ Y_B(q(t_{N_s}), \dot{q}(t_{N_s}), \ddot{q}(t_{N_s})) \end{bmatrix} \in \mathbb{R}^{nN_s \times n_B}, \tag{7}$$

and the stacked torque vector

$$\bar{\tau} = \mathrm{col}(\tau(t_1), \tau(t_2), \cdots, \tau(t_{N_s})) \in \mathbb{R}^{nN_s}. \tag{8}$$

It is assumed that $n\,N_s > n_B$ applies and considering the sampled values $\mathrm{rank}\bar{Y}_B = n_B$ is valid. If furthermore $\mathrm{rank}\left[\bar{Y}_B\ \bar{\tau}\right] = n_B$ is fulfilled then (6) can be unambiguously solved for θ_B, such that (5) also applies for θ_B. But since (6) is overdetermined, it can become unsolvable if $\mathrm{rank}\left[\bar{Y}_B\ \bar{\tau}\right] > n_B$, such that an error $\varepsilon \in \mathbb{R}^{n\,N_s}$ needs to be introduced, which yields

$$\bar{Y}_B\, \theta_B = \bar{\tau} + \varepsilon. \tag{9}$$

However, the optimal solution $\hat{\theta}_B$ of (9) which minimizes the Euclidean norm of ε can be obtained by the ordinary least squares (OLS) method given by

$$\hat{\theta}_B = \bar{Y}_B^{\dagger} \bar{\tau}, \tag{10}$$

with the pseudoinverse $\bar{Y}_B^{\dagger}$ which can be calculated explicitly by

$$\bar{Y}_B^{\dagger} = (\bar{Y}_B^{\top} \bar{Y}_B)^{-1} \bar{Y}_B^{\top} \in \mathbb{R}^{n_B \times nN_s} \tag{11}$$

if $\mathrm{rank}\bar{Y}_B = n_B$. If the rank constraint is not fulfilled, the inverse of the Gram matrix ($\bar{Y}_B^{\top} \bar{Y}_B$ does not exist, so the formula (11) is not applicable. However, the pseudoinverse still exists, but is not unique. It is still necessary to verify whether the least squares optimal solution $\hat{\theta}_B$ actually meets the conditions (3). If on the other hand $\mathrm{rank}\left[\bar{Y}_B\ \bar{\tau}\right] = n_B$ and $\|\varepsilon\| = 0$ applies for the error, the least squares optimal solution solves (5) exactly, i.e. $\hat{\theta}_B = \theta_B$.

For the evaluation of (5) and also of (6) it is necessary that the time derivatives $\dot{q}$ and $\ddot{q}$ are available. However, since these can not be measured, it is necessary to reconstruct them, which inevitably leads to errors even if measurement noise and unmodelled dynamics are not taken into account. This makes the identification of θ_B more challenging. Summarizing, the three main issues that make a meaningful identification of the parameters difficult can be identified for parameter identification (see [6]):

I) The time derivatives $\dot{q}$ and $\ddot{q}$ of the positions q are not available for measurement and therefore have to be reconstructed, which accordingly leads to an error in (5) and thus to $\|\varepsilon\| > 0$ even if measurement noise and unmodelled dynamics are neglected.
II) Rank deficiencies and numerical ill-conditioning due to inadequate data may cause the inversion of $\bar{Y}_B^{\top} \bar{Y}_B$ to be numerically unstable. Furthermore, the required data is large and requires much memory space.
III) The identified base parameters θ_B must be physically feasible.

Therefore, the following section explains how the polynomial approximation presented in [7] can be used to obtain an equation for θ_B depending only on measurable signals but containing no equation error. Then, the discussed sampling approach can be applied, in order to obtain θ_B by the least squares method. Subsequently, an algorithm is presented, which allows the numerically stable computation of the pseudoinverse without a complicated singular value decomposition and at the same time results in a lower memory usage with sequentially acquired data.

III. POLYNOMIAL APPROXIMATION

In the following it is described how the application of the polynomial approximation operator $\mathcal{P}_{N,t_d}\{\cdot\}$, introduced in [7], can be used to obtain an equation for θ_B by integral transformation, such that no equation error results. For this reason, the procedure of the polynomial approximation is briefly reviewed in this section. For a detailed explanation of the procedure, the reader is refered to [7].

For a quadratically Lebesgue integrable signal $x(t)$ the bijective transformation $\phi_T : \tilde{\mathcal{I}} = [-1,1] \mapsto \mathcal{I}_{t,T}$ and the corresponding inverse mapping ϕ_T^{-1}, can be used to define $\bar{x} = x \circ \phi_T$ on a Hilbert space $\mathcal{H} = L_2(-1,1)$ with the inner product

$$\langle \varphi_i, \varphi_j \rangle = \int_{-1}^{1} \varphi_i(\tau)\varphi_j(\tau) w^{(\alpha,\beta)}(\tau)\mathrm{d}\tau, \quad \forall \varphi_i, \varphi_j \in \mathcal{H}, \tag{12}$$

and the corresponding induced norm

$$\|\varphi\| = \sqrt{\langle \varphi, \varphi \rangle}, \quad \forall \varphi \in \mathcal{H}. \tag{13}$$

The weight function $w^{(\alpha,\beta)}$, which is given by

$$w^{(\alpha,\beta)}(\tau) = \begin{cases} (1-\tau)^{\alpha}(1+\tau)^{\beta}, & \tau \in [-1,1] \\ 0, & \tau \notin [-1,1]\,, \end{cases} \tag{14}$$

with real exponential coefficients $\alpha, \beta > -1$ allows to introduce an orthonormal basis $P_i^{(\alpha,\beta)}, i \in \mathbb{N}_0$ for $\mathcal{H}$ by the normalized Jacobi polynomials $P_i^{(\alpha,\beta)}$ (see [10, Sec. 4.3]). According to the projection theorem (see, e.g., [11]) the best fitting (in the least squares sense) approximation of N-th order $\hat{x} \in \mathcal{H}$ of $\bar{x}$ always exists unambiguously, and can be evaluated at any time $t - t_d \in \mathcal{I}_{t,T}, t_d > 0$ by

$$\hat{x}(t-t_d) = \sum_{i=0}^{N} \underbrace{\langle \bar{x}, P_i^{(\alpha,\beta)} \rangle}_{c_i} P_i^{(\alpha,\beta)} \circ \phi_T^{-1}(t-t_d), \tag{15}$$

with the constant coefficients $c_i = \langle x \circ \phi_T, P_i^{(\alpha,\beta)} \rangle$. The added delay $t_d \geq 0$ is chosen as a zero $p_{N+1}^{(\alpha,\beta)}$ of the Jacobi polynomial $P_{N+1}^{(\alpha,\beta)}$, in order to reduce the order of the approximation error by one (see [12]). The delayed polynomial approximation of x based on (15) can thus be written as an integral within the original time window $\mathcal{I}_{t,T}$ by

$$\hat{x}(t-t_d) = \int_0^T x(t-\tau) g_{N,t_d}(\tau)\mathrm{d}\tau =: \mathcal{P}_{N,t_d}\{x\}(t), \tag{16}$$

with

$$R_{N,t_d}^{(\alpha,\beta)}(\tau) = \sum_{i=0}^{N} P_i^{(\alpha,\beta)}(\tau)\,(P_i^{(\alpha,\beta)} \circ \phi_T^{-1}(t-t_d)) \tag{17}$$

and with the time independent kernel

$$g_{N,t_d}(\bar{\tau}) = \frac{2}{T}(R_{N,t_d}^{(\alpha,\beta)}\, w^{(\alpha,\beta)}) \circ \phi_T^{-1}(t-\tau)\,, \tag{18}$$

yielding the definition of the polynomial approximation operator $\mathcal{P}_{N,t_d}\{\cdot\}$ (see [7]). As already introduced in [7] the polynomial approximation operator is linear an has further properties, which allow further simplification of differential algebraic expressions. The polynomial approximation of the k-th time derivative of x with $k < \alpha, \beta$ can be determined by the differentiation approximation operator $\mathcal{P}_{N,t_d}^{(k)}\{x\}(t)$, which is given by

$$\mathcal{P}_{N,t_d}\{x^{(k)}\}(t) = \int_0^T x(t-\tau) g_{N,t_d}^{(k)}(\tau)\mathrm{d}\tau =: \mathcal{P}_{N,t_d}^{(k)}\{x\}(t) \tag{19}$$

with the derivative of the kernel given by

$$g_{N,t_d}^{(k)}(\tau) = (-1)^k \left(\frac{2}{T}\right)^{k+1} (R_{N,t_d}^{(\alpha,\beta)} w^{(\alpha,\beta)})^{(k)} \circ \phi_T^{-1}(t-\tau). \tag{20}$$

Furthermore, in order to consider nonlinear expressions the composition of x by a Lipschitz continuous function $\psi : \mathbb{R} \mapsto \mathbb{R}$ can be considered by

$$\mathcal{P}_{N,t_d}\{\psi(x)\}(t) \approx \psi\left(\mathcal{P}_{N,t_d}\{x\}(t)\right) = \psi(\hat{x})\,. \tag{21}$$

The polynomial approximation of the product $x_1 x_2$ of two signals $x_1, x_2 \in L_2([t-T,t])$ and $x_1 \in \pi_{N^*}$ with $N^* \in \mathbb{N}$ can be defined by

$$\mathcal{P}_{N,t_d}\{x_1\, x_2\}(t) = \sum_{i=0}^{N^*} c_i \mathcal{P}_{N,t_d}\{(P_i^{(\alpha,\beta)} \circ \phi_T^{-1})\, x_2\}(t) \tag{22}$$

$$= \sum_{i=0}^{N^*} \mathcal{P}_{N,0,c_i}\{x_1\}(t) \tilde{\mathcal{P}}_{N,t_d,i}\{x_2\}(t) \tag{23}$$

with the modified polynomial approximation operator given by

$$\begin{aligned} \tilde{\mathcal{P}}_{N,t_d,i}\{x\}(t) &= \mathcal{P}_{N,t_d}\{(P_i^{(\alpha,\beta)} \circ \phi_T^{-1}) x\}(t) \\ &= \int_0^T x(t-\tau)\tilde{g}_{N,t_d,i}(\tau)\mathrm{d}\tau, \end{aligned} \tag{24}$$

with the kernel

$$\tilde{g}_{N,t_d,i}(\tau) = \frac{2}{T}(P_i^{(\alpha,\beta)}\, R_{N,t_d}^{(\alpha,\beta)}\, w^{(\alpha,\beta)}) \circ \phi_T^{-1}(t-\tau)\,. \tag{25}$$

The coefficients c_i can be obtained by

$$c_i = \int_0^T x(t-\tau) g_{c_i}(\tau)\mathrm{d}\tau =: \mathcal{P}_{N,0,c_i}\{x\}(t), \tag{26}$$

with the kernel

$$g_{c_i}(\tau) = \frac{2}{T}(P_i^{(\alpha,\beta)}\, w^{(\alpha,\beta)}) \circ \phi_T^{-1}(t-\tau). \tag{27}$$

Thus, multiplicative differential algebraic expressions can also be polynomially approximated without the necessity of numerically differentiating the signals, since the kernels (25) (27) have compact support (see [7]). Note that the application of the polynomial approximation operator corresponds to the evaluation of time-discrete FIR filters with subsequent processing of the filtered values (see [7]).

In the following, the polynomial approximation operator is applied in the context of the presented problem in order to evaluate (5) consistently solely by the measurable signals q and τ.

IV. POLYNOMIAL APPROXIMATION OF THE REGRESSION

The approach to obtain an expression for θ_B only depending on measurable signals is achieved by the application of the polynomial approximation operator to both sides of the equation (4) and yields

$$\mathcal{P}_{N,t_d}\{Y_B(q,\dot{q},\ddot{q})\}(t)\theta_B = \mathcal{P}_{N,t_d}\{\tau\}(t). \tag{28}$$

By further decomposition according to (19)-(27) it is possible to evaluate (28) solely on the basis of measurable values q and τ, which will be explained in the following.

If the polynomial approximation operator $\mathcal{P}_{N,t_d}\{\cdot\}$ is applied to the signals q and τ, the approximated values $\hat{q}$ and $\hat{\tau}$ can be represented as a truncated series of Jacobi polynomials to the order N

$$\hat{q} = \mathcal{P}_{N,t_d}\{q\}(t) = \sum_{i=0}^{N} c_{q,i} P_i^{(\alpha,\beta)} \circ \phi_T^{-1}(t-t_d) \tag{29}$$

and

$$\hat{\tau} = \mathcal{P}_{N,t_d}\{\tau\}(t) = \sum_{i=0}^{N} c_{\tau,i} P_i^{(\alpha,\beta)} \circ \phi_T^{-1}(t-t_d). \tag{30}$$

An analog representation for the time derivatives $\dot{q}$ and $\ddot{q}$ can be obtained by applying the differentiation approximation operator to q, which is given by

$$\hat{\dot{q}} = \mathcal{P}^{(2)}_{N,t_d}\{q\}(t) = \sum_{i=0}^{N} c_{q,i} P_i^{(\alpha,\beta)} \circ \phi_T^{-1}(t-t_d) \tag{31}$$

and

$$\hat{\ddot{q}} = \mathcal{P}^{(2)}_{N,t_d}\{q\}(t) = \sum_{i=0}^{N} c_{\tau,i} P_i^{(\alpha,\beta)} \circ \phi_T^{-1}(t-t_d). \tag{32}$$

It is furthermore possible to apply the polynomial approximation operator directly to the differential algebraic expressions in the j-th column of $Y_B(q,\dot{q},\ddot{q})$. By using the partial approximation (22) and composition (21) with subsequent integration by parts can therefore also be represented as the truncated series

$$\mathcal{P}_{N,t_d}\{Y_{B,j}(q,\dot{q},\ddot{q})\}(t) = \sum_{i=0}^{N} c_{j,i} P_i^{(\alpha,\beta)} \circ \phi_T^{-1}(t-t_d), \tag{33}$$

for $j = 1, ..., n_B$, in which the right hand side depends solely on the measurable variable q (see [7]).

By substituting the columns of $Y_B(q,\dot{q},\ddot{q})$ and the torque vector τ by the approximated expressions (33) and (30) in (5) the truncated series expansion

$$\begin{aligned}
&\theta_{B,1} \underbrace{\sum_{i=0}^{N} c_{1,i} P_i^{(\alpha,\beta)} \circ \phi_T^{-1}(t-t_d)}_{=\mathcal{P}_{N,t_d}\{Y_{B,1}(q,\dot{q},\ddot{q})\}(t)} + \cdots \\
&\quad + \theta_{B,n_B} \underbrace{\sum_{i=0}^{N} c_{n_B,i} P_i^{(\alpha,\beta)} \circ \phi_T^{-1}(t-t_d)}_{=\mathcal{P}_{N,t_d}\{Y_{B,n_B}(q,\dot{q},\ddot{q})\}(t)} \\
&\quad = \underbrace{\sum_{i=0}^{N} c_{\tau,i} P_i^{(\alpha,\beta)} \circ \phi_T^{-1}(t-t_d)}_{=\mathcal{P}_{N,t_d}\{\tau\}(t)}.
\end{aligned} \tag{34}$$

results. This series is equivalent to the projection of both sides of (5) onto the orthonormal base $\{P_i^{(\alpha,\beta)}\}_{i=0}^{N}$, which introduces no equation error. Furthermore, all components of (34) only depend on the measurable variables q and τ. The resulting equation can be brought into the matrix form

$$Y_B^* \theta_B = \tau^*, \tag{35}$$

with the projected regressor matrix

$$Y_B^* = \begin{bmatrix} \mathcal{P}_{N,t_d}\{Y_{B,1}\}(t) & \cdots & \mathcal{P}_{N,t_d}\{Y_{B,n_B}\}(t) \end{bmatrix} \in \mathbb{R}^{n \times n_B}, \tag{36}$$

and the projected torque vector $\tau^* = \mathcal{P}_{N,t_d}\{\tau\}(t) \in \mathbb{R}^n$. Analogous to (6) the equation can now be evaluated for the times $t_1, ..., t_{N_S}$ yielding the stacked regressor matrix $\bar{Y}_B^*$ and stacked torque vector $\bar{\tau}^*$. The least squares optimal estimation of the parameters $\hat{\theta}_B$ thus results in

$$\hat{\theta}_B = (\bar{Y}_B^*)^{\dagger} \bar{\tau}^*, \tag{37}$$

which is the exact solution for (35) and for (5). Since for the evaluation of the integrals of each individual approximation operator (15), (19), (24) and (26) the respective kernels are each independent of time t, these integral transformations can be realized for each element of the stacked regressor matrix as well as the torque vector as a time-discrete FIR filter (see [7]), which can be evaluated either online or offline solely on the basis of the measurable positions q and the torques τ.

Note furthermore that the effect of measurement noise on the respective elements in the regressor matrix and the torque vector can be influenced by the parameters N, T, α and β, in order to achieve damping and even exact suppression of specific frequencies (see [13]).

Since the pseudoinverse needs to be calculated for (37), the Gram matrix $(\bar{Y}_B^*)^\top \bar{Y}_B^*$ must be inverted according to (11), if $\mathrm{rank}\bar{Y}_B^* = n_B$ is assumed. The inversion becomes numerically unstable if the matrix is ill-conditioned. For this reason, the following section provides an algorithm which allows the base parameters θ_B to be estimated on the basis of an initial guess $\theta_{B,0}$, by calculating the pseudoinverse numerically stable without a singular value decomposition.

V. MODIFIED RECURSIVE LEAST SQUARES

In order to overcome problem II, the insufficiency of the measured data and the limitation of the memory space, an algorithm is presented in this section, which allows the computation of the estimated parameters iteratively and thereby ensures the numerical stability during the inversion of the matrix $(\bar{Y}_B^*)^\top \bar{Y}_B^*$ for the pseudoinverse. The presented procedure is similar the approach of hybrid recursive least squares and also based on sequentially obtained data blocks (see [14]). The inversion, however is calculated explicitly to avoid error accumulation as in recursive algorithms (see, e.g., [6]).

Note further that the procedure described below can be used independently of the polynomial approximation presented. Therefore, the stacked regressor matrix, the parameter and the stacked torque vector are defined generally by $\bar{Y} \in \mathbb{R}^{nN_s \times n_p}$,$\theta \in \mathbb{R}^{n_p}$ and $\bar{\tau} \in \mathbb{R}^{nN_s}$, which can be respectively substituted by the polynomial approximations and/or the base parameters.

It should be mentioned at this point that the ordinary least squares OLS method always produces the best results in least squares sense if the measured values are recorded for the whole trajectory, which must excite all parameter values (see, e.g., [15]). It is furthermore not possible to consider a priori known parameter values. However, it is not always possible to guarantee both prerequisites, which is the reason for the necessity of a procedure which leads iteratively to meaningful parameter values even with data determined in sections.

Assuming that the data is obtained sequentially, so that both the stacked regressor matrix $\bar{Y}$ and the stacked torque vector $\bar{\tau}$ can be split into $n_s \in \mathbb{N}$ individual sequences

$$\bar{\tau} = \mathrm{col}(\bar{\tau}_1, \bar{\tau}_2, \cdots, \bar{\tau}_{n_s}) \tag{38}$$

and

$$\bar{Y} = \mathrm{col}(\bar{Y}_1, \bar{Y}_2, \cdots, \bar{Y}_{n_s}) \tag{39}$$

with the respective data sequence $\bar{Y}_k \in \mathbb{R}^{nn_k \times n_p}$, $\bar{\tau}_k \in \mathbb{R}^{n\,n_k}$, $n_k \in \mathbb{N}$, such that $\sum_{k=0}^{n_s} n_k = N_s$. Furthermore, the accumulated regressor matrix $\bar{Y}_k$ is defined by $\bar{Y}_k = \mathrm{col}(\bar{Y}_1, ..., \bar{Y}_k)$, which contains the information received up to the k-th step. The Gram matrix $K = \bar{Y}^\top \bar{Y} \in \mathbb{R}^{n_p \times n_p}$ can then be calculated recursively in each step according to

$$\begin{aligned} K_{k+1} &= K_k + \bar{Y}_{k+1}^\top \bar{Y}_{k+1} = \bar{Y}_{k+1}^\top \bar{Y}_{k+1} \\ &= \begin{bmatrix} \langle \bar{Y}_{k+1}^1, \bar{Y}_{k+1}^1 \rangle & \langle \bar{Y}_{k+1}^1, \bar{Y}_{k+1}^2 \rangle & \cdots & \langle \bar{Y}_{k+1}^1, \bar{Y}_{k+1}^{n_p} \rangle \\ \langle \bar{Y}_{k+1}^1, \bar{Y}_{k+1}^2 \rangle & \langle \bar{Y}_{k+1}^2, \bar{Y}_{k+1}^2 \rangle & \cdots & \langle \bar{Y}_{k+1}^2, \bar{Y}_{k+1}^{n_p} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \bar{Y}_{k+1}^1, \bar{Y}_{k+1}^{n_p} \rangle & \langle \bar{Y}_{k+1}^2, \bar{Y}_{k+1}^{n_p} \rangle & \cdots & \langle \bar{Y}_{k+1}^{n_p}, \bar{Y}_{k+1}^{n_p} \rangle \end{bmatrix} \end{aligned} \tag{40}$$

with the inner vector product denoted by $\langle \cdot, \cdot \rangle$ and $\bar{Y}_{k+1}^j$ denoted as the j-th column of $\bar{Y}_{k+1}$, $j = 1, ..., n_p$. Based on an initial guess θ_0 the parameters may be updated in each step by

$$\theta_{k+1} = \theta_k + K_{k+1}^{-1} \bar{Y}_{k+1}^\top (\bar{\tau}_k - \bar{Y}_{k+1} \theta_k). \tag{41}$$

The inverse $K_{k+1}^{-1} = \left(\bar{Y}_{k+1}^\top \bar{Y}_{k+1} \right)^{-1}$ must be calculated in each step. There are two possible cases which can lead to instability of the inversion. On the one hand, if a column in $\bar{Y}_{k+1}$ is not sufficiently excited and on the other hand if the columns are linearly dependent. The first case can easily be checked using the principal diagonal entries of the matrix K_{k+1}, since the values $\langle \bar{Y}_{k+1}^j, \bar{Y}_{k+1}^j \rangle, j = 1, ..., n_p$ are given here. The investigation of the second issue, however, is more complicated. It is necessary to obtain the orthogonal components of the respective columns. One strategy to achieve this is given by the Gram-Schmidt process, which can be used to form the orthogonal base to the columns of $\bar{Y}_{k+1}$. The orthogonal basis vectors can be determined successively using

$$v_1 = \bar{Y}_{k+1}^1 \tag{42a}$$

$$v_2 = \bar{Y}_{k+1}^2 - \frac{\langle \bar{Y}_{k+1}^2, v_1 \rangle}{\langle v_1, v_1 \rangle} v_1 \tag{42b}$$

$$\vdots$$

$$v_{n_p} = \bar{Y}_{k+1}^{n_p} - \sum_{i=1}^{n_p - 1} \frac{\langle \bar{Y}_{k+1}^{n_p}, v_i \rangle}{\langle v_i, v_i \rangle} v_i. \tag{42c}$$

If the j-th column $\bar{Y}_{k+1}^j$ depends strongly on the others, this can be determined by the fact that the squared absolute value $\|v_j\|^2 = \langle v_j, v_j \rangle$ of the vector v_j becomes very small. More precisely, the value of $\langle v_j, v_j \rangle$ can be compared with a minimum value $\sigma_j^2 \in \mathbb{R}^+$ of the corresponding column $\bar{Y}_{k+1}^j$ to check whether the orthogonal part is sufficiently excited. Then, $\bar{Y}_{k+1}$ must be removed, since it does not provide any relevant information for identification and hence, identifying the j-th parameter is therefore not reasonable. Thus, when calculating the inverse K_{k+1}^{-1}, the j-th row and column will be ignored and set to zero so that the j-th parameter is taken at its initial value $\theta_{j,0}$. For this, the absolute values of the orthogonal vectors $v_j, j = 1, ..., n_p$ have to be determined respectively. According to the relation (42), the orthogonality of the vectors, i.e. $\langle v_k, v_l \rangle = 0, k, l = 1, ..., j, k \neq l$ and the commutativity of the inner product, i.e. $\langle v_k, v_l \rangle = \langle v_l, v_k \rangle, \forall k, l \in \mathbb{N}$, the squared absolute value can be expressed by

$$\langle v_j, v_j \rangle = \langle \bar{Y}_{k+1}^j, \bar{Y}_{k+1}^j \rangle - \sum_{i=1}^{j-1} \frac{\langle \bar{Y}_{k+1}^j, v_i \rangle^2}{\langle v_i, v_i \rangle}. \tag{43}$$

Furthermore, based on (42) the inner product of any column $\bar{Y}_{k+1}^j, j = 1, ..., n_p$ with a orthogonal vector $v_l, l = 1, ..., j$ can be reformulated to

$$\langle v_l, \bar{Y}_{k+1}^j \rangle = \langle \bar{Y}_{k+1}^l, \bar{Y}_{k+1}^j \rangle - \sum_{i=1}^{l-1} \frac{\langle \bar{Y}_{k+1}^j, v_i \rangle \langle \bar{Y}_{k+1}^l, v_i \rangle}{\langle v_i, v_i \rangle}. \tag{44}$$

Assuming that $\langle v_i, v_i \rangle \neq 0, i = 1, ..., n_p$, an upper triangular form $\tilde{K}_{k+1}$ of K_{k+1} can be obtained by the Gauss elimina-

tion and yields

$$\tilde{K}_{k+1} = \begin{bmatrix} \langle v_1, v_1 \rangle & \langle v_1, \bar{Y}^2_{k+1} \rangle & \cdots & \langle v_1, \bar{Y}^{n_p}_{k+1} \rangle \\ 0 & \langle v_2, v_2 \rangle & \cdots & \langle v_2, \bar{Y}^{n_p}_{k+1} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \langle v_{n_p}, v_{n_p} \rangle \end{bmatrix}, \quad (45)$$

such that for the i-th row of $\tilde{K}_{k+1}$ the first $i-1$ elements are zero, the i-th element is $\langle v_i, v_i \rangle$ and the $l = i+1, ..., n_p$ elements are $\langle v_i, Y^l_{k+1} \rangle$. The proof is provided by induction. The base case is fulfilled for $i = 1$ by the first row of K_{k+1}, since $v_1 = \bar{Y}^1_{k+1}$. Consider the calculation of the $i+1$-th row of the matrix $\tilde{K}_{k+1}$ based on $\tilde{K}_{k+1}$ as the induction step. The calculation of the first $j = 1, ..., i$ elements $\tilde{K}^{i,j}_{k+1}$ of the matrix $\tilde{K}_{k+1}$ based on K_{k+1} results in

$$\begin{aligned} \tilde{K}^{i+1,j}_{k+1} &= K^{i+1,j}_{k+1} - \sum_{l=1}^{j-1} a_l \langle v_l, \bar{Y}^j_{k+1} \rangle - a_j \langle v_j, v_j \rangle \\ &= \langle \bar{Y}^j_{k+1}, \bar{Y}^{i+1}_{k+1} \rangle - \sum_{l=1}^{j-1} a_l \langle v_l, \bar{Y}^j_{k+1} \rangle - a_j \langle v_j, v_j \rangle \overset{!}{=} 0, \end{aligned} \quad (46)$$

with the values $a_l, l = 1, ..., j \in \mathbb{R}$ which need to be determined. These can be set to

$$a_l = \frac{\langle v_l, \bar{Y}^{i+1}_{k+1} \rangle}{\langle v_l, v_l \rangle} \quad (47)$$

which is also proven by induction. The base case for $j = 1$ is fulfilled, since

$$\tilde{K}^{i+1,1}_{k+1} = \langle \bar{Y}^1_{k+1}, \bar{Y}^{i+1}_{k+1} \rangle - \frac{\langle v_1, \bar{Y}^{i+1}_{k+1} \rangle}{\langle v_1, v_1 \rangle} \langle v_1, v_1 \rangle = 0. \quad (48)$$

The inductive step for $j+1$ is given by

$$\begin{aligned} \tilde{K}^{i+1,j+1}_{k+1} = & \underbrace{\langle \bar{Y}^{j+1}_{k+1}, \bar{Y}^{i+1}_{k+1} \rangle - \sum_{l=1}^{j} \frac{\langle v_l, \bar{Y}^{i+1}_{k+1} \rangle}{\langle v_l, v_l \rangle} \langle v_l, \bar{Y}^{j+1}_{k+1} \rangle}_{\overset{(44)}{=} \langle v_{j+1}, \bar{Y}^{i+1}_{k+1} \rangle} \\ & - \underbrace{\frac{\langle v_{j+1}, \bar{Y}^{i+1}_{k+1} \rangle}{\langle v_{j+1}, v_{j+1} \rangle} \langle v_{j+1}, v_{j+1} \rangle}_{= \langle v_{j+1}, \bar{Y}^{i+1}_{k+1} \rangle} \overset{!}{=} 0. \end{aligned} \quad (49)$$

Thus, for all $j = 1, ..., i$ the elements of $\tilde{K}^{i+1,j}_{k+1}$ vanish, if the values $a_l, l = 1, ..., i$ are chosen according to (47). The values a_l are the factors of the $l = 1, ..., i$ rows which need to be substracted from the $i+1$-th. Based on these results, it can be shown that the $i+1$-th element $\tilde{K}^{i+1,i+1}_{k+1}$ is determined by

$$\begin{aligned} \tilde{K}^{i+1,i+1}_{k+1} &= K^{i+1,i+1}_{k+1} - \sum_{l=1}^{i} a_l \langle v_l, \bar{Y}^{i+1}_{k+1} \rangle \\ &\overset{(47)}{=} \langle \bar{Y}^{i+1}_{k+1}, \bar{Y}^{i+1}_{k+1} \rangle - \sum_{l=1}^{i} \frac{\langle v_l, \bar{Y}^{i+1}_{k+1} \rangle}{\langle v_l, v_l \rangle} \langle v_l, \bar{Y}^{i+1}_{k+1} \rangle \quad (50) \\ &\overset{(43)}{=} \langle v_{i+1}, v_{i+1} \rangle, \quad (51) \end{aligned}$$

which also fulfills the induction step for $i+1$. The $j = i+2, ..., n_p$ elements of the row are given by

$$\begin{aligned} \tilde{K}^{i+1,j}_{k+1} &= K^{i+1,j}_{k+1} - \sum_{l=1}^{i} a_l \langle v_l, \bar{Y}^j_{k+1} \rangle \\ &\overset{(47)}{=} \langle \bar{Y}^{i+1}_{k+1}, \bar{Y}^j_{k+1} \rangle - \sum_{l=1}^{i} \frac{\langle v_l, \bar{Y}^{i+1}_{k+1} \rangle}{\langle v_l, v_l \rangle} \langle v_l, \bar{Y}^j_{k+1} \rangle \quad (52) \\ &\overset{(44)}{=} \langle v_{i+1}, \bar{Y}^j_{k+1} \rangle. \quad (53) \end{aligned}$$

Therefore, by induction it can be proven that the $i+1$-th row and therefore for all $i = 1, ..., n_p$ the i-th rows of the matrix $\tilde{K}_{k+1}$ have the form (45). Furthermore the form (45) is a necessary partial result of the inversion of K_{k+1} if the Gauss-Jordan algorithm is used.

For this purpose the Gauss-Jordan algorithm is modified in order fulfill the parameter identification task and simultaneously evaluate $\langle v_i, v_i \rangle, i = 1, ..., n_p$. A confidence parameter $s_i, i = 1, ..., n_B$ can thus added and assigned to each specific parameter, which marks whether the specific parameter has been identified ($s_i = 1$), or whether the value of the initial parameter was left due to insufficient excitation ($s_i = 0$) or due to linear dependency with other columns in the regressor matrix ($s_i = -1$). The parameter s_i can be used subsequently to interpret the reason for the ill-conditioning of K_{k+1}. The presented algorithm is shown in Fig. 1. Note further that the parameter values are only adapted and thus identified if the corresponding column in the regressor is not linearly dependent to the others and the respective excitation is sufficient. Otherwise, if $s_i = 0$ or $s_i = -1$, the parameter value θ_i remains at its initial value $\theta_{0,i}$. The determined confidence parameters allow a subsequent interpretation if the dependency of several parameters is too similar or if a value was not excited during the movement. Thus, either a simplification of the parameter values or an adjustment of the identification trajectory can be made subsequently.

VI. FEASIBILITY OF PARAMETERS

To ensure that the base parameters can be correctly identified, it is necessary to check whether obtained estimate $\hat{\theta}_B$ is valid w.r.t. (3). Considering the iterative adaptation of the parameters by the presented modified least squares method, it is possible to verify the corresponding parameters in each adaptation step. In this paper it is assumed that the limitations of the base parameters can be described by box constraints, analogous to [16]. In this case, the constraints of any parameter θ can each be described by

$$\theta_i \in \left[\underline{\theta}_i, \overline{\theta}_i \right] =: \mathcal{I}_{\theta_i}, \quad \forall i = 1, ..., n_p. \quad (54)$$

By applying set arithmetic, the limits $\underline{\theta}_i, \overline{\theta}_i$ can be transfered to the base parameters θ_B yielding $\underline{\theta}_{B,i}, \overline{\theta}_{B,i}$. Thus, the parameter adaption in line 37 in Fig. 1 can be limited to the corresponding upper and lower bounds $\underline{\theta}_{B,i}$ and $\overline{\theta}_{B,i}$. Thereby, the parameters are always determined feasible, but can become suboptimal in least squares sense. Note that complex constraints which result for example for the inertia matrix J^i can be rewritten according to the parallel axis

Initialization:

for $i = 1$ **to** n_p **do**
  $s_i = 0;$ ▷ Initialize all s_i with zero
end for
$\theta_0 = \theta_0;$ ▷ Set parameter value to initial guess
$K_0 = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix};$ ▷ Set initial value of K
$k = 0;$

Iteration:

$K_{k+1} = K_k + \bar{Y}_{k+1}^\top \bar{Y}_{k+1}$ ▷ Update K
$M = [K_{k+1} | I]$ ▷ Set augmented matrix M
▷ Calculate upper triangular form
for $i = 1$ **to** n_p **do**
  if $M(i,i) > \sigma^2$ **then** ▷ Evaluate $\langle v_j, v_j \rangle > \sigma^2$
    $s_i = 1;$ ▷ Parameter is identifiable
    for $j = i+1$ **to** n_p **do**
      $M(j,:) = M(j,:) - \frac{M(j,i)}{M(i,i)} M(i,:);$
    end for
  else ▷ Parameter is not identifiable
    if $K_{k+1}(i,i) > \sigma^2$ **then**
      $s_i = -1;$ ▷ Due to linear dependency
    else
      $s_i = 0;$ ▷ Due to insufficient excitation
    end if
    $M(i,:) = 0;$
    $M(:,i) = 0;$
  end if
end for
▷ Invert matrix
for $i = 1$ **to** n_p **do**
  if $M(i,i) > 0$ **then**
    $M(i,:) = M(i,:)/M(i,i);$
    for $j = 1$ **to** $i-1$ **do**
      $M(j,:) = M(j,:) - \frac{M(j,i)}{M(i,i)} M(i,:);$
    end for
  end if
end for
$K_{k+1}^{-1} = M(1:n_p, n_p+1:2n_p);$ ▷ Set inverse
▷ Update parameters
$\theta_{k+1} = \theta_k + K_{k+1}^{-1} \bar{Y}_{k+1}^\top (\bar{\tau}_k - \bar{Y}_{k+1}\theta_k);$
$k = k+1;$

Fig. 1. Iterative parameter adaption by the modified recursive least squares method

theorem (see [8]) and thus be limited by the maximum and minimum values of the masses and moments, which can be obtained from geometric information or from CAD data for example (see, e.g., [3]). Note also that if the corresponding constraints can not be considered as fixed box constraints, it is still possible to apply a nonlinear optimization subsequently in order to determine physically feasible parameters θ that match the base parameters $\hat{\theta}_B$. Possible approaches for this were presented in [17] and [3].

VII. RESULTS

The presented methods were used to identify the dynamic parameters of a closed-loop delta robot of autonox24. This is operated by three Siemens motors of the type 1FK7033-4CK71-1RH2, a SIMATIC 1517 TF controller and SINAMICS S120 drives. The experimental setup is shown in Fig. 2.

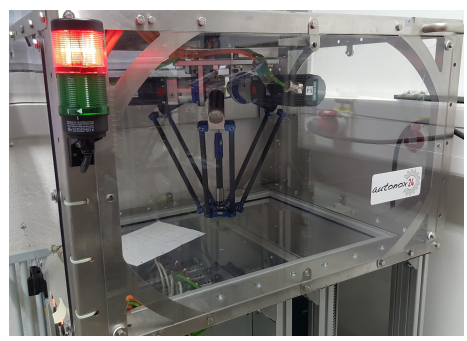

Fig. 2. Experimental setup of the delta robot to be identified.

The identification was done offline based on 20000 samples with the corresponding position and torque values of the three axes based on an example trajectory and the sample time $T_S = 4\text{ms}$. The identified base parameters are shown in Tab. I for the ordinary least squares method with zero phase Savitzky Golay filters (OLS filtered), for the ordinary least squares method with the presented polynomial approximation (OLS with PA) and for the presented modified recursive least squares method with polynomial approximation (MRLS with PA). The window size of the Savitzky Golay filters has been chosen to $L = 11$ and second order polynomials have been considered. The parameters of the presented polynomial approximation have been chosen to $T = 44\text{ms}$, $N = 1, N^* = 2$ and $t_d = 14.7\text{ms}$. It is shown in Tab. I that the identified parameter values of the MRLS procedure converge to those of the OLS, since the trajectory has been chosen such that $\text{rank}\bar{Y}_B = n_B$. In order to rate the identified values, a simulation model was implemented and excited by the actually measured torque. Thus, the different parameter sets can be evaluated by comparing them with the real measured values q_m.

Based on the graph in Fig. 3 it is shown that both parameter values allow a good approximation of the system behaviour, but do not exactly reproduce it, due to unmodelled dynamics. However, as shon in Fig. 3 the parameters

TABLE I

IDENTIFIED PARAMETER VALUES

parameter	unit	OLS filtered	OLS with PA	MRLS with PA
$c_{fc,1}$	N	2.1006	2.2910	2.2910
$c_{fc,2}$	N	2.2885	2.4565	2.4565
$c_{fc,3}$	N	2.2406	2.4722	2.4722
$c_{fv,1}$	Nsrad^{-1}	1.1139	0.9937	0.9937
$c_{fv,2}$	Nsrad^{-1}	1.2097	1.0965	1.0965
$c_{fv,3}$	Nsrad^{-1}	1.0165	0.8747	0.8747
$J_{m1} + J^1$	kgm^2	0.0691	0.0716	0.0716
$J_{m2} + J^2$	kgm^2	0.0711	0.0732	0.0732
$J_{m3} + J^3$	kgm^2	0.0690	0.0712	0.0712
$m1\|r_1\|$	kgm	0.0526	0.0526	0.0526
$m2\|r_2\|$	kgm	0.0442	0.0446	0.0446
$m3\|r_3\|$	kgm	0.0567	0.0552	0.0552
$m4$	kg	0.4586	0.4802	0.4802

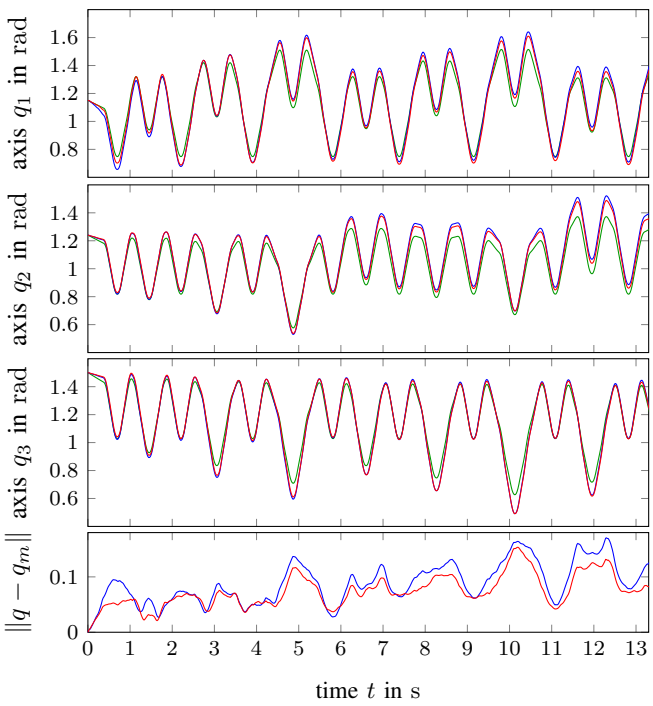

Fig. 3. Simulation results of the positions q_1, q_2 and q_3 with the identified parameters (—— OLS filtered) and (—— OLS with PA) and the actual measured values q_m (——) as well as $\|q - q_m\|$ for (—— OLS filtered) and (—— OLS with PA) the delta robot.

according to the presented method (OLS with PA) achieve better results w.r.t. the euclidian norm $\|q - q_m\|$. It should also be noted that in contrast to the method with Savitzky Golay filters, the identified parameters in OLS or MRLS with PA lead to similar parameter values even for a different parametrization (i.e. N, t_d, α and β) of the FIR filter. The presented method based on the polynomial approximation is therefore more robust w.r.t. the parametrization of the polynomial approximation.

VIII. CONCLUDING REMARKS

In this paper a method was presented to identify the dynamic and inertial parameters of a rigid robot. The equation to obtain the parameters can be evaluated solely on the basis of measurable values by means of polynomial approximation. Furthermore, an algorithm was presented, with the help of which the parameters can be determined physically feasible and sequentially, despite possibly inadequate data. The presented methods can be applied independent of each other and without any restriction to other parameter identification problems. In further research work, additional enhancements in identification and the extension of the procedure to problems of fault detection w. r. t. parameter faults will be investigated.

ACKNOWLEDGMENT

The authors kindly express their gratitude to the industrial research partner Siemens AG, Digital Industries Operating Company Erlangen for funding and supporting this project.

REFERENCES

[1] M. Gautier, "Numerical calculation of the base inertial parameters of robots," in *Proceedings., IEEE International Conference on Robotics and Automation*, 1990, pp. 1020–1025.

[2] M. Gautier and W. Khalil, "Direct calculation of minimum set of inertial parameters of serial robots," *IEEE Transactions on Robotics and Automation*, vol. 6, pp. 368–373, 1990.

[3] C. Gaz, F. Flacco, and A. De Luca, "Extracting feasible robot parameters from dynamic coefficients using nonlinear optimization methods," in *2016 IEEE International Conference on Robotics and Automation (ICRA)*, 2016, pp. 2075–2081.

[4] J. Swevers, C. Ganseman, D. B. Tukel, J. de Schutter, and H. Van Brussel, "Optimal robot excitation and identification," *IEEE Transactions on Robotics and Automation*, vol. 13, pp. 730–740, 1997.

[5] D. Kostic, Bram de Jager, M. Steinbuch, and R. Hensen, "Modeling and identification for high-performance robot control: an rrr-robotic arm case study," *IEEE Transactions on Control Systems Technology*, vol. 12, pp. 904–919, 2004.

[6] B. Siciliano and O. Khatib, *Handbook of Robotics*. Berlin, Heidelberg: Springer, 2007.

[7] A. Lomakin and J. Deutscher, "Algebraic fault detection and identification for rigid robots," in *Submitted to International Conference on Robotics and Automation ICRA , Paris, France*, 2019, available at http://arxiv.org/abs/1911.06065.

[8] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, *Robotics: Modelling, Planning and Control*, 3rd ed. Springer, London, 2008.

[9] K. Ayusawa, G. Venture, and Y. Nakamura, "Identifiability and identification of inertial parameters using the underactuated base-link dynamics for legged multibody systems," *The International Journal of Robotics Research*, vol. 33, pp. 446–468, 2013.

[10] G. Szegö, *Orthogonal Polynomials*, 23rd ed. American Mathematical Society, Providence, 1959.

[11] D. G. Luenberger, *Optimization by Vector Space Methods*. John Wiley & Sons, Inc., New York, 1997.

[12] M. Mboup, C. Join, and M. Fliess, "Numerical differentiation with annihilators in noisy environment," *Numerical Algorithms*, vol. 50, pp. 439–467, 2009.

[13] L. Kiltz and J. Rudolph, "Parametrization of algebraic numerical differentiators to achieve desired filter characteristics," in *Proceedings of the 52nd IEEE Conference on Decision and Control, CDC 2013, Firenze, Italy*, 2013, pp. 7010–7015.

[14] N. Wang, J.-C. Sun, M. Er, and Y.-C. Liu, "Hybrid recursive least squares algorithm for online sequential identification using data chunks," *Neurocomputing*, vol. 174, pp. 651–660, 2015.

[15] M. Gautier and W. Khalil, "Exciting trajectories for the identification of base inertial parameters of robots," *The International Journal of Robotics Research*, vol. 11, pp. 362–375, 1992.

[16] M. Gautier and G. Venture, "Identification of standard dynamic parameters of robots with positive definite inertia matrix," in *IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2013, pp. 5815–5820.

[17] C. Gaz, M. Cognetti, A. Oliva, P. Giordano, and A. Luca, "Dynamic identification of the franka emika panda robot with retrieval of feasible parameters using penalty-based optimization," *IEEE Robotics and Automation Letters*, pp. 4147–4154, 2019.