2020 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)
October 25-29, 2020, Las Vegas, NV, USA (Virtual)

Guaranteed Parameter Estimation of Hunt-Crossley Model with Chebyshev Polynomial Approximation for Teleoperation

Daniel Budolak and Alexander Leonessa*

Abstract— **In haptic time delayed teleoperation as the time delay from the communication channel increases, teleoperation system stability and performance degrade. To increase performance and provide better stability margins, various estimation methods and observers have been implemented in literature to more accurately capture the force exerted by the remote system. Previously, solutions focused on environment force estimation methods that primarily rely on linearization of the Hunt-Crossley (HC) contact model, which has limiting assumptions for use. This work addresses the shortcomings of the aforementioned methods by investigating alternative HC parameter estimation techniques. A new application of Chebyshev polynomial approximation for adaptive parameter estimation of the HC model is proposed. This approximation is compared to current linearization methods as well as nonlinear estimation methods that are not well covered in literature. Moreover, the Chebyshev approximation is used in a new estimation approach that provides control via backstepping with adaptive parameter estimation using Lyapunov methods. This method reduces excitation requirements by using nonlinear swapping and the data accumulation concept to guarantee parameter convergence. A simulated full teleoperation system with time delay demonstrates the effectiveness of this approach.**

I. INTRODUCTION

For adequate performance and stability in a teleoperation system, the master (local) side needs sufficient knowledge of the remote environment, particularly for model based controllers. To achieve this, estimation methods can be implemented to obtain the remote side dynamics or update the local side model. Many have implemented observers in the control scheme to accomplish this such as in [1], [2], with variations using adaptive methods [3], or implementation that focuses on remote side impedance matching [4].

Alternatively, it may be more desirable to estimate the environment contact force based on its material properties. The most common and effective way of achieving this is through the use of a contact model, such as the Hunt-Crossley (HC) [5] model. For contact models to be effective, the parameters of the material needs to be known a priori. However, if the material is unknown, or the task changes, there needs to be a method to determine these parameters online such that the model adapts to the material without prior knowledge. Most notably in [6] a new linearization approach for the HC model is implemented that builds off of previous work of the Haddadi method as first introduced in [7]. This work uses a log linearization with an exponentially weighted recursive least squares estimator (EWRLS) that provides promising results. However, to satisfy the linearization assumptions, the velocity of the contact is limited to a threshold value with a minimum deformation depth. A variation of the Haddadi method was studied in [8], where EWRLS was used with a modified Hiddadi linearization using Taylor series expansion. In this approach, the Lankarani and Nikravesh contact model is used instead of HC. Similarly, a variation of a Taylor series approximation of the HC model is also examined in [9]. In this implementation a modified Kalman filter based observer is used for estimation. In [10], a self-perturbing recursive least squares estimation was used to identify the parameters of the HC as well as the linear Kelvin-Voigt model. The estimator switched between the two models given the contact velocity to not violate the linearization of the HC method provided by [7]. However, this work did not incorporate any time delay. Another method in [11] proposed approximating the nonlinear exponential term in the HC model with a quadratic polynomial. This avoids the drawbacks of the Haddadi method, however, the performance is limited to the number of polynomial terms used for estimation.

To avoid limitations of linear approximation, others have pursued nonlinear parameter and state estimation. The work in [12] showed promising results using an EKF for a nonlinear tissue dynamics model. More recent work in [13] used an uncented Kalman filter (UKF) for estimating the contact force of the HC model. These methods show promise in accurate force estimation but there is no discussion on parameter convergence. This limits their use for predictive based controllers. A different approach that maintains the nonlinearities of the HC model was investigated using a neural network (NN) in [14], [15]. However, this requires sufficient training of the NN. The implementation of any other nonlinear estimation technique for the HC model is scarce in the literature.

It must be noted that for all of these estimation techniques, sufficient excitation is required to achieve parameter convergence. Additionally, with the exception of adaptive methods the estimators alone do not provide a control law, and stability is achieved through another control technique. Thus, a more robust solution is desired that addresses both environment estimation and control of the remote system. Moreover, previous studies using online parameter estimation for teleoperation have not fully addressed the problems in its implementation. For instance, although there is large support in the use of the HC model for teleoperation in particular, few have demonstrated a way to estimate the parameters while maintaining the nonlinearities of the HC model. Due to the challenge of nonlinear parameter identification, an alternate

Department of Mechanical Engineering, Virginia Tech, Blacksburg, VA 24061, USA.
* Corresponding author: aleoness@vt.edu

approach is to linearize the HC model to make it suitable for estimation with least squares techniques such as in [6]-[11]. However, linearization assumptions can limit the application of these methods, and naturally have increased error as they do not truly capture the nonlinearities.

To address the shortcomings of the aforementioned methods, this paper provides a new linearization approach using Chebyshev polynomial approximation of the HC model. As a means of providing both trajectory tracking of the remote system and guaranteed parameter convergence, this paper proposes the use of adaptive backstepping with guaranteed parameter estimation using nonlinear swapping and data accumulation. This guaranteed adaptive parameter estimation technique, termed GuAPE, is a novel application and extension of the work first presented in [16] and [17] for a new use in teleoperation. This paper makes the additional contribution of examining additional online nonlinear estimation techniques using gradient descent and Levenberg–Marquardt (LM) algorithms. Both the proposed Chebyshev approximation and nonlnear methods are compared to the most prevalent estimation techniques for the HC model in simulation. The proposed GuAPE method with Chebyshev approximation is then evaluated for a time delayed haptic teleoperation system using a classical Smith predictor in a simulation study.

II. Chebyshev Polynomial Approximation

The Chebyshev polynomials form an orthonormal functional basis and are particularly well suited for approximating polynomials with periodicity. In this case for approximating the exponential term in the HC model with sinusoidal excitation of the environment. Applying the Chebyshev approximation to the exponential term in the HC model gives

$$F_{\text{env}} = K\delta^n + B\delta^n\dot{\delta} \simeq \sum \alpha_i T_i(\bar{x}) + \sum \beta_i T_i(\bar{x})\dot{\delta}, \quad (1)$$

where $\bar{x}$ is a normalization ensuring the psudo penetration δ is on the interval [-1,1] for the Chebyshev polynomials of the first kind $T_i(\bar{x})$, and $i = 0, 1, 2, \ldots, n$. The coefficients α_i and β_i are computed by the projections

$$\alpha_i = N_\delta N_\pi K \int_0^{L_\delta} \frac{\delta(\bar{x})^n T_i(\bar{x})}{\sqrt{1-\bar{x}^2}} d\bar{x},$$
$$\beta_i = N_\delta N_\pi B \int_0^{L_\delta} \frac{\delta(\bar{x})^n T_i(\bar{x})}{\sqrt{1-\bar{x}^2}} d\bar{x},$$

where L_δ is an arbitrarily defined penetration limit for a given application, $N_\delta = 2/L_\delta$ is a scaling for the normalization of the penetration, $\bar{x} = \delta N_\delta - 1$, ($\delta(\bar{x}) = \frac{\bar{x}+1}{N_\delta}$), and N_π is a scaling for the orthogonality property with $N_\pi = 1/\pi$ when $i = 0$ and $N_\pi = 2/\pi$ otherwise.

The Chebyshev approximation can easily be put into a form that is convenient for parameter identification

$$F_{env} = \theta^\top \varphi,$$

with

$$\theta = [\alpha_i, \ldots, \alpha_n, \beta_i, \ldots, \beta_n]^\top,$$
$$\varphi = [T_i(\bar{x}), \ldots, T_n(\bar{x}), T_i(\bar{x})\dot{\delta}, \ldots, T_n(\bar{x})\dot{\delta}]^\top.$$

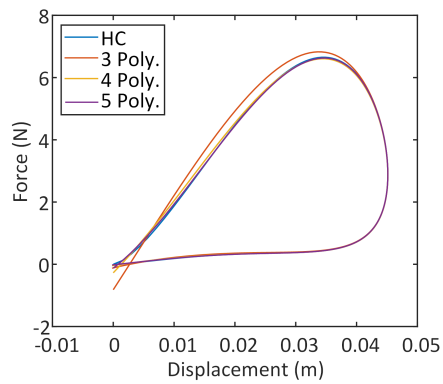

Fig. 1: Comparison of Chebyshev polynomial approximation of the HC contact model using the first 3, 4, and 5 polynomials of the first kind, K = 300, B = 122, n = 1.5.

Naturally, a better fit is achieved with a greater number of terms as depicted in Fig. 1. However, this in turn comes at the cost of requiring a greater excitation and longer convergence times. To alleviate the need of persistent excitation, GuAPE is applied to this approximation as presented in IV.

III. Comparison of Estimation methods

To evaluate the performance of both linear and nonlinear HC estimation methods a simulation study is carried out for a simplified second order remote system in contact with a stationary environment in MATLAB and Simulink. The trajectory used for the contact was $x_{\text{d}} = 0.1 + 0.05\sin(2t)$. Soft material parameter values of K = 300, B = 122, and n = 1.5 were arbitrarily chosen for the simulation. Table I presents the results of the compared estimation methods, providing the estimation error of the contact force and whether convergence was achieved.

The nonlinear estimation methods examined involved output error optimization and Kalman filtering. Specifically the methods studied were gradient descent, Levenberg-Marquardt, EKF, and UKF. For the output error optimization, the quadratic cost function being minimized was

$$r = (h - F)^2,$$

where h is the measured or simulated force, and $F = K\delta^n + B\delta^n\dot{\delta}$ is the modeled force. At each time step an update to the parameter vector $\theta = [K, B, n]^\top$ is calculated.

For the Kalman methods, simultaneous state and parameter estimation was implemented by augmenting the state matrix as $x = [\delta, \dot{\delta}, K, B, n]^{\text{T}}$.
In discrete form the next update is given as

$$x_k = \begin{bmatrix} \delta_{k-1} + \dot{\delta}_{k-1}\Delta t \\ \dot{\delta}_{k-1} \\ K_{k-1} \\ B_{k-1} \\ n_{k-1} \end{bmatrix} + q_k,$$

with the observation equation

$$h_k = \begin{bmatrix} K_{k-1}\delta_{k-1}^{n_{k-1}} + B_{k-1}\delta_{k-1}^{n_{k-1}}\dot{\delta}_{k-1} \\ \delta_{k-1} + \dot{\delta}_{k-1}\Delta t \end{bmatrix} + r_k.$$

No convergence was achieved with the EKF and UKF. The influence of the exponential term particularly proved too sensitive for the method to estimate. Various cases were run for different magnitudes and parameter values as well as permutations of having one, two, or all parameters unknown (n known with K & B unknown vs. K known with n & B unknown etc.). The trials resulted in convergence as long as at least one parameter is known. However, when all three parameters are estimated, the tuning of the covariance matrices proved to be too sensitive to achieve convergence. Although the parameters did not converge, in all cases the environmental force of the HC model was estimated with high accuracy. Perhaps this is why the results of [13] only show the estimated force, and the work in [18] assumes a value for n.

On the other hand, both the gradient descent and LM optimization were able to recover the parameters with varying degrees of error. However, a few modifications to the algorithms had to be made to achieve sufficient results. Using a constant learning rate did not achieve good performance and resulted in small parameter updates that would require very long estimation times. Thus, a varying learning rate α was used that updated at each iteration to optimize α along a line $F(\theta_k + \alpha_k p_k)$, where p_k is the search direction [19]. This results in the learning rate being calculated at each step as

$$\alpha_k = \left.\frac{J^\top J}{J^\top H J}\right|_{\theta_k},$$

where J and H are the Jacobian and Hessian matrices of the cost function respectively. This improved the update step, but due to the different magnitudes of the parameters, a scaling factor had to be applied to the update. Since the gradient descent method is susceptible to converging at a local result, the initial conditions were chosen on the same relative magnitude as the true parameters, θ_0 = [100 100 1]. The number of iterations for the gradient descent was kept constant at one thousand iterations per time step. The LM algorithm is implemented using the built in LSQNONLIN MATLAB function. However, having one data point per time step results in low robustness to noise. To overcome this, a mini-batch approach is used. This is implemented with a tapped delay line of 500 samples for each iteration of the LM algorithm. The parameter convergence and estimation error for both the gradient descent and the mini batch LM is depicted in Fig. 2.

For the linearized HC methods, the proposed Chebyshev polynomial approximation, utilizing both RLS and KF for the estimation, was compared to the Haddadi and Quad Poly linearization. Since the Chebyshev approximation uses a set of polynomials for both $K\delta^n$ and $B\delta^n$, there are more parameters to identify than in the Haddadi and Quad Poly methods. However, since the Chebyshev Polynomial approximation has a convenient algebraic form, it is suitable for estimation with a Kalman filter as well as RLS. This results in faster convergence and less error using the former in comparison to the Quad Poly method. When compared to the Haddadi method, although the proposed approach has slightly longer convergence times, it has the benefit of not being limited by a minimum penetration depth and loading rate with a comparable error in force estimation. This allows for a greater range of application in practical use. The parameter trajectories as well as the resulting error for these methods is shown in Fig. 3.

Ultimately, the mini-batch LM method has the fastest

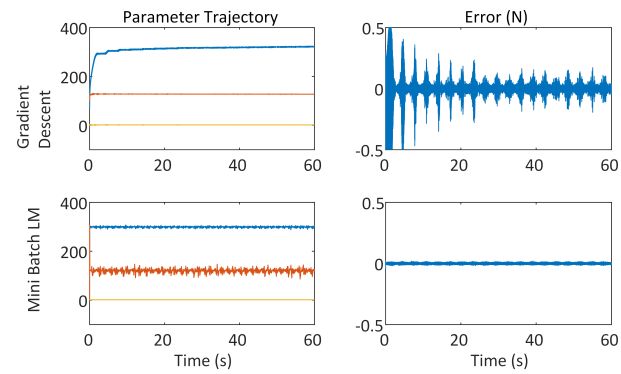

Fig. 2: Comparison of estimation techniques for Nonlinear HC parameter identification.

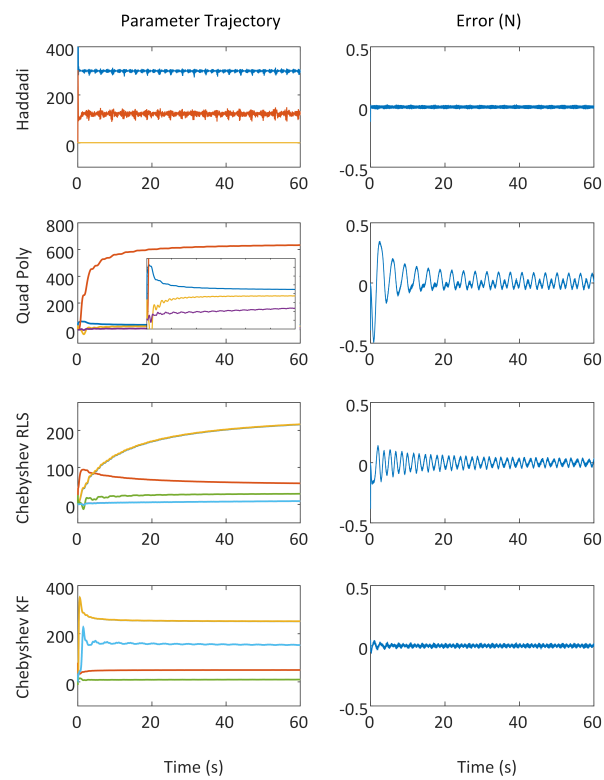

Fig. 3: Comparison of estimation techniques for linearized HC parameter identification. K = 300, B = 122, n = 1.5

TABLE I: Simulation results of linearized and nonlinear HC parameter estimation methods.

Method	Convergence		Force MSE
Haddadi	$K_\mu = 299.231$	$s_K^2 = 1.633$	1.219E-5
	$B_\mu = 121.285$	$s_B^2 = 4.207$	
	$n_\mu = 1.499$,	s_n^2 = 1.350E-6	
Quad Poly	yes		8.064E-3
Chebyshev RLS	yes		1.922E-3
Chebyshev KF	yes		5.820E-5
Gradient Descent	$K_\mu = 312.532$	s_K^2=1.379E-2	1.551E-2
	$B_\mu = 127.644$	s_B^2=2.233E-5	
	$n_\mu = 1.513$,	s_n^2=5.574E-6	
Mini Batch LM	$K_\mu = 299.081$	$s_K^2 = 1.207$	1.104E-5
	$B_\mu = 119.974$	$s_B^2 = 2.365$	
	$n_\mu = 1.499$	s_n^2 =1.011E-6	
EKF	no		2.014E-7
UKF	no		1.758E-5

convergence times and lowest error out of all the estimators used. Since it maintains the nonlinear form of the HC parameters as well, it is the best candidate from a performance standpoint. In comparison, the proposed Chebyshev approximation can be a simplified alternative with Kalman filtering that has similar accuracy and reasonable parameter convergence times, with no limiting assumptions for the linearization. Thus, depending on bandwidth and convergence criteria, either of the proposed Chebyshev approximation or LM method are improved alternative estimation methods for the HC model that can be used in a teleoperation system.

IV. Guaranteed Adaptive Parameter Estimation

Although the estimation methods above provide good parameter convergence and accuracy, they have the drawback of requiring sufficient excitation for parameter convergence. Moreover, they do not address the control of the remote side system. To address these shortcomings of the above estimation techniques, the use of adaptive parameter estimation with nonlinear swapping and data accumulation is proposed as a solution. This method was originally introduced in [17] for state parameter estimation. This work modifies the method to estimate the contact force model parameters influenced by the control instead of state parameters as originally formulated. The proposed method as applied here will be refered to as the guaranteed adaptive parameter estimation (GuAPE) method. The following presents the derivation for applying the adaptive parameter estimation method to a general second order system with an externally applied force.

Given a system of the form

$$\dot{x}_1 = x_2,$$
$$\dot{x}_2 = -a_1 x_1 - a_2 x_2 + b(u - F_\mathrm{e}),$$

with states x_1 and x_2 as the position and velocity, the external force F_e is modeled as the Chebyshev polynomial approximation of the nonlinear HC environment force as defined in equation 1. Nonlinear backstepping is applied to develop a controller for the system to track a reference trajectory with the error defined as

$$z_1 = x_1 - y_r,$$

where $y_r = x_{r1}$ from a second order reference system

$$\dot{x}_r = A_r x_r + B_r r,$$

with $x_r = [x_{r1}\ x_{r2}]^\top$ and A_r Hurwitz. x_2 is used as a virtual control with the stabilizing function chosen as

$$\alpha_1 = -c_1 z_1 + \dot{y}_r.$$

Defining $z_2 = x_2 - \alpha_1$, the error dynamics become

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r = x_2 - \alpha_1 - c_1 z_1 = z_2 - c_1 z_1,$$
$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}_1 = -a_1 x_1 - a_2 x_2 + bu - b\theta^\top \varphi - \dot{\alpha}_1,$$

where $\dot{\alpha}_1 = -c_1 \dot{z}_1 + \ddot{y}_r = -c_1 x_2 + c_1 \dot{y}_r + \ddot{y}_r$ making

$$\dot{z}_2 = -a_1 x_1 - a_2 x_2 + bu - b\theta^\top \varphi + c_1 x_2 - c_1 \dot{y}_r - \ddot{y}_r.$$

Choosing the Lyapunov function as

$$V_1(z_1, z_2) = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2,$$
$$\begin{aligned}\dot{V}_1 &= z_1 \dot{z}_1 + z_2 \dot{z}_2 \\ &= z_1 z_2 - c_1 z_1^2 + z_2(-a_1 x_1 - a_2 x_2 + \\ &\quad bu - b\theta^\top \varphi + c_1 x_2 - c_1 \dot{y}_r - \ddot{y}_r).\end{aligned}$$

To ensure that $\dot{V}$ is negative definite the control input is selected as

$$\begin{aligned}u = \frac{1}{b}(&-z_1 - c_2 z_2 + a_1 x_1 + a_2 x_2 + \\ &b\theta^\top \varphi - c_1 x_2 + c_1 \dot{y}_r + \ddot{y}_r).\end{aligned}$$

Substituting the estimates in the control the error dynamics are now

$$\dot{z}_1 = z_2 - c_1 z_1,$$
$$\dot{z}_2 = -z_1 - c_2 z_2 - b\tilde{\theta}^\top \varphi,$$

with $\tilde{\theta} = \theta - \hat{\theta}$. The parameter update is now obtained from the Lypunov function

$$\begin{aligned}V_2(V_1, \tilde{\theta}) &= \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} \tilde{\theta}^\top \Gamma_\theta^{-1} \tilde{\theta}, \\ \dot{V}_2 &= z_1 \dot{z}_1 + z_2 \dot{z}_2 + \tilde{\theta}^\top \Gamma_\theta^{-1} \dot{\tilde{\theta}} \\ &= -c_1 z_1^2 - c_2 z_2^2 - \tilde{\theta}^\top (z_2 b \varphi + \Gamma_\theta^{-1} \dot{\hat{\theta}}),\end{aligned} \tag{2}$$

with tuning matrix Γ_θ, giving the update law

$$\dot{\hat{\theta}} = -\Gamma_\theta z_2 b \varphi. \tag{3}$$

From here, the nonlinear swapping filters below are applied with the estimator dynamics specifically for this system now defined as

$$\dot{\hat{x}} = Ax + B(u - \varphi^\top \hat{\theta}) - \Lambda_x (X - \hat{X}) + K_x^{-1} W_x \dot{\hat{\theta}},$$
$$\dot{W}_x = \Lambda_x W_x + K_x G,$$

such that

$$\dot{\tilde{\epsilon}} = \Lambda_x \tilde{\epsilon}, \quad \tilde{\epsilon}(0) = e_x(0),$$

where $e_x = x - \hat{x}$, $W_x \in \mathbb{R}^{n \times p}$, $\tilde{\epsilon} \in \mathbb{R}^{n \times 1}$, and the tuning matrices $K_x = \mathrm{diag}(k_{x_1}, \ldots, k_{x_n})$ and $\Lambda_x = \mathrm{diag}(\lambda_{x_1}, \ldots, \lambda_{x_n})$ are positive definite and negative definite

respectively. All filter signals are globally bounded and $W_x\tilde{\theta} = \varsigma_x$ with the relation $\varsigma_x = K_x(e_x - \tilde{\epsilon})$. The coordinate transform $\psi = Q\theta$ is performed with data accumulation such that

$$\dot{Q} = \Lambda_r(Q_r - Q)W_x^\top R_x W_x, \quad Q(0) = 0,$$
$$\dot{\psi} = -\Lambda_r(Q_r - Q)W_x^\top R_x(\varsigma_x + W_x\hat{\theta}), \quad \psi(0) = 0,$$

and

$$\hat{\psi} = Q\hat{\theta},$$

where R_x is positive definite, Λ_r is negative definite, Q_r is a constant full rank reference matrix, and $\tilde{\psi} = \psi - \hat{\psi} = Q\tilde{\theta}$. The parameter update law in equation 3 is now modified to give

$$\dot{\hat{\theta}} = \Gamma_\theta(-z_2 b\varphi + \mu),$$

where the modifier μ is defined through known quantities as

$$\mu = M\tilde{\theta} = W_x^\top \Gamma_x \varsigma_x + Q^\top \Gamma_\psi \tilde{\psi}.$$

The purpose of this modifier is to augment the parameter update such that when Q achieves full rank, M is positive definite. This makes the Lyapunov derivative in equation 2 become

$$\dot{V}_2 = -z^\top C z - \tilde{\theta}^\top M \tilde{\theta}, \tag{4}$$

where C is a positive definite matrix of control gains c_1 & c_2 for $z = [z_1 \; z_2]^\top$. Since $\dot{V}_2$ is negative definite, the estimation error converges to zero according to the LaSalle-Yoshizawa theorem [20].

V. Master Side Model Update

In this study a Smith predictor is used on the master side for the main teleoperation control strategy to deal with the time delay. The block diagram in Fig. 4 shows the described system with the proposed estimation method. When the HC parameters utilized in the Smith predictor are updated from an initial a priori estimate, care must be taken that this transition does not disrupt operation with a jump in the estimated force. To provide a smooth transition and maintain continuity in the signal, the use of a Sigmoid function is proposed. The function is defined as

$$\alpha = 1 - \frac{1}{1 + e^{\gamma(t-\beta)}},$$

where γ and β are a scaling factor and offset, respectively, to change the slope and timing of the parameter transition. Once a new parameter set is converged, α is used to transition from the initial parameters to the new parameters as follows

$$F_1 = \theta_1^\top \varphi,$$
$$F_2 = \theta_2^\top \varphi,$$
$$F = \alpha F_2 + (1-\alpha)F_1,$$

where F is the haptic force being sent to the user. The criteria for when a new parameter set is ready to be used can be heuristically determined based on application. This can be very helpful depending what estimation method is used by allowing an updated force to be sent smoothly

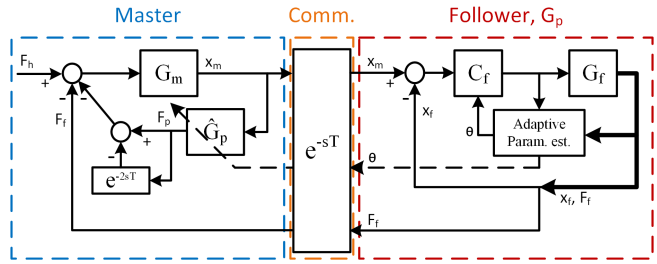

Fig. 4: Block diagram of implemented adaptive parameter estimation method with a Smith predictor.

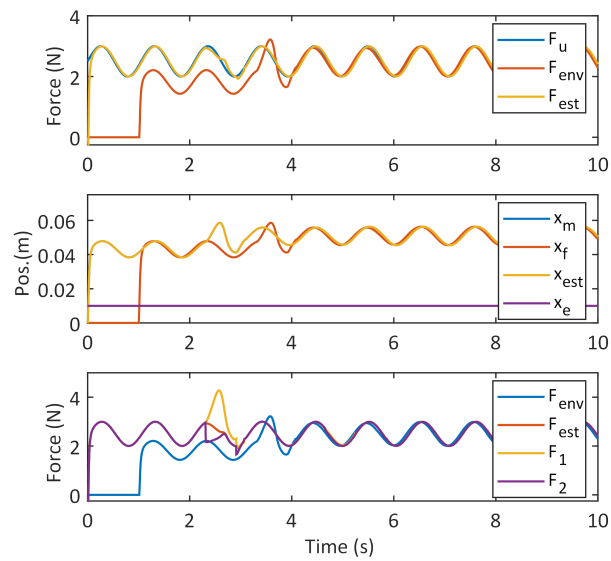

Fig. 5: 2s RTT GuAPE simulation. Subscripts u, m, f, and e refer to the user, master, follower, and contact location.

even while parameters are changing to minimize the delay between contact and updated force rendering. The update criteria used here is when the root mean square error between the estimated and measured force is less then the desired tolerance.

VI. Teleoperation Simulation

The GuAPE approach is applied to a time delayed haptic teleoperation system described in Fig. 4 in simulation using Simulink. The master and remote devices are generalized as identical second order mass spring damper systems, and the derivation in IV is used for the control and update laws. The predictive controller is implemented at an arbitrarily chosen 1k Hz. The contact location is assumed to be known a priori at $x_e = 0.01$m and the user input force is chosen as $F_u = 2.5 + 0.5\sin(6t)$. The tuning parameters for the simulation are $\Gamma_\theta = 10$, $\Gamma_x = R_x = 30I_{[2\times2]}$, $\Gamma_\psi = 250I_{[6\times6]}$, $Q_r = 20I_{[6\times6]}$, $K_x = 2I_{[2\times2]}$, $\Lambda_r = -10I_{[6\times6]}$, and $\Lambda_x = -20I_{[2\times2]}$.

Fig. 5 shows the system performance with the proposed control architecture for a round trip time (RTT) delay of 2 seconds. The adaptive backsteping control law achieves

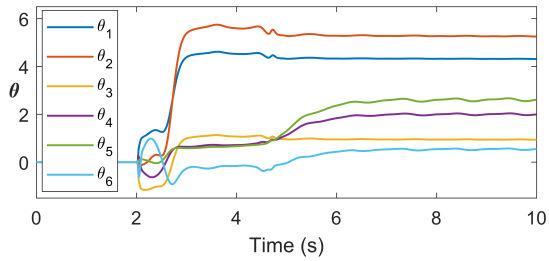

Fig. 6: Parameter Convergence for 2s RTT using GuAPE.

asymptotic position tracking of the master signal. An initial guess for parameters was chosen greater than the true parameters resulting in an overestimation of the environment contact force. This initial guess was corrected at approximately 2.5s when the estimated force error was sufficiently small to send the parameter update. The estimated force transition can be seen in the bottom subplot of Fig. 5. After the parameters adapt sufficiently, the estimated force converges to the environment force. Using the Smith predictor corrected force along with the reflected force results in a larger error, therefore only the initial estimated force of the predictor is used for force feedback.

The true HC parameters were $K = 300$, $B = 122$, and $n = 1.5$. Projecting this onto the Chebyshev polynomial approximation using the first 3 polynomials results in the true parameters $\theta = [4.026, 4.831, 0.690, 1.63738, 1.964, 0.280]$ for $N_\delta = 20$.

The parameter adaptation is shown in Fig. 6. The final values after 10s for the estimates are $\hat{\theta} = [4.315, 5.255, 0.940, 1.999, 2.614, 0.548]$, with a force estimation MSE error of 4.425E-7 N. The difference in the parameter values is in part a result of using only the first 3 polynomials for the estimation as the lower number of terms must be weighted differently to make up for the higher order nonlinearity matching. M was observed to be positive definite for the entire duration of estimation, thus ensuring parameter convergence. Overall, these results show promise for the GuAPE method to be used for estimation of the Chebyshev approximated HC parameters. Of note is the stability and performance of this method in a RTT of 2s. Moreover, the ability for the Smith predictor to smoothly transition between parameter updates to make up for incorrect initial values gives confidence in the robustness of this architecture for use in application.

VII. CONCLUSIONS

This work provided a quantitative analysis comparing linear and nonlinear estimation methods for the HC contact model. Additionally, a new linearization method that uses Chebyshev polynomial approximation was proposed and compared to existing methods. Simulation results show its capability with good convergence time and accuracy while not being limited to the linearization assumptions of current methods. Moreover, an adaptive parameter estimation technique that provides control through backstepping with guaranteed parameter estimation was proposed and evaluated in a full teleoperation system simulation. Results show good tracking of the desired master position with parameter convergence resulting in high accuracy force estimation.

REFERENCES

[1] J. M. Daly and D. W. L. Wang, "Time-Delayed Output Feedback Bilateral Teleoperation With Force Estimation for n-DOF Nonlinear Manipulators," *IEEE Transactions on Control Systems Technology*, vol. 22, pp. 299–306, 1 2014.

[2] L. Chan, F. Naghdy, and D. Stirling, "Position and force tracking for non-linear haptic telemanipulator under varying delays with an improved extended active observer," *Robotics and Autonomous Systems*, vol. 75, pp. 145–160, 1 2016.

[3] Rui Cortesao, Jaeheung Park, and O. Khatib, "Real-time adaptive control for haptic manipulation with active observers," in *Proceedings 2003 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS 2003)*, vol. 3, pp. 2938–2943, IEEE, 2003.

[4] J. Song, Y. Ding, Z. Shang, and J. Liang, "Model-mediated teleoperation with improved stability," *International Journal of Advanced Robotic Systems*, vol. 15, p. 172988141876113, 3 2018.

[5] K. H. Hunt and F. R. E. Crossley, "Coefficient of Restitution Interpreted as Damping in Vibroimpact," *Journal of Applied Mechanics*, vol. 42, pp. 440–445, 6 1975.

[6] A. Haddadi and K. Hashtrudi-Zaad, "Real-Time Identification of Hunt–Crossley Dynamic Models of Contact Environments," *IEEE Transactions on Robotics*, vol. 28, pp. 555–566, 6 2012.

[7] A. Haddadi and K. Hashtrudi-Zaad, "A New Method for Online Parameter Estimation of Hunt-Crossley Environment Dynamic Models," in *2008 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 981–986, IEEE, 9 2008.

[8] J. Ma, L. Qian, and G. Chen, "Parameter estimation of the Lankarani-Nikravesh contact force model using a new modified linear method," in *2015 IEEE International Conference on Advanced Intelligent Mechatronics (AIM)*, vol. 2015-Augus, pp. 494–499, IEEE, 7 2015.

[9] A. Pappalardo, A. Albakri, C. Liu, L. Bascetta, E. De Momi, and P. Poignet, "Hunt–Crossley model based force control for minimally invasive robotic surgery," *Biomedical Signal Processing and Control*, vol. 29, pp. 31–43, 8 2016.

[10] A. Achhammer, C. Weber, A. Peer, and M. Buss, "Improvement of model-mediated teleoperation using a new hybrid environment estimation technique," in *2010 IEEE International Conference on Robotics and Automation*, pp. 5358–5363, IEEE, 5 2010.

[11] R. Schindeler and K. Hashtrudi-Zaad, "Online Identification of Environment Hunt–Crossley Models Using Polynomial Linearization," *IEEE Transactions on Robotics*, vol. 34, pp. 447–458, 4 2018.

[12] A. Sie, M. Winek, and T. M. Kowalewski, "Online identification of abdominal tissues in vivo for tissue-aware and injury-avoiding surgical robots," in *2014 IEEE/RSJ International Conference on Intelligent Robots and Systems*, pp. 2036–2042, IEEE, 9 2014.

[13] J. Shin, Y. Zhong, D. Oetomo, and C. Gu, "Random Weighting, Strong Tracking, and Unscented Kalman Filter for Soft Tissue Characterization," *Sensors*, vol. 18, p. 1650, 5 2018.

[14] A. C. Smith and K. Hashtrudi-Zaad, "Smith Predictor Type Control Architectures for Time Delayed Teleoperation," *The International Journal of Robotics Research*, vol. 25, pp. 797–818, 8 2006.

[15] H. J. Choi and S. Jung, "Neural network-based Smith predictor design for the time-delay in a tele-operated control system," *Artificial Life and Robotics*, vol. 14, pp. 578–583, 12 2009.

[16] M. Krstic and P. Kokotovic, "Adaptive nonlinear design with controller-identifier separation and swapping," *IEEE Transactions on Automatic Control*, vol. 40, pp. 426–440, 3 1995.

[17] R. Mishkov and S. Darmonski, "NONLINEAR ADAPTIVE CONTROL AND EXACT PARAMETER ESTIMATION," in *International Conference AUTOMATICS AND INFORMATICS'2013 3-7 October 2013, Sofia, Bulgaria*, no. October, pp. 285–288, 2013.

[18] J. Kim and J. Kim, "Viscoelastic Characterization of Mouse Zona Pellucida," *IEEE Transactions on Biomedical Engineering*, vol. 60, pp. 569–575, 2 2013.

[19] M. T. Hagan, H. B. Demuth, M. H. Beale, and O. De Jesús, *Neural Network Design 2nd Edition*. Martin Hagan, 2014, 2 ed., 2014.

[20] M. Krstic, P. V. Kokotovic, and I. Kanellakopoulos, *Nonlinear and Adaptive Control Design*. John Wiley & Sons, Inc., 1995.